ISSN 2231-346X (Print)
JUSPS-A Vol. 29(2), 58-65 (2017). Periodicity-Monthly
Section A
(Online) ISSN 2319-8044

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org
Estd. 1989

Nano Quotient Mappings

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http://dx.doi.org/10.22147/jusps-A/290202

Acceptance Date 24th Dec., 2016, Online Publication Date 2nd Jan., 2017

Abstract

The purpose of this study is to introduce, define and study several classes of nano quotient map in nano topological spaces. We have initiated the several types of mappings such as nano α-quotient map, nano strongly α-quotient map and nano α^*-quotient map in nano topological space and its properties are discussed. Also we have made comparisions among them.

Key words : Nano topology, nano quotient map, nano α-open, nano α-irresolute, nano α^*-quotient map.
2010 Ams Subject Classification: 54B05,54CO8

1 Introduction

Lellis Thivagar *et al.*[4] introduced nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. It has achieved a large amount of applications in various fields. Equivalence relation is the building block for the nano approximations. It is named as nano topology, because of its size. Whatever may be the size of the universe, it has atmost five nano open sets. Lellis Thivagar[2] defined various forms of quotient mappings in topological spaces. Many authors assorted several forms of quotient mappings in terms of various forms of open sets in topological spaces. This paper initiates the concept of nano quotient map, nano α-quotient map, nano strongly α-quotient map and nano α^*-quotient map in nano topological spaces and its properties are studied. Some examples are also given to illustrate the results.

2 Preliminaries :

The following recalls requisite ideas and preliminaries necessitated in the sequel of our work.

Definition 2.1 [4]: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe R be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with

one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower appproximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,

$L_R(X) = \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$U_R(X) = \bigcup_{X \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by

$B_R(X) = U_R(X) - L_R(X)$

Definition 2.2[4]: Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$ and $\tau_R(X)$ satisfies the following axioms.

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$

(ii) The union of elements of any subcollection $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. That is $\tau_R(X)$ forms a topology $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. A set A is said to be nano closed if its complement is nano open.

Definition 2.3[4] : If $(\mathcal{U}, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathcal{U}$ and if $A \subseteq \mathcal{U}$, then nano interior of A is defined as the union of all nano open sets contained in A and its denoted by $\mathcal{N}Int(A)$. That is $\mathcal{N}Int(A)$ is the largest nano open subsets contained in **A.**

The nano closure of A is defined as the intersection of all nano closed sets containig A and its denoted by $\mathcal{N}Cl(A)$. That is $\mathcal{N}Cl(A)$ is the smallest closed set containing **A.**

Definition 2.4[4] : Let $(\mathcal{U}, \tau_R(X))$ be a nano topological spaces and $A \subseteq \mathcal{U}$ then A is said to be

1. nano semi-open if $A \subseteq \mathcal{N}cl(\mathcal{N}Int(A))$.
2. nano α -open if $A \subseteq \mathcal{N}Int(\mathcal{N}cl(\mathcal{N}Int(A)))$.
3. nano pre-open if $A \subseteq \mathcal{N}Int(\mathcal{N}cl(A))$.

Definition 2.5[5] : Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ be a nano topological spaces, then the mapping $f : (\mathcal{U}, \tau_R(X)) \to (\mathcal{V}, \tau_R{}'(Y)$ is called

1. nano α -continuous (resp. semi-continuous, pre-continuous) if the inverse image of each nano open set in $\mathcal{V}$ is an nano α -open(resp. nano semi-open set, nano pre-open set) in $\mathcal{U}$.
2. nano α -open mapping (resp. nano semi-open mapping, nano pre-open mapping) if the image of each nano open set in $\mathcal{U}$ is an nano α -open set (resp. nano semi-open set, nano pre-open set) in $\mathcal{V}$.
3. nano α -irresolute (resp. nano semi-irresolute, nano pre-irresolute) if the inverse image of every nano α -open set (resp. nano semi-open, nano pre-open set) in $\mathcal{V}$ is an nano α -open set (resp. nano semi-open set, nano pre-open set) in $\mathcal{U}$.

Throughout this paper, $\mathcal{U}$ and $\mathcal{V}$ are non empty finite universes, $X \subseteq \mathcal{U}$ and $Y \subseteq \mathcal{V}$ and where R and R are equivalence relations on $\mathcal{U}$ and $\mathcal{V}$ respectively. $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ are the nano topological space with

respect to X and Y respectively.

3 Nano α -quotient mappings :

In this section we define some notions of nano quotient mappings and their properties were discussed.

