



(Print)

JUSPS-A Vol. 29(2), 77-82 (2017). Periodicity-Monthly

Section A

(Online)



Estd. 1989

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
 An International Open Free Access Peer Reviewed Research Journal of Mathematics
 website:- www.ultrascientist.org

Computation of Separation Axioms in N-Topology¹M. LELLIS THIVAGAR, ²M. AROCKIA DASAN ³V. RAMESH

School of Mathematics, Madurai Kamaraj University Madurai-625 021, Tamil Nadu, India

Corresponding Author ²E-mail : dassfredy@gmail.com<http://dx.doi.org/10.22147/jusps-A/290206>

Acceptance Date 29th Dec., 2016,

Online Publication Date 2nd Feb., 2017

Abstract

In this paper we introduce and characterize $N\tau\tilde{g}$ -open sets and derive that this class of $N\tau\tilde{g}$ -open sets forms a topology on X. We also define the necessary and sufficient condition for a set to be $N\tau\tilde{g}$ -open set. Further we induce various kinds of spaces in N-topology and its applications.

Key words: N-topology, $N\tau g$ -closed, $N\tau\tilde{g}$ -closed, $N\tau *_g$ -closed, $N\tau\#_{gs}$ -closed.

MSC 2010: 54A05, 54A99, 54C10

1. Introduction

Norman Levine⁵ initiated the concept of semi open sets in 1963. He⁶ further investigated the properties of g-closed sets. Dontchev¹ established some properties of separation axioms in α -topology. P. Sundaram *et al.*⁷ introduced some weak forms of closed sets and its separation axioms. Later on Lellis Thivagar *et al.*² initiated the concept of N-topological space and defined its open sets. He^{3, 4} also developed some weak forms of open sets and generalized closed sets in N-topological space. Here we introduce and develop the properties of $N\tau\tilde{g}$ -open sets. Also we derive the necessary and sufficient condition for a set to be $N\tau\tilde{g}$ -open set. Further we induce various kinds of spaces in N-topology and characterize its properties.

2. Preliminaries :

In this section we recall some known results of N-topological spaces which are used in the following sections. By a space $(X, N\tau)$, we mean N-topological space with N-topology $N\tau$ on X on which no separation axioms are assumed unless explicitly stated.

*Definition 2.1.*² Let X be a non-empty set, $\tau_1, \tau_2, \tau_3, \dots, \tau_N$ be N-arbitrary topologies defined on X and let the collection $N\tau$ be defined by

$$N\tau = \{S \subseteq X : S = (\cup_{i=1}^N A_i) \cup (\cap_{i=1}^N B_i), A_i, B_i \in \tau_i\}$$

satisfying the following axioms:

- (i) $X, \emptyset \in N\tau$.
- (ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $S_i \in N\tau$.
- (iii) $\bigcap_{i=1}^n S_i \in N\tau$ for all $S_i \in N\tau$.

Then the pair $(X, N\tau)$ is called a N -topological space on X and the elements of the collection $N\tau$ are known as $N\tau$ -open sets on X . A subset A of X is said to be $N\tau$ -closed on X if the complement of A is $N\tau$ -open on X . The set of all $N\tau$ -open sets on X and the set of all $N\tau$ -closed sets on X are respectively denoted by $N\tau O(X)$ and $N\tau C(X)$.

*Definition 2.2.*² Let $(X, N\tau)$ be a N -topological space and S be a subset of X . Then

- (i) the $N\tau$ -interior of S , denoted by $N\tau\text{-int}(S)$, and is defined by
 $N\tau\text{-int}(S) = \bigcup \{G : G \subseteq S \text{ and } G \text{ is } N\tau\text{-open}\}.$
- (ii) the $N\tau$ -closure of S , denoted by $N\tau\text{-cl}(S)$, and is defined by
 $N\tau\text{-cl}(S) = \bigcap \{F : S \subseteq F \text{ and } F \text{ is } N\tau\text{-closed}\}.$

*Remark 2.3.*³ A subset A of a N -topological space $(X, N\tau)$ is called a

- (i) $N\tau$ - α open set if $A \subset N\tau\text{-int}(N\tau\text{-cl}(N\tau\text{-int}(A)))$.
- (ii) $N\tau$ -semi open set if $A \subset N\tau\text{-cl}(N\tau\text{-int}(A))$.
- (iii) $N\tau$ -pre open set if $A \subset N\tau\text{-int}(N\tau\text{-cl}(A))$.
- (iv) $N\tau$ - β open set if $A \subset N\tau\text{-cl}(N\tau\text{-int}(N\tau\text{-cl}(A)))$.

