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A Generalized Fixed Point Theorem In dislocated Quasi-Metric Space

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Abstract

In this work, we discuss the existence of fixed points of continuous contracting mappings defined on dislocated quasi-metric space. This work is a continuity of the previous works of Isufati.

Key words: Fixed point, dislocated quasi-metric space.
AMS Subject Classification : 47H10, 54H25.

1 Introduction

The Polish mathematician Stefan Banach[1] proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. It is well known as a Banach fixed point theorem. The existence of a fixed point playes an important role in several area of mathematics, physics and chemistry. Isufati[6] and Zeyada[5] have extended, generalized and improved Banach fixed point theorem in different ways.

The aim of this paper is to obtain a fixed point theorem in the generalized form for continuous contracting mappings in dislocated quasi-metric space.

2 Preliminaries:

Definition 2.1[5] Let X be a non empty set and let $d : X \times X \to [0, \infty)$ be a function satisfying following conditions:

(i) $d(x, y) = d(y, x) = 0$, implies $x = y$,

(ii) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a dislocated quasi-metric on X. If d satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

Definition 2.2[5] A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X, d) is called Cauchy sequence if for, given $\varepsilon > 0$, there exist $n_0 \in N$, such that $\forall$ m, $n \geq n_0$, implies $d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$ i.e. $\min \{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$.

Definition 2.3[5] A sequence $\{x_n\}$ dislocated quasi-convergent to x if

$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$

In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \to x$.

Definition 2.4[5] A dq-metric space (X, d) is called complete if every Cauchy sequence in it is a dq-convergent.

Definition 2.5[5] Let (X, d) be a dq-metric space. A map $T : X \to X$ is called contraction if there exists $0 \leq \lambda \leq 1$

such that

$d(Tx, Ty) \leq \lambda\, d(x, y)$, for all $x,y \in X$.

3 Main results

Theorem 3.1 Let (X, d) be a complete dq-metric space and let $T : X \to X$ be a continuous mapping satisfying the following conditions

$$d(Tx, Ty) \leq \alpha \frac{d(y,Ty)[1+d(x,Ty)]}{1+d(x,y)} + \beta\, d(x,y) + \gamma\, d(Tx, y)$$

for all $x,y \in X$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\alpha + \beta + \gamma < 1$. Then T has a unique fixed point.

Proof : Let $x_o \in X$ and define a sequence $\{x_n\}$ in X such that

$T(x_0) = x_1,\ T(x_1)= x_2$ ...., $T(x_n) = x_{n+1}$............

Consider, $d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$

$$\leq \alpha \frac{d(x_n, Tx_n)[1+d(x_{n-1}, Tx_{n-1})]}{1+d(x_{n-1}, x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(Tx_{n-1}, x_n)$$

$$\leq \alpha \frac{d(x_n, x_{n+1})[1+d(x_{n-1}, x_n)]}{1+d(x_{n-1}, x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(x_n, x_n)$$

Therefore, $d(x_n, x_{n+1}) \leq \dfrac{\beta}{1-\alpha} d(x_{n-1}, x_n)$

$= \lambda\, d(x_{n-1}, x_n)$

Where $\lambda = \dfrac{\beta}{1-\alpha}$ with $0 \leq \lambda \leq 1$. In a similar way we will show that

$d(x_{n-1}, x_n) \leq \lambda\, d(x_{n-2}, x_{n-1})$

and $d(x_n, x_{n+1}) \leq \lambda^2 d(x_{n-2}, x_{n-1})$

Thus $d(x_n, x_{n+1}) \leq \lambda^n d(x_1, x_0)$

Since $0 \leq \lambda < 1$, as $n \to \infty$, $\lambda^n \to 0$. Hence $\{x_n\}$ is a dq-cauchy sequence in X. Thus $\{x_n\}$ dislocated quasi-convergences to some t_0. Since T is continuous, we have

$T(t_0) = \lim T(x_n) = \lim x_{n+1} = t_0$.

Thus $T(t_0) = t_0$. Hence T has a fixed point.

Uniqueness : Let x be a fixed point of T. Then by given condition, we have

$d(x, x) \leq d(Tx, Tx)$

$$\leq \alpha \frac{d(x,Tx)[1+d(x\ ,Tx)]}{1+d(x,x)} + \beta\, d(x, x) + \gamma\, d(Tx, x)$$

$\leq (\alpha + \beta + \gamma)d(x, x)$. Which gives $d(x, x) = 0$, since $0 \leq \alpha + \beta + \gamma < 1$ and $d(x, x) \geq 0$. Thus $d(x, x) \geq 0$, if x is fixed point of T.

Let $x,y \in X$ be fixed points of T, i.e. $Tx = x$, $Ty = y$.

Then by given condition,

$d(x, y) = d(Tx, Ty)$

$$\leq \alpha \frac{d(x,Tx)[1+d(x,Tx)]}{1+d(x,y)} + \beta\, d(x, y) + \gamma\, d(Tx, y)$$

$\leq (\beta + \gamma)d(x, y)$. Which gives $d(x, y) = 0$, since $0 \leq \beta + \gamma < 1$ and $d(x, y) \geq 0$. Similarly $d(y, x)= 0$ and hence $x = y$.

Thus fixed point of T is unique.

Remark:

(i) If we put $\gamma = 0$ we obtained Theorem 3.1 of [5].

(ii) If we put $\alpha = \gamma = 0$ we obtain Theorem 2.8 of [4].

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