Definition 3.1 : Let $f:\mathcal{U}\to\mathcal{V}$ be a surjective map. Then f is said to be nano-quotient map: if f is nano continuous and $f^{-1}(V)$ is nano open in $\mathcal{U}$ implies V is an nano open set in $\mathcal{V}$.

Example 3.2 : Let $\mathcal{U}=\{a,b,c,d,e\}$ with $\mathcal{U}/R=\{\{a\},\{b,c\},\{d,e\}\}$. Let $X=\{a,d\}\subseteq\mathcal{U}$, then $\tau_R(X)=\{\mathcal{U},\varnothing,\{a\},\{a,d,e\},\{d,e\}\}$. $\tau_R^{\alpha}(X)=\{\mathcal{U},\varnothing,\{a\},\{a,d,e\},\{d,e\},\{a,b,d,e\},\{a,c,d,e\}\}$. $\mathcal{V}=\{x,y,z,u,v\}$ with $\mathcal{V}/R'=\{\{x\},\{y,z\},\{u,v\}\}$. Let $Y=\{x,y\}\subseteq\mathcal{V}$, then $\tau_{R'}(Y)=\{\mathcal{V},\varnothing,\{x\},\{x,y,z\},\{y,z\}\}$. $\tau_{R'}^{\alpha}(Y)=\{\mathcal{V},\varnothing,\{x\},\{x,y,z\},\{y,z\},\{x,y,z,v\},\{x,y,z,u\}\}$. Define $f:\mathcal{U}\to\mathcal{V}$ as $f(a)=x$, $f(b)=u$, $f(c)=v$, $f(d)=y$, $f(e)=z$. Then f is clearly a nano quotient map.

Definition 3.3 : The function $f:\mathcal{U}\to\mathcal{V}$ is called nano quasi α -open if the image of every nano α -open set in $\mathcal{U}$ is nano open in $\mathcal{V}$.

Example 3.4 : Let $\mathcal{U}=\{a,b,c,d\}$ with $\mathcal{U}/R=\{\{a\},\{b,c,d\}\}$. Let $X=\{a,b\}\subseteq\mathcal{U}$, then $\tau_R(X)=\{\mathcal{U},\varnothing,\{a\},\{b,c,d\}\}$. $\tau_R^{\alpha}(X)=\{\mathcal{U},\varnothing,\{a\},\{b,c,d\}\}$. $\mathcal{V}=\{x,y,z,u\}$ $\mathcal{V}/R'=\{\{y\},\{x,z,u\}\}$. Let $Y=\{y,u\}\subseteq\mathcal{V}$, then $\tau_{R'}(Y)=\{\mathcal{V},\varnothing,\{y\},\{x,z,u\}\}$. $\tau_{R'}^{\alpha}(Y)=\{\mathcal{V},\varnothing,\{y\},\{x,z,u\}\}$. Define $f:\mathcal{U}\to\mathcal{V}$ as $f(a)=y$, $f(b)=x$, $f(c)=z$, $f(d)=u$. Then $f\{a\}=\{y\}$, $f\{b,c,d\}=\{x,z,u\}$. Thus image of nano α -open set in $\mathcal{U}$ is nano open in $\mathcal{V}$, hence f is a nano quasi-α -open.

Definition 3.5 : Let $f:\mathcal{U}\to\mathcal{V}$ be a surjective map. Then f is said to be

1. nano α -quotient map: if f is nano α -continuous and $f^{-1}(V)$ is nano open in $\mathcal{U}$ implies V is an nano α -open set in $\mathcal{V}$
2. nano semi-quotient map: if f is nano semi-continuous and $f^{-1}(V)$ is nano open in $\mathcal{U}$ implies V is nano semi-open in $\mathcal{V}$
3. nano pre-quotient map: if f is nano pre-continuous and $f^{-1}(V)$ is nano open in $\mathcal{U}$ implies V is nano pre-open in $\mathcal{V}$

Example 3.6 : Let $\mathcal{U}=\{a,b,c,d,e\}$, with $\mathcal{U}/R=\{\{a,c\},\{b\},\{d,e\}\}$. Let $X=\{a,d,e\}\subseteq\mathcal{U}$, then $\tau_R(X)=\{\mathcal{U},\varnothing,\{d,e\},\{a,c,d,e\},\{a,c\}\}$, and also $\tau_R^{\alpha}(X)=\{\mathcal{U},\varnothing,\{d,e\},\{a,c,d,e\},\{a,c\}\}$. Let $\mathcal{V}=\{x,y,z,w\}$, with $\mathcal{V}/R'=\{\{x\},\{y,z\},\{w\}\}$ and Y= $\{x,z\}\subseteq\mathcal{V}$, then $\tau_R'(Y)=\{\mathcal{V},\varnothing,\{x\},\{x,y,z\},\{y,z\}\}$ $\tau_{R'}{}^{\alpha}(y)=\{\mathcal{V},\varnothing,\{x\},\{x,y,z\},\{y,z\}\}$. Now define $f:U\to V$ as $f(a)=x$, $f(b)=w$, $f(c)=x$, $f(d)=z$, $f(e)=y$. Hence $f^{-1}\{x\}=\{a,c\}$, $f^{-1}\{x,y,z\}=\{a,c,d,e\}$, $f^{-1}\{y,z\}=\{d,e\}$, $f^{-1}\{\mathcal{V}\}=\{\mathcal{U}\}$, $f^{-1}\{\varnothing\}=\{\varnothing\}$. Hence clearly, f is an nano-α quotient map.

