ISSN 2231-346X (Print)

JUSPS-A Vol. 29(2), 74-76 (2017). Periodicity-Monthly

Section A

ISSN 2319-8044 (Online)

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Mathematics

website:- www.ultrascientist.org

Estd. 1989

A New Type Of Generalized Fixed Point Theorem On Compact Metric Space

MANOJ GARG

P.G. Department of Mathematics, Nehru (P.G.) College, Chhibramau, Kannauj (U.P.)

Corresponding Author E-mail : garg_manoj1972@yahoo.co.in

http://dx.doi.org/10.22147/jusps-A/290205

Acceptance Date 30th Dec., 2016, Online Publication Date 2nd Feb., 2017

Abstract

In this paper, we prove a fixed point theorem for self mappings satisfying a new contractive type condition in a compact metric space.

Key words: Fixed point, compact metric space, fixed point theorem.

Introduction

In 1922, the Polish mathematician Stefan Banach proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. His result is called Banachs fixed point theorem. This result provides a technique for solving variety of applied problems in mathematical science and engineering. Many authors like Edalstien[1], Kanon[3], Soni[4] and Sahu[7] have extended, generalized and improved Banach fixed point theorem in different ways. In this paper, we extend the work of Sahu for self mapping satisfying a new contractive type condition in a compact metric space.

Throughout this paper the compact metric space (X, d) is denoted by X.

Main Result

Theorem : Let F be a continuousself mapping defined on a compact metric space X with

$$d(Fx, Fy) \le k_1 \frac{d(x,Fx)d(y,Fx)+d(y,Fy)d(x,Fy)}{d(x,Fx)+d(y,Fx)+d(y,Fy)+d(x,Fy)} +$$

$$k_2 \frac{d(x,Fx)d(y,Fy)+d(x,Fy)d(y,Fx)}{d(x,Fx)+d(y,Fy)+d(x,Fy)+d(y,Fx)} + k_3 \frac{d(x,Fx)d(x,Fy)+d(y,Fx)d(y,Fy)}{d(x,Fx)+d(x,Fy)+d(y,Fx)+d(y,Fy)}$$

$$+ k_4[d(x, Fx) + d(y, Fx)] + k_5[d(x, Fy) + d(y, Fx)] + k_6\, d(x, y) \qquad (1)$$

$\forall x, y \in X, x \neq y$ and $k_1 + k_2 + k_3 + 2k_4 + 4k_5 + 2k_6 < 2$, then F has a fixed point. Further when $k_2 + 2k_4 + 4k_5 + 2k_6 < 2$. Then F has a unique fixed point.

Proof: First we define a function G on X as follows, Gx = d(x, Fx).
Since d and Fare continuous on X, therefore G is also continuous on X.
Since X is compact, there exist a point $p \in X$ such that
Gp = inf {Gx : $x \in X$} (2)

If Gp = 0, then d(p, Fp) = 0 i.e. p = Fp

So G(Fp) = d(Fp, F(Fp))

Now $d(Fp, F(Fp)) \le k_1 \dfrac{d(p,Fp)d(Fp,Fp)+d(Fp,F(Fp))d(p,F(Fp))}{d(p,Fp)+d(Fp,Fp)+d(Fp,F(Fp))+d(p,F(Fp))}$

$$+ k_2 \frac{d(p,Fp)d(Fp,F(Fp))+d(p,F(Fp))d(Fp,Fp)}{d(p,Fp)+d(Fp,F(Fp))+d(p,F(Fp))+d(Fp,Fp)}$$

$$+ k_3 \frac{d(p,Fp)d(p,F(Fp))+d(Fp,Fp)d(Fp,F(Fp))}{d(p,Fp)+d(p,F(Fp))+d(Fp,Fp))+d(Fp,F(Fp))}$$

