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Truss Design Optimization with Imprecise Load and Stress in Intuitionistic Fuzzy Environment

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Abstract

In this paper, we have introduced a intuitionistic fuzzy mathematical programming with intuitionistic fuzzy number as co-efficient of objectives. Real world engineering problems are usually designed with imprecise parameter by the presence of many conflicting objectives. In this paper we develop an approach to solve multi-objective structural design using probabilistic operator. Here total integral values of triangular intuitionistic fuzzy number,and it has been used for imprecise applied load and material density in the test problem. In this paper we have considered a multi objective structural optimization model with weight and deflection as objectives and stress as constraint function. Here design variables are considered as cross sectional area of bars. This classical truss design example is presented here in to demonstrate the significance of our proposed optimization approach. Numerical example is given here to illustrate this structural model through this approximation method.

Key word: Triangular Intuitionistic Fuzzy Number, Total Integral Value, Ranking of Intuitionistic Fuzzy Number, Multi-Objective Intuitionistic Optimization, Structural Optimization.

1. Introduction

Optimization techniques for structural optimal design consisting of deterministic optimization and non-deterministic optimization methods have been widely used in practice,. The former aims to search for the optimum solution under given constraints without consideration of uncertainties. However, in so many engineering structures, Deterministic optimization approaches are unable to handle structural performances exhibit variations such as the fluctuation of external loads, the variation of material properties, e.t.c due to the presence of uncertainties and thus the so-called optimum solution obtained may lie in the infeasible region when uncertainties are present. Thus, so many realistic design approaches must be able to deal with the imprecise nature of structures. Several non-deterministic structural design optimization approaches which are reliability-based design optimization (RBDO) by D.M Frangopol *et.al.*[18] and M. Papadrakakis[19] considering structural impreciseness have been reported in the literature. In the former optimum solution has been obtained under given reliability constraints, while the latter aims to minimize the variation of the objective function. Moreover in the practical optimization problems usually more than one objective is required to be optimized such as minimum cost, maximum stiffness ,minimum displacement at specific structural points ,maximum natural frequency of free vibration and optimum structural strain energy. This makes it necessary to formulate a multi-objective optimization problem. The application of different optimization technique to structural problem has attracted the interest of many researchers. For example Ray Optimization[6], artificial bee colony algorithm[16], Particle Swarm Optimization[6,7,12,14,15,17], genetic Algorithm, meta heuristic algorithm (Kaveh, A. Motie, S. Mohammed, A., Moslehi, M.(2013)), others (Shih, C.j. and Chang, C.J.(1994), Hajela, P. and Shih,C.J. (1990), Wang , D., Zhang, W.H. and Jiang, J.S.(2004), Wang, D., Zhang, W.H. and Jiang, J.S. (2002)., Kripakaran, P., Gupta, A. and Baugh Jr, J.W. (2007)). Fuzzy as well