Theorem 3.7 : If $f:\mathcal{U}\to\mathcal{V}$ is surjective nano α -continuous and nano $\alpha-$ open, then f is an nano α -quotient map.

Proof : Let $f^{-1}(V)$ be nano open in $\mathcal{U}$. Then $f(f^{-1}(V))$ is an nano α -open set. since f is an nano α -open set. Hence V is an nano α -open set, as f is surjective, $f(f^{-1}(V))=V$. Thus f is an nano α -quotient map.

Theorem 3.8 : If $f:\mathcal{U}\to\mathcal{V}$ be an nano open surjective nano α -irresolute and g : $\mathcal{V}\to\mathcal{W}$ be an nano α -quotient map. Then gof is an nano α -quotient map.

Proof : Let V be any nano open set in $\mathcal{W}$. Then $g^{-1}(V)$ is an nano α -open, since g is an nano α -quotient map. And also since f is nano α -irresolute, $f^{-1}(g^{-1}(v))$ is an nano α -open set. Hence $(gof)^{-1}(V)$ is an nano α -open set implies gof is an nano α -open set. Hence gof is an nano α -continuous. Also, assume that $(gof)^{-1}(V)$ be nano open in $\mathcal{U}$ for V$\subseteq\mathcal{W}$, that is $f^{-1}(g^{-1})(V))$ is nano open in $\mathcal{U}$. since $f:\mathcal{U}\to\mathcal{V}$ is nano open, $f(f^{-1}g^{-1}(V))$ is nano open in $\mathcal{V}$. It follows that $g^{-1}(V)$ is nano open in $\mathcal{V}$ f is surjective. since g is an nano α -quotient map, V is an nano α -quotient map, V is an nano α -open set. Thus gof is an nano α -quotient map.

Remark 3.9 : The following example reveals the above theorem.

Example 3.10 : Let $\mathcal{U}=\{a,b,c,d,e\}$ with $\mathcal{U}/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$. Let $X=\{a,d,e\}\subseteq\mathcal{U}$, then $\tau_R(X)=\{\mathcal{U},\varnothing,\{d,e\},\{a,c,d,e\},\{a,c\}\}$. $\mathcal{V}=\{x,y,z,w\}$ with $\mathcal{V}/R'=\{\{x\},\{y,z\},\{w\}\}$. Let $Y=\{x,y,z\}\subseteq\mathcal{V}$. then $\tau_R'(Y)=\{\mathcal{V},\varnothing,\{x\},\{x,y,z\},\{y,z\}\}$. Now define $f:\mathcal{U}\to\mathcal{V}$ as $f(a)=x$, $f(b)=w$, $f(c)=x$, $f(d)=$ z, $f(e)=y$. Hence $f^{-1}\{x\}=\{a,c\}$, $f^{-1}\{x,y,z\}=\{a,c,d,e\}$, $f^{-1}\{y,z\}=\{d,e\}$, $f^{-1}\{\mathcal{V}\}=\{\mathcal{U}\}$, $f^{-1}\{\varnothing\}=\{\varnothing\}$. Hence clearly f is nano open, surjective and nano α -irresolute. $\mathcal{W}=\{p,q,r,s\}$ with $\mathcal{W}/R''=\{\{p,q\},\{r\},\{s\}\}$. Let $Z=\{p,s\}\subseteq\mathcal{W}$, then $\tau_{R''}(Z)=\{\mathcal{W},\varnothing,\{s\},\{p,q,s\},\{p,q\}\}$. Now define g: $\mathcal{V}\to\mathcal{W}$ as $g(x)=s, g(y)=p, g(z)=q$, $g(w)=r$. Hence $g^{-1}\{s\}=\{x\}$, $g^{-1}\{p,q,s\}=\{x,y,z\}$, $g^{-1}\{p,q\}=\{y,z\}$, where g is a nano α -quotient map. Then $(gof)^{-1}\{s\}=f^{-1}g^{-1}(s)=f^{-1}\{x\}=\{a,c\}$. $(gof)^{-1}\{p,q,s\}=f^{-1}g^{-1}(p,q,s)=f^{-1}\{x,y,z\}=\{a,c,d,e\}$. $(gof)^{-1}\{p,q\}=f^{-1}g^{-1}(p,q)=f^{-1}\{y,z\}=\{d,e\}$. Thus gof is an nano α -quotient map.