*Definition 2.4.*⁴ A subset A of a N -topological space $(X, N\tau)$ is called

- (i) a $N\tau$ -generalized closed (briefly $N\tau g$ -closed) set if $N\tau\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ -open in $(X, N\tau)$.
- (ii) a $N\tau$ - α generalized closed (briefly $N\tau\alpha g$ -closed) set if $N\tau\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ -open in $(X, N\tau)$.
- (iii) a $N\tau$ -generalized α closed (briefly $N\tau g\alpha$ -closed) set if $N\tau\text{-}\alpha\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ - α open in $(X, N\tau)$.
- (iv) a $N\tau$ -generalized semi closed (briefly $N\tau gs$ -closed) set if $N\tau\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ -open in $(X, N\tau)$.
- (v) a $N\tau$ -semi generalized closed (briefly $N\tau sg$ -closed) set if $N\tau\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ -semi open in $(X, N\tau)$.
- (vi) a $N\tau\tilde{g}$ -closed set if $N\tau\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau$ -semi open in $(X, N\tau)$.
- (vii) a $N\tau^*g$ -closed set if $N\tau\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau\tilde{g}$ -open in $(X, N\tau)$.
- (viii) a $N\tau^\#gs$ -closed set if $N\tau\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^*g$ -open in $(X, N\tau)$.
- (ix) a $N\tau\tilde{g}$ -closed set if $N\tau\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N\tau^\#gs$ -open in $(X, N\tau)$.

Remark 2.5. (i) ⁴ The complement of a $N\tau g$ -closed (resp. $N\tau\alpha g$ -closed, $N\tau g\alpha$ -closed, $N\tau gs$ -closed, $N\tau sg$ -closed, $N\tau\hat{g}$ -closed, $N\tau^*g$ -closed, $N\tau^\#gs$ -closed, $N\tau\tilde{g}$ -closed) set is called $N\tau g$ -open (resp. $N\tau\alpha g$ -open, $N\tau g\alpha$ -open, $N\tau gs$ -open, $N\tau sg$ -open, $N\tau\tilde{g}$ -open, $N\tau^*g$ -open, $N\tau^\#gs$ -open, $N\tau\hat{g}$ -open) set.

(ii) [4] The family of all $N\tau g$ -closed (resp. $N\tau\alpha g$ -closed, $N\tau g\alpha$ -closed, $N\tau gs$ -closed, $N\tau sg$ -closed, $N\tau\hat{g}$ -closed, $N\tau^*g$ -closed, $N\tau^\#gs$ -closed, $N\tau\tilde{g}$ -closed) sets of $(X, N\tau)$ is denoted by $N\tau GC(X)$ (resp. $N\tau\alpha GC(X)$, $N\tau G\alpha C(X)$, $N\tau GSC(X)$, $N\tau SGC(X)$, $N\tau\hat{G}C(X)$, $N\tau^*GC(X)$, $N\tau^\#GC(X)$, $N\tau\tilde{G}C(X)$) and the family of all $N\tau g$ -open (resp. $N\tau\alpha g$ -open, $N\tau g\alpha$ -open, $N\tau gs$ -open, $N\tau sg$ -open, $N\tau\hat{g}$ -open, $N\tau^*g$ -open, $N\tau^\#gs$ -open, $N\tau\tilde{g}$ -open) sets of $(X, N\tau)$ is denoted by $N\tau GO(X)$ (resp. $N\tau\alpha GO(X)$, $N\tau G\alpha O(X)$, $N\tau GSO(X)$, $N\tau SGO(X)$, $N\tau\hat{G}O(X)$, $N\tau^*GO(X)$, $N\tau^\#GO(X)$, $N\tau\tilde{G}O(X)$).

3. Characterization of $N\tau\hat{g}$ -open sets :

In this section we discuss some more properties of $N\tau\tilde{g}$ -open set and the relationships between this open set and other existing open sets.