$+ k_4[d(p, Fp) + d(Fp, Fp)] + k_5[d(p, F(Fp)) + d(Fp, Fp)] + k_6\, d(p, Fp).$

Thus $d(Fp, F(Fp)) \le \frac{k_1}{2} d(Fp, F(Fp)) + \frac{k_2}{2} d(Fp, F(Fp)) + \frac{k_3}{2} d(p, Fp) + k_4 d(p, Fp) + k_5\, d(p, F(Fp)) + k_6\, d(p, Fp).$

i.e. $d(Fp, F(Fp))[1 - (\frac{k_1}{2} + \frac{k_2}{2} + k_5)] \le d(p, Fp)[\frac{k_3}{2} + k_4 + k_5 + k_6].$

i.e. $d(Fp, F(Fp)) \le \dfrac{\frac{k_3}{2}+k_4+k_5+k_6}{1-(\frac{k_1}{2}+\frac{k_2}{2}+k_5)}\, d(p, Fp)$

i.e. $d(Fp, F(Fp)) \le k\ d(p, Fp).$

where $k = \dfrac{\frac{k_3}{2}+k_4+k_5+k_6}{1-(\frac{k_1}{2}+\frac{k_2}{2}+k_5)} < 1$, Since $k_1 + k_2 + k_3 + 2k_4 + 4k_5 + 2k_6 < 2$.

Thus G(Fp)< Gp, which is contradiction to the condition (2).
So Fp = p. Consequently p is a fixed point of F in X.
Now we show that p is unique. For suppose q be other fixed point such that Fq = q,

Then $d(p, q) = d(Fp, Fq) \le k_1 \dfrac{d(p,Fp)d(q,Fp)+d(q,Fq))d(p,Fq)}{d(p,Fp)+d(q,Fp)+d(q,Fq))+d(p,Fq))}$

$$+ k_2 \frac{d(p,Fp)d(q,Fq)+d(p,Fq)d(q,Fp)}{d(p,Fp)+d(q,Fq))+d(p,Fq))+d(q,Fp)}$$

$$+ k_3 \frac{d(p,Fp)d(p,Fq)+d(q,Fp)d(q,Fq)}{d(p,Fp)+d(p,Fq)+d(q,Fp))+d(q,Fq)}$$

$+ k_4[d(p, Fp) + d(q, Fp)] + k_5[d(p, Fq) + d(q, Fp)] + k_6\, d(p, q).$

Thus $d(p, q) < (\frac{k_2}{2} + k_4 + 2k_5 + k_6)\, d(p, q).$

Which is a contradiction because $k_2 + 2k_4 + 4k_5 + 2k_6 < 2$.
Thus $d(p, q) = 0$ i. e. $p = q$.
Hence F has a unique fixed point.

Remark:

(i) If $k_1 = k_2 = k_3 = k_4 = k_5 = 0$ and $k_6 = 1$then the theorem reduce to Edelstien[1].

(ii) If $k_5 = 0$ then the theorem reduce to Sahu[7].

References

1. Edelstein, M.; On fixed and periodic under contractive mappings, *J. Land. Math. Sec. 37,* 74-79 (1960).
2. Edelstein, M.; An extention of Banach's contraction principal, *Proc. Amer. Math. Soc., 12,* 7-10 (1961).
3. Kanon, S. and Sharma P.l.; Fixed point mapping for contract metric space, *The Mathematical Education,* 1 (1990).
4. Soni, G.K.; Fixed point theorem in compact metric space, *The Mathematical Education,* XXVII, 4 (1993).
5. Soni, G.K.; Fixed point theorem in compact metric space, *Acta Ciencia Indica,* XXVIM, *4,* 295 (2000).
6. Fisher, B., Namdeo, R.K.; Related fixed point theorem for two pairs of set valued mapping on compact metric space, *Ind. Jour. Of Math., 46(2-3),* 161-171 (2004).
7. Sahu, M.K. and Kaurav, P.S.; A Fixed point theorem on compact metric space, *Acta Ciencia Indica,* XXXIIIM, *3,* 827-829 (2004).