Theorem 3.11: The function $f:\mathcal{U}\to\mathcal{V}$ is an nano α -quotient iff it is a nano semi-quotient map and a nano pre-quotient map.

Proof : Let f be an nano α -quotient map. To prove that f is nano semi-quotient map. Since f is a nano α -quotient map, $f^{-1}(V)$ is an nano α -open set, hence it is nano semi-open and nano pre-open in $\mathcal{U}$. That is, V is any nano open set in $\mathcal{V}$ implies $f^{-1}(V)$ is nano semi-open in $\mathcal{U}$. Hence f is nano semi-continuous. Let $f^{-1}(V)$ be an nano open set in $\mathcal{U}$. Since f is a nano α -quotient map, V is an nano α -open set in $\mathcal{V}$, which is nano semi-open and nano pre-open in $\mathcal{V}$, that is, $f^{-1}(V)$ is nano-open in $\mathcal{U}$ implies V is nano semi-open in $\mathcal{V}$. Hence f is nano semi-quotient map. Similarly we can prove that f is a nano pre-quotient map. Conversely, let f be a nano semi-quotient map and a pre-quotient map. Let V be any nano open set in $\mathcal{V}$. Since f is both a nano semi-quotient and a nano pre-quotient map, $f^{-1}(V)$ is both nano semi-open and nano pre-open in $\mathcal{U}$, so that $f^{-1}(V)$ is nano α -open set. Hence f is nano α -continuous. Since f is a nano semi-quotient map and a pre-quotient map, V is nano semi-open and nano pre-open in $\mathcal{V}$ so that V is nano α -open in $\mathcal{V}$. Thus f is an nano α -quotient map.

Example 3.12: Let $\mathcal{U}=\{a,b,c,d\}$ with $\mathcal{U}/R=\{\{a\},\{b,c\},\{d\},\}$. Let $X=\{a,d\}\subseteq\mathcal{U}$, then $\tau_R(X)=\{\mathcal{U},\varnothing,\{a,d\}\}$. $\tau_R^{\alpha}(X)=\{\mathcal{U},\varnothing,\{a,d\},\{a,b,d\},\{a,c,d\}\}$. $\tau_R^{SO}(X)=\{\mathcal{U},\varnothing,\{a,d\},\{a,b,d\},\{a,c,d\}\}$. $\tau_R^{PO}(X)=\{\mathcal{V},\varnothing,\{a\},\{d\},\{a,b\},\{a,d\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$. Let $\mathcal{V}=\{x,y,z,w\}$

with $\mathcal{V}/R' = \{\{x,z\},\{y,w\}\}$. Let $Y = \{x,z\} \subseteq \mathcal{U}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x,z\}\}$. $\tau^{\alpha}_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x,z\}, \{x,y,z\}, \{x,z,w\}\}$. $\tau^{SO}_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x,z\}, \{x,y,z\}, \{x,z,w\}\}$.

$\tau^{PO}_{R'}(Y) = \{\mathcal{V} \varnothing, \{x\}, \{z\}, \{x,y\}, \{x,z\}, \{x,w\}, \{y,z\}, \{z,w\}, \{x,y,z\}, \{x,y,w\}, \{x,z,w\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = x, f(b) = y, f(c) = w, f(d) = z$. Then $f^{-1}\{x,z\} = \{a,d\}$, which are nano α-open, nano semi-open and nano pre-open. Therefore f is nano α-quotient, nano semi-quotient and nano pre-quotient.

4. Nano strongly α-quotient Mappings :

Here we introduce the concept of some sorts of nano strong quotient maps and its properties were characterised.

Definition 4.1: Let $f : \mathcal{U} \to \mathcal{V}$ be a on-to map. Then f is called nano strongly α-quotient (resp. strongly semi-quotient, strongly pre-quotient) map provided a set U of $\mathcal{V}$ is nano-open in $\mathcal{V}$ if and only if $f^{-1}(U)$ is nano α-open (resp. nano semi-open set, nano pre-open set) in $\mathcal{U}$.