Proposition 3.1 Let $(X, N\tau)$ be a N-topological space. Then

- (i) every $N\tau$ -open set is $N\tau g$ -open set but not conversely.
- (ii) every $N\tau$ -semi open set is $N\tau^\#gs$ -open set but not conversely.
- (iii) every $N\tau\alpha$ -open set is $N\tau^\#gs$ -open set but not conversely.
- (iv) every $N\tau g$ -open set is $N\tau\alpha g$ -open set but not conversely.
- (v) every $N\tau g$ -open set is $N\tau gs$ -open set but not conversely.
- (vi) every $N\tau sg$ -open set is $N\tau\beta$ -open set but not conversely.
- (vii) every $N\tau g\alpha$ -open set is $N\tau$ -pre open set but not conversely.
- (viii) every $N\tau$ -open set is $N\tau\tilde{g}$ -open set but not conversely.
- (ix) every $N\tau\tilde{g}$ -open set is $N\tau\hat{g}$ -open set but not conversely.
- (x) every $N\tau\hat{g}$ -open set is $N\tau g$ -open set but not conversely.
- (xi) every $N\tau\tilde{g}$ -open set is $N\tau\alpha g$ -open set but not conversely.
- (xii) every $N\tau\tilde{g}$ -open set is $N\tau sg$ -open set but not conversely.
- (xiii) every $N\tau\tilde{g}$ -open set is $N\tau\beta$ -open set but not conversely.
- (xiv) every $N\tau\tilde{g}$ -open set is $N\tau g\alpha$ -open set but not conversely.
- (xv) every $N\tau\tilde{g}$ -open set is $N\tau$ -pre open set but not conversely.
- (xvi) every $N\tau\tilde{g}$ -open set is $N\tau gs$ -open set but not conversely.

Proof: Proof is the analogue of Proposition 3.3, Proposition 3.7⁴. Also the converse follows from Examples 3.4, 3.5, 3.6, 3.8⁴.

Proposition 3.2 An arbitrary union of $N\tau\tilde{g}$ -open sets is $N\tau\tilde{g}$ -open.

Proof: It follows from the Theorem 4.7⁴.

Proposition 3.3 If A and B are $N\tau\tilde{g}$ -open sets, then $A \cap B$ is $N\tau\tilde{g}$ -open set.

Proof: Proof is analogue to the Theorem 4.9⁴.

Remark 3.4 From the Propositions 3.2 and Proposition 3.3, we observe that the class of $N\tau\tilde{g}$ -open sets forms a topology.

Theorem 3.5 A set A is $N\tau\tilde{g}$ -open if and only if $F \subseteq N\tau\text{-int}(A)$ whenever F is $N\tau^\#gs$ -closed and $F \subseteq A$.

Proof: *Necessity:* Let A be $N\tau\tilde{g}$ -open and $F \subseteq A$, where F is $N\tau^\#gs$ -closed. By definition, $X - A$ is $N\tau\tilde{g}$ -closed. Also $X - A \subset X - F$. This implies $N\tau\text{-cl}(X - A) \subset X - F$. But $N\tau\text{-cl}(A^c) = X - N\tau\text{-int}(A)$, then $X - N\tau\text{-int}(A) \subset X - F$. That is $F \subset N\tau\text{-int}(A)$.

Sufficiency: If A is a $N\tau^\#gs$ -closed with $F \subseteq N\tau\text{-int}(A)$ whenever $F \subseteq A$, it follows that $X - A \subset X - F$ and $X - N\tau\text{-int}(A) \subset X - F$. That is, $N\tau\text{-cl}(X - A) \subset X - F$. Hence $X - A$ is $N\tau\tilde{g}$ -closed and A becomes $N\tau\tilde{g}$ -open.

Theorem 3.6 If $N\tau\text{-int}(A) \subseteq B \subseteq A$ and A is $N\tau\tilde{g}$ -open, then B is $N\tau\tilde{g}$ -open.

Proof: By hypothesis $A^c \subseteq B^c \subseteq (N\tau\text{-int}(A))^c$. That is, $A^c \subseteq B^c \subseteq (X - N\tau\text{-cl}(A^c))^c = N\tau\text{-cl}(A^c)$. Since A^c is $N\tau\tilde{g}$ -closed and by Theorem 4.13⁴, B^c is $N\tau\tilde{g}$ -closed. Hence B is $N\tau\tilde{g}$ -open.

Theorem 3.7 A set A is $N\tau\tilde{g}$ -closed if and only if $N\tau\text{-cl}(A) \setminus A$ is $N\tau\tilde{g}$ -open set.

Proof: *Necessity:* Suppose that A is $N\tau\tilde{g}$ -closed in $(X, N\tau)$. Let F be a $N\tau^\#gs$ -closed subset of $N\tau\text{-cl}(A) \setminus A$. By Theorem 4.12⁴, $F = \emptyset$. Therefore, $F \subseteq N\tau\text{-int}(N\tau\text{-cl}(A) \setminus A)$ and by Theorem 3.5, $N\tau\text{-cl}(A) \setminus A$ is $N\tau\tilde{g}$ -open.

Sufficiency: Let $A \subseteq G$ where G is a $N\tau^\#gs$ -open set. Then $N\tau\text{-cl}(A) \cap G^c \subseteq N\tau\text{-cl}(A) \cap A^c = N\tau\text{-cl}(A) \setminus A$. Since $N\tau\text{-cl}(A) \subseteq G^c$ is $N\tau^\#gs$ -closed set and $N\tau\text{-cl}(A) \setminus A$ is $N\tau\tilde{g}$ -open, by Theorem 3.5, we have $N\tau\text{-cl}(A) \cap G^c \subseteq N\tau\text{-int}(N\tau\text{-cl}(A) \setminus A) = \emptyset$. Hence A is $N\tau\tilde{g}$ -closed in $(X, N\tau)$.

Theorem 3.8 For a subset A of $(X, N\tau)$, the following are equivalent:

- (i) A is $N\tau\tilde{g}$ -closed.
- (ii) $N\tau\text{-cl}(A) \setminus A$ contains no non empty $N\tau^\#gs$ -closed set.
- (iii) $N\tau\text{-cl}(A) \setminus A$ is $N\tau\tilde{g}$ -open.

Proof: Follows from Theorem 3.12⁴ and Theorem 3.7.

4. Some Separation Axioms in N -Topology :

In this section we introduce some new class of spaces via $N\tau\tilde{g}$ -closed set. The relationships between these spaces and other existing spaces are also discussed.

Definition 4.1 An N -topological space $(X, N\tau)$ is called a

- (i) $N\tau T_{1/2}$ -space, if every $N\tau g$ -closed set is $N\tau$ -closed.
- (ii) $N\tau_{gs}T_{1/2}^\#$ -space, if every $N\tau^\#gs$ -closed set is $N\tau$ -closed.
- (iii) $N\tau\alpha$ -space, if every $N\tau\text{-}\alpha$ closed set is $N\tau$ -closed.
- (iv) $N\tau_\alpha T_b$ -space, if every $N\tau\alpha g$ -closed set is $N\tau$ -closed.
- (v) $N\tau_\alpha T_d$ -space, if every $N\tau\alpha g$ -closed set is $N\tau g$ -closed.
- (vi) $N\tau T_b$ -space, if every $N\tau gs$ -closed set is $N\tau$ -closed.
- (vii) $N\tau^*T_{1/2}$ -space, if every $N\tau g$ -closed set is $N\tau^*g$ -closed.
- (viii) $N\tau T_{\tilde{g}}$ -space, if every $N\tau\hat{g}$ -closed set is $N\tau$ -closed.
- (ix) $N\tau_\alpha T_{\tilde{g}}$ -space, if every $N\tau\alpha g$ -closed set is $N\tau\tilde{g}$ -closed.
- (x) $N\tau T_{\tilde{g}}$ -space, if every $N\tau\tilde{g}$ -closed set is $N\tau$ -closed.
- (xi) $N\tau_g T_{\tilde{g}}$ -space, if every $N\tau g$ -closed set is $N\tau\tilde{g}$ -closed.

Theorem 4.2 Every $N\tau T_{1/2}$ -space is a $N\tau T_{\tilde{g}}$ -space but not conversely.

Proof: Let $(X, N\tau)$ be a $N\tau T_{1/2}$ -space and A be a $N\tau\tilde{g}$ -closed subset of X . By Proposition 3.7⁴, A is $N\tau g$ -closed. Since X is a $N\tau T_{1/2}$ -space, A is a $N\tau$ -closed subset of X . Therefore X is a $N\tau T_{\tilde{g}}$ -space.