Example 4.2 : Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a,b\},\{c,d\},\{e\},\}$. Let $X = \{a,c,d\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{c,d\}, \{a,b,c,d\}, \{a,b\}\}$. $\tau^{\alpha}_R(X) = \{\mathcal{U}, \varnothing, \{c,d\}, \{a,b,c,d\}, \{a,b\}\}$. Let $\mathcal{V} = \{p,q,r,s\}$ with $\mathcal{V}/R' = \{\{p\},\{q\},\{r,s\},\}$. Let Y = $\{p,r\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}\}$. $\tau^{\alpha}_{R'}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = r, f(b) = s, f(c) = p = f(d), f(e) = t$. clearly f is nano strongly α-quotient map.

Theorem 4.3 : Every nano strongly α-quotient map is a nano α-quotient map

Proof : Let V be an nano open set in $\mathcal{V}$. Since f is nano strongly α-quotient, $f^{-1}(V)$ is an nano α-open set in $\mathcal{U}$. Let $f^{-1}(V)$ be nano open in $\mathcal{U}$, then $f^{-1}(V)$ is an nano α-set in $\mathcal{U}$. Hence f is an nano α-quotient map.

Remark 4.4 : Converse of the above theorem is false.

Example 4.5 : Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a\},\{b,d\},\{c,e\},\}$. Let $X = \{a,e\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{a\}, \{a,c,e\}, \{c,e\}\}$. $\tau^{\alpha}_R(X) = \{\mathcal{U}, \varnothing, \{a\}, \{a,c,e\}, \{c,e\}, \{a,b,c,e\}, \{a,c,d,e\}\}$. $\mathcal{V} = \{x,y,z,u,v\}$ with $\mathcal{V}/R' = \{\{x\},\{y,v\},\{z,u\}\}$. Let $Y = \{x,y\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x\}, \{x,y,v\}, \{y,v\}\}$. $\tau^{\alpha}_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x\}, \{x,y,v\}, \{y,v\}, \{x,y,z,v\}, \{x,y,u,v\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = x, f(b) = u, f(c) = y, f(d) = z, f(e) = v$. Clearly f is nano α-quotient map but not nano strongly α-quotient since $f^{-1}\{x,y,z,v\} = \{a,c,d,e\}$ is an nano α-open set in $\mathcal{U}$, but $\{x,y,z,v\} \notin \tau_{R'}(Y)$.

Theorem 4.6 : A function $f : \mathcal{U} \to \mathcal{V}$ is nano strongly semi-quotient and nano strongly pre-quotient, then f is nano strongly α-quotient map.

Proof : Let V be an nano open set in $\mathcal{V}$. Since f is nano strongly semi-quotient and nano strongly pre-quotient, $f^{-1}(V)$ is nano semi-open as well as nano pre-open, that is, $f^{-1}(V) \in \mathcal{N}so(X) \cup \mathcal{N}po(X)$. Hence $f^{-1}(V) \in \tau^{\alpha}_R(X) = \mathcal{N}so(X) \cup \mathcal{N}po(X)$, so that $f^{-1}(V) \in \mathcal{N}so(X)$. Since f is nano strongly semi-

quotient map, V is nano open in $\mathcal{V}$. Similarly, f is nano strongly pre-quotient map, V is nano open in $\mathcal{V}$. Hence V is nano open in $\mathcal{V}$ iff $f^{-1}(V) \in \mathcal{N}so(X) \cup \mathcal{N}|po(X) = \tau_R^{\alpha}(X)$. Hence f is nano strongly α-map.

Remark 4.7 : Converse of the above theorem is not true, which can be shown by the following example.

Example 4.8: Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a\},\{b,c\},\{d,e\}\}$. Let $X = \{a,d\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{a\}, \{a,d,e\}, \{d,e\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U}, \varnothing, \{a\}, \{a,d,e\}, \{d,e\}, \{a,b,d,e\}, \{a,c,d,e\}\}$. $\tau_R^{so}(X) = \{\mathcal{U}\ \varnothing, \{a\}, \{a,d,e\}, \{d,e\}, \{a,b,d,e\}, \{a,c,d,e\}, \{a,b,c\}, \{b,c,d,e\}\}$. $\mathcal{V} = \{p,q,r,s\}$ with $\mathcal{V}/R' = \{\{p\},\{q,s\},\{r\}\}$. Let $Y = \{p,q\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{U}, \varnothing, \{p\}, \{p,q,s\}, \{q,s\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,q,s\}, \{q,s\}\}$. $\tau_{R'}^{so}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r\}, \{p,q,s\}, \{q,r,s\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = p$, $f(b) = f(c) = r$, $f(d) = q$, $f(e) = s$. Then f is nano strongly α-quotient map, but not nano strongly semi quotient map. Since $f^{-1}\{p,r\} = \{a,b,c\} \in \tau_R^{so}(X)$, but $\{p,r\} \notin \tau_{R'}(Y)$.