Example 4.3 If $N=5$, $X=\{a, b, c, d\}$, $\tau_1=\{\emptyset, X, \{a\}\}$, $\tau_2=\{\emptyset, X, \{b\}\}$, $\tau_3=\{\emptyset, X, \{a\}, \{a, b\}\}$, $\tau_4=\{\emptyset, X, \{b\}, \{a, b\}\}$ and $\tau_5=\{\emptyset, X, \{a, b\}\}$. Then $5\tau O(X)=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $5\tau GC(X)=\{\emptyset, X, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$ and $5\tau\tilde{g}C(X)=\{\emptyset, X, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Clearly, $(X, 5\tau)$ is $5\tau T_{\tilde{g}}$ -space but not $5\tau T_{1/2}$ -space.

Theorem 4.4 Every $N\tau T_{\tilde{g}}$ -space is a $N\tau T_g$ -space but not conversely.

Proof: Proof trivially follows from Proposition 3.7⁴.

Example 4.5 If $N=2$, $X=\{a, b, c\}$, $\tau_1=\{\emptyset, X, \{a\}\}$ and $\tau_2=\{\emptyset, X, \{b, c\}\}$. Then $2\tau O(X)=\{\emptyset, X, \{a\}, \{b, c\}\}$, $2\tau\tilde{g}C(X)=\{\emptyset, X, \{a\}, \{b, c\}\}$ and $2\tau\hat{g}C(X)=P(X)$, the power set of X . Thus $(X, 2\tau)$ is $2\tau T_{\tilde{g}}$ -space but not $2\tau T_g$ -space.

Theorem 4.6 Every $N\tau T_b$ -space is a $N\tau T_{\tilde{g}}$ -space but not conversely.

Proof: Proof follows from Proposition 3.7⁴.

Example 4.7 The space $(X, 5\tau)$ in the Example 4.3 is $5\tau T_{\tilde{g}}$ -space but not a $5\tau T_b$ -space.

Remark 4.8 $N\tau T_{\tilde{g}}$ -space and a $N\tau\alpha$ -space are independent as shown in the following examples.

Example 4.9 The space $(X, 5\tau)$ in the Example 4.3 is $5\tau T_{\tilde{g}}$ -space but not a $5\tau\alpha$ -space.

Example 4.10 If $N=5$, $X=\{a, b, c, d\}$, $\tau_1=\{\emptyset, X, \{a\}\}$, $\tau_2=\{\emptyset, X, \{b, c\}\}$, $\tau_3=\{\emptyset, X, \{a, b, c\}\}$, $\tau_4=\{\emptyset, X, \{b, c\}, \{a, b, c\}\}$ and $\tau_5=\{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then $5\tau O(X)=\{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$, $5\tau C(X)=\{\emptyset, X, \{d\}, \{a, d\}, \{b, c, d\}\}$ and $5\tau\tilde{g}C(X)=\{\emptyset, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Here $(X, 5\tau)$ is

$5\tau\alpha$ -space but not $5\tau T_{\tilde{g}}$ -space.

Remark 4.11 $N\tau T_{\tilde{g}}$ -space and $N\tau T_{1/2}$ -space are independent as shown in the following example.

Example 4.12 The space $(X, 2\tau)$ in the Example 4.5 is $2\tau T_{\tilde{g}}$ -space but not a $2\tau T_{1/2}$ -space. Also consider if $N = 2, X = \{a, b, c, d\}, \tau_1 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}, \{a, b, c\}\}$. Then $2\tau O(X) = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $2\tau \tilde{G}C(X) = \{\emptyset, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. Here $(X, 2\tau)$ is $2\tau T_{1/2}$ -space but not a $2\tau T_{\tilde{g}}$ -space.

Theorem 4.13 For a N-topological space $(X, N\tau)$, the following are equivalent:

- (i) $(X, N\tau)$ is a $N\tau T_{\tilde{g}}$ -space.
- (ii) Every singleton of $(X, N\tau)$ is either $N\tau^{\#}gs$ -closed or $N\tau$ -open.