5. Nano α^*-quotient mappings :

In this section we have discussed about nano α^*-quotient mapping in nano topological spaces.

Definition 5.1 : Let $f : \mathcal{U} \to \mathcal{V}$ be a onto map. Then f is called nano α^*-quotient (resp. nano semi *-quotient, nano pre*-quotient) map if f is nano α-irresolute (resp. nano semi-irresolute, nano pre-irresolute) and $f^{-1}(U)$ is nano α-open (resp. nano semi-open, nano pre-open) in $\mathcal{U}$ implies U is nano-open in $\mathcal{V}$.

Example 5.2 : Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a,b\},\{c,e\},\{d\}\}$. Let $X = \{a,d\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{d\}, \{a,b,d\}, \{a,b\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U}, \varnothing, \{d\}, \{a,b,d\}, \{a,b\}, \{a,b,c,d\}, \{a,b,d,e\}\}$. $\mathcal{V} = \{p,q,r,s\}$ with $\mathcal{V}/R' = \{\{p\},\{q\},\{r,s\}\}$. Let $Y = \{p,r\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = r$, $f(b) = s$, $f(d) = p$, $f(c) = f(e) = q$. clearly f is nano α-irresolute. And $f^{-1}\{p\} = \{d\} \in \tau_R^{\alpha}(X)$ and $\{p\} \in \tau_{R'}(Y)$. $f^{-1}\{p,r,s\} = \{a,b,d\} \in \tau_R^{\alpha}(X)$ and $\{p,r,s\} \in \tau_{R'}(Y)$. $f^{-1}\{r,s\} = \{a,b\} \in \tau_R^{\alpha}(X)$ and $\{r,s\} \in \tau_{R'}(Y)$. Hence f is nano α^*-quotient map.

Remark 5.3 : It is sufficiently important that a nano α-irresolute function need not be a nano α^*-quotient map. This can be shown by the following example.

Example 5.4 : Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a,d\},\{c,e\},\{b\}\}$. Let $X = \{a,b\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{b\}, \{a,b,d\}, \{a,d\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U}\ \varnothing, \{b\}, \{a,b,d\}, \{a,b\}, \{a,b,c,d\}, \{a,b,d,e\}\}$. $\mathcal{V} = \{x,y,z,w\}$ with $\mathcal{V}/R' = \{\{x,z\},\{y,w\}\}$. Let $Y = \{x,z\} \subseteq \mathcal{U}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{x,z\}, \{x,y,z\}, \{x,z,w\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{V}, \varnothing, \{x,z\}, \{x,y,z\}, \{x,z,w\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = f(d) = z$, $f(b) = x$, $f(c) = w$, $f(e) = y$. Clearly f is nano irresolute but not nano α^*-quotient map, since $f^{-1}\{x,y,z\} = \{a,b,d,e\} \in \tau_R^{\alpha}(X)$ and $\{x,y,z\} \notin \tau_{R'}(Y)$.

Definition 5.5 : A function $f : \mathcal{U} \to \mathcal{V}$ is called nano strongly α-open map if the image of every nano α-open set in $\mathcal{U}$ is a nano α-open set in $\mathcal{V}$.

Example 5.6 : Let $\mathcal{U} = \{a,b,c,d,e\}$ with $\mathcal{U}/R = \{\{a,b\},\{c,d\},\{e\}\}$. Let $X = \{a,c,d\} \subseteq \mathcal{U}$, then

$\tau_R(X) = \{\mathcal{U}, \varnothing, \{c,d\}, \{a,b,c,d\}, \{a,b\}\}$. $\tau_R^{\alpha}(Y) = \{\mathcal{U}, \varnothing, \{c,d\}, \{a,b,c,d\}, \{a,b\}\}$. $\mathcal{V} = \{p,q,r,s,t\}$ with $\mathcal{V}/R' = \{\{p\}, \{q\}, \{r,s\}, \{t\}\}$. Let $Y = \{p,r\} \subseteq \mathcal{U}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}, \{p,q,r,s\}, \{p,r,s,t\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{V}, \varnothing, \{p\}, \{p,r,s\}, \{r,s\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = r$, $f(b) = s$, $f(c) = p$, $f(d) = p$, $f(e) = t$. Then the image of every nano α-open set in $\mathcal{U}$ is nano α-open set in $\mathcal{V}$ and hence f is nano strongly α-open map.

Theorem 5.7 : Let $f : \mathcal{U} \to \mathcal{V}$ be a surjective nano strongly α-open map and nano α-irresolute map and $g : \mathcal{V} \to \mathcal{W}$ be a nano α^*-quotient map. Then gof is an nano α^*-quotient map.