Proof: (i) \Rightarrow (ii): Let $x \in X$ and if $\{x\}$ is not $N\tau^{\#}gs$ -closed, then $X - \{x\}$ is not $N\tau^{\#}gs$ -open. Since X is the only $N\tau^{\#}gs$ -open set containing $X - \{x\}$. Then $X - \{x\}$ is $N\tau \tilde{g}$ -closed set in X . By assumption, $X - \{x\}$ is a $N\tau$ -closed set of X or equivalently $\{x\}$ is $N\tau$ -open.

(ii) \Rightarrow (i): Let $\{x\}$ be a $N\tau \tilde{g}$ -closed set and always $A \subset N\tau\text{-cl}(A)$. Let $x \in N\tau\text{-cl}(A)$. By assumption, $\{x\}$ is either $N\tau^{\#}gs$ -closed or $N\tau$ -open.

Case (i): If $\{x\}$ is $N\tau^{\#}gs$ -closed and $x \notin A$, then $N\tau\text{-cl}(A) - A$ contains a non-empty $N\tau^{\#}gs$ -closed set $\{x\}$, which is a contradiction to Proposition 4.12 [4]. Thus $x \in A$.

Case (ii): If $\{x\}$ is $N\tau$ -open, since $x \in N\tau\text{-cl}(A)$ then $\{x\} \cap A \neq \emptyset$. So $x \in A$. Thus in both cases $x \in A$ and so $N\tau\text{-cl}(A) \subset A$. Therefore $A = N\tau\text{-cl}(A)$. That is, A is a $N\tau$ -closed. Thus X is a $N\tau T_{\tilde{g}}$ -space.

Theorem 4.14 Every $N\tau T_{1/2}$ -space is a $N\tau_g T_{\tilde{g}}$ -space but not conversely.

Proof: Let $(X, N\tau)$ be a $N\tau T_{1/2}$ -space and A be a $N\tau_g$ -closed subset of X . Since X is a $N\tau T_{1/2}$ -space, A is a $N\tau$ -closed subset of X and by Proposition 3.7 [4], A is $N\tau \tilde{g}$ -closed. Therefore X is a $N\tau_g T_{\tilde{g}}$ -space.

Example 4.15 The space $(X, 5\tau)$ in the Example 4.10 is $5\tau_g T_{\tilde{g}}$ -space but not a $5\tau T_{1/2}$ -space.

Remark 4.16 $N\tau T_{\tilde{g}}$ -space and $N\tau_g T_{\tilde{g}}$ -space are independent as shown in the following example.

Example 4.17 The space $(X, 5\tau)$ in the Example 4.10 is $5\tau_g T_{\tilde{g}}$ -space but not a $5\tau T_{\tilde{g}}$ -space. The space $(X, 5\tau)$ in the Example 4.3 is $5\tau T_{\tilde{g}}$ -space but not a $5\tau_g T_{\tilde{g}}$ -space.

Theorem 4.18 If $(X, N\tau)$ is a $N\tau_g T_{\tilde{g}}$ -space, then every singleton subset of $(X, N\tau)$ is either $N\tau_g$ -closed or $N\tau \tilde{g}$ -open.

Proof: Let $x \in X$ and if $\{x\}$ is not $N\tau_g$ -closed, then $X - \{x\}$ is not $N\tau_g$ -open. Then X is the only $N\tau$ -open set containing $X - \{x\}$. So $X - \{x\}$ is $N\tau_g$ -closed. Since X is a $N\tau_g T_{\tilde{g}}$ -space, the set $X - \{x\}$ is $N\tau \tilde{g}$ -closed or equivalently $\{x\}$ is $N\tau \tilde{g}$ -open.

Example 4.19 The converse of the Theorem 4.18 need not be true. Consider the space $(X, 5\tau)$ in the Example 4.3, here the sets $\{c\}, \{d\}$ are $5\tau_g$ -closed and the sets $\{a\}, \{b\}$ are $5\tau \tilde{g}$ -open. But it is not a $5\tau_g T_{\tilde{g}}$ -space.

Theorem 4.20 A space $(X, N\tau)$ is $N\tau T_{1/2}$ if and only if $(X, N\tau)$ is $N\tau T_{\tilde{g}}$ and $N\tau_g T_{\tilde{g}}$.