Proof : Let V be a nano α-open set in $\mathcal{W}$. Then $g^{-1}(V)$ is an nano α-open set in $\mathcal{V}$. Since f is nano α-irresolute, $f^{-1}(g^{-1}(V))$ is an nano α-open set in $\mathcal{U}$ implies gof is nano α-irresolute. Suppose $(gof)^{-1}(V)$ is an nano α-open set in $\mathcal{U}$ for $V \subseteq \mathcal{W}$, that is, $f^{-1}(g^{-1}(V))$ is an nano α-open in $\mathcal{U}$. Since f is nano strongly α-open, $f(f^{-1}(g^{-1}(V)))$ is an nano α-open set in $\mathcal{V}$, and since f is surjective, $g^{-1}(V)$ is an nano α-open set in $\mathcal{V}$. Since g is a nano α^*-quotient map, V is a nano open set in $\mathcal{W}$. Thus gof is an nano α^*-quotient map.

Theorem 5.8 : If $f : \mathcal{U} \to \mathcal{V}$ is nano semi*-quotient map and nano pre*-quotient map and then f is nano α^*-quotient map.

Proof : Let V be a nano α-open set in $\mathcal{V}$. Since f is nano semi*-quotient map and nano pre*-quotient map, $f^{-1}(V)$ is nano semi-open and nano pre-open in $\mathcal{U}$, so $f^{-1}(V)$ is also nano α-open in $\mathcal{U}$. Hence f is nano α-irresolute. Let $f^{-1}(V)$ be an nano α-open set in $\mathcal{U}$, since f is nano semi*-quotient map and nano pre*-quotient map, V is an nano open set in $\mathcal{V}$. Hence f is a nano α^*-quotient map.

Remark 5.9 : The converse of above theorem need not be true which can be explained by the following example.

Example 5.10 : Let $\mathcal{U} = \{a,b,c,d\}$ with $\mathcal{U}/R = \{\{a,b\}, \{b\}, \{c\}\}$. Let $X = \{a,c\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \varnothing, \{c\}, \{a,c,d\}, \{a,d\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U}, \varnothing, \{c\}, \{a,c,d\}, \{a,d\}\}$, $\tau_R^{so}(X) = \{\mathcal{U}, \varnothing, \{c\}, \{a,c,d\}, \{a,d\}, \{b,c\}, \{a,b,d\}\}$ $\tau_R^{po}(X) = \{\mathcal{U}, \varnothing, \{a\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{c,d\}, \{b,c\}, \{a,c,d\}, \{a,b,c\}, \{b,c,d\}\}$. $\mathcal{V} = \{x,y,z,w\}$ with $\mathcal{V}/R' = \{\{x,z\}, \{y\}, \{w\}\}$. Let $Y = \{x,w\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V}, \varnothing, \{w\}, \{x,z,w\}, \{x,z\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{V}, \varnothing, \{w\}, \{x,z,w\}, \{x,z\}\}$. $\tau_{R'}^{so}(Y) = \{\mathcal{U}, \varnothing, \{w\}, \{x,z\}, \{y,w\}, \{x,y,z\}, \{x,z,w\}\}$, $\tau_{R'}^{po}(Y) = \{\mathcal{U}, \varnothing, \{x\}, \{z\}, \{w\}, \{x,z\}, \{x,w\}, \{y,w\}, \{z,w\}, \{x,y,w\}, \{y,z,w\}, \{x,z,w\}\}$. $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = x$, $f(b) = y$, $f(c) = w$, $f(d) = z$, $f(e) = y$. Clearly f is nano irresolute and also a nano α^*-quotient map. And also f is nano semi-irresolute, but $f^{-1}\{z,w\} = \{c,d\} \in \tau_R^{so}(X)$ and $\{z,w\} \notin \tau_{R'}(Y)$. This shows that f is nano α^*-quotient map, but not a nano semi-quotient map.

6. Comparisons :

In this section we have made comparisions among the several classes of nano quotient mappings in nano topological spaces.

Theorem 6.1 : Every nano α^*-quotient map is nano strongly α-quotient map.

Proof : If f is nano α-irresolute and V is nano open set in $\mathcal{V}$ then $f^{-1}(V)$ is nano α-open set in $\mathcal{U}$. Suppose $f^{-1}(V)$ is nano α-open set in $\mathcal{U}$, since f is an nano α^*-quotient map, V is a nano open set in $\mathcal{V}$. Hence f is nano strongly α-quotient map.

Theorem 6.2 : Every nano quotient map is a nano α-quotient map.