Proof: Necessity: It follows from Theorem 4.2 and Theorem 4.13.

Sufficiency: Suppose that X is both $N\tau T_{\tilde{g}}$ and $N\tau_g T_{\tilde{g}}$. Let A be a $N\tau_g$ -closed subset of X , then A is $N\tau_g$ -closed. Then A is $N\tau \tilde{g}$ -closed and $N\tau$ -closed. Therefore $(X, N\tau)$ is a $N\tau T_{1/2}$ -space.

Theorem 4.21 Every $N\tau_{\alpha} T_b$ -space is $N\tau_{\alpha} T_{\tilde{g}}$ -space but not conversely.

Proof: Proof follows trivially from Proposition 3.7⁴.

Example 4.22 The space $(X, 5\tau)$ in the Example 4.10 is $5\tau_{\alpha} T_{\tilde{g}}$ -space but not a $5\tau_{\alpha} T_b$ -space.

Theorem 4.23 Every $N\tau_{\alpha} T_{\tilde{g}}$ -space is $N\tau_{\alpha} T_d$ -space but not conversely.

Proof: Let $(X, N\tau)$ be a $N\tau_{\alpha} T_{\tilde{g}}$ -space and A be a $N\tau_{\alpha} g$ -closed subset of X . Then A is a $N\tau \tilde{g}$ -closed subset of

X and by Proposition 3.7⁴, A is $N\tau g$ -closed. Therefore X is a $N\tau_\alpha T_d$ -space.

Example 4.24 The space $(X, 5\tau)$ in the Example 4.3 is $5\tau_\alpha T_d$ -space but not a $5\tau_\alpha T_{\tilde{g}}$ -space.

Theorem 4.25 If $(X, N\tau)$ is a $N\tau_\alpha T_{\tilde{g}}$ -space, then every singleton subset of $(X, N\tau)$ is either $N\tau\alpha g$ -closed or $N\tau\tilde{g}$ -open.

Proof: Proof is similar as Theorem 4.15⁴.

Example 4.26 Converse of the Theorem 4.25 need not be true. Consider the space $(X, 5\tau)$ in the Example 4.3, here the sets $\{c\}$, $\{d\}$ are $5\tau\alpha g$ -closed and the sets $\{a\}$, $\{b\}$ are $5\tau\tilde{g}$ -open. But $(X, 5\tau)$ is not a $5\tau_\alpha T_{\tilde{g}}$ -space.

Conclusion

Thus in this paper we introduced and investigated the properties of some generalized open sets. Further we successfully developed various kinds of spaces and discussed the necessary and sufficient conditions. This theory has wider scope of developing into other applicable research areas of topology such as Nano topology, Fuzzy topology, Intuitionistic topology, Digital topology and so on.

References

1. Dontchev J., On some separation axioms associated with the α -topology, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 18, 31-35 (1997).
2. Lellis Thivagar M., Ramesh V., Arockia Dasan M.; On new structure of N- topology, Cogent Mathematics (Taylor and Francis) Vol. 3, 1204104 (2016).
3. Lellis Thivagar M., Arockia Dasan M., Ramesh V.; New classes of weak open sets in N-topology, Proceedings of International conference on Mathematical Sciences 2016, Carmel College for Women, Goa, India.
4. Lellis Thivagar M., Arockia Dasan M., Ramesh V.; Innovative generalized closed sets of N-topology, (Communicated).
5. Levine N., Semi-open sets and semi-continuity in topological spaces, Amer Math. Monthly, 70, 36-41 (1963).
6. Levine N., Generalized closed sets in topology, Rend. Circ. Mat. Palermo., 19, 89-96 (1970).
7. Sundaram P., Maki H. and Balachandran K., Semi-generalized continuous maps and semi- $T_{1/2}$ -spaces, Bull. Fukuoka Univ. Edu., Part III, 40, 33-40 (1991).
8. Veerakumar M.K.R.S., $\#g$ -semi closed sets in topological spaces, Antarctica J. Math, 2(2), 201-222 (2005).
9. Mashhour. A.S., Abd El-Monsef M.E., El-Deeb.S.N., On pre continuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt, 53, 47-53 (1982).
10. Njåstad O., On some classes of nearly open sets, Pacific J. Math., 15, 961-970 (1965).