Proof : Suppose f is a nano quotient map. Let V be an nano open set in $\mathcal{V}$, since f is nano quotient map $f^{-1}(V)$ is nano α - open in $\mathcal{U}$. Therefore, f is nano α -continuos. Let V $\subseteq \mathcal{V}$ and $f^{-1}(V)$ be nano open in $\mathcal{V}$. Since f is a nano quotient map, V is a nano open set in $\mathcal{V}$, that is nano α-open set in $\mathcal{V}$. Hence f is nano α-quotient map.

Remark 6.3 : Converse of the above theorem is not true which can be shown by the following example.

Example 6.4 : Let $\mathcal{U} = \{a,b,c,d\}$ with $\mathcal{U}/R = \{\{a,d\},\{b,c\}\}$. Let $X = \{a,d\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\varnothing,\{a,d\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U},\varnothing,\{a,d\},\{a,c,d\},\{a,b,d\}\}$. $\mathcal{V} = \{x,y,z\}$ with $\mathcal{V}'/R' = \{\{x,z\},\{y\}\}$. Let $Y = \{x,z\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V},\varnothing,\{x,z\}\} = \tau_{R'}^{\alpha}(Y)$. Define $f : \mathcal{U} \to \mathcal{V}$ defined by $f(a) = x$, $f(b) = y$, $f(c) = f(d) = z$. Here f is both nano α^*-quotient map and nano strongly α -quotient map but not a nano quotient map. Since $f^{-1}\{x,z\} = \{a,c,d\} \notin \tau_R(X)$.

Remark 6.5 : The following example shows that a nano quotient map is neithernano α^*-quotient nor nano strongly α -quotient map.

Example 6.6 : Let $\mathcal{U} = \{a,b,c,d\}$ with $\mathcal{U}/R = \{\{a\},\{b,c,d\}\}$. Let $X = \{a\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U},\varnothing,\{a\}\}$. $\tau_R^{\alpha}(X) = \{\mathcal{U},\varnothing,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$. $\mathcal{V} = \{p,q,r\}$ with $\mathcal{V}/R' = \{\{p\},\{q,r\}\}$. Let $Y = \{p\} \subseteq \mathcal{V}$, then $\tau_{R'}(Y) = \{\mathcal{V},\varnothing,\{p\}\}$. $\tau_{R'}^{\alpha}(Y) = \{\mathcal{U},\varnothing,\{p\},\{p,q\},\{p,r\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ defined by $f(a) = p$, $f(b) = f(c) = q$, $f(d) = r$. Clearly, f is a nano quotient map but is neither nano α^*-quotient map nor nano strongly α -quotient map. since $f^{-1}\{p,q\} = \{a,b,c\} \in \tau_R^{\alpha}(X)$, but $\{p,q\} \notin \tau_{R'}(Y)$.

Remark 6.7 : The following table shows the relationships of nano quotient map with other sorts of nano quotient maps. The symbol"1" in a cell means that a set implies the other maps and the symbol"0" means that a set does not imply the other sets.

Functions	A	B	C	D
A	1	1	1	0
B	0	1	1	0
C	0	0	1	0
D	0	0	1	1

(A). Nano α^*-quotient map (B). Nano strongly α -quotient map (C). Nano α -quotient map (D). Nano quotient map.

Conclusion 6.8 : The study of quotient mappings is applicable in most areas of pure and applied mathematics. This study would open up the academic flood gates and new vistas in the field of Nano quotient spaces and bitopology for further research studies.

References

1. Ganster. M. and Reily I.L., *Locally closed sets and LC-Continuous functions, Internat. J. Math. Math. Sci*., 417-424 (1989).
2. Lellis Thivagar. M., *A Note On Quotient Mappings*, Bull. Malayasian Math. Soc. (Second Series), *14*, 21-30 (1991).
3. Lellis Thivagar. M., *On some Bitopological γ - seperation axioms*, Cubo Mathematical journal Vol.10 No: *3*, 137-144 (2008).
4. Lellis Thivagar.M and Carmel Richard., *On Nano Forms of Weakly Open sets*, International Journal Of Pure and Applied Mathematics, Vol. *101*, No.5, 893-904 (2015).
5. Lellis Thivagar. M and Carmel Richard., *Weak Forms of Nano Continuity*, IISTE., Vol. *3*, No.7, (2013).
6. Mashour. A.S., Abd El-Monsef. M.E. and El-Deeb S.N., *On Pretopological spaces*, Bull. Math. de la Soc. R.S.de Roumanie, *28*, 3945 (1984).
7. Pawlak.Z., *Rough Sets*, International journal of computer and Information Sciences, *11*, (341-356) (1982).
8. Ramakrishnan. P.V. and Regina Mary. A., *On Fuzzy Quotient Mappings*, Bulletin of the Mathematical Sciences Society, *28, (2)*, 215-221 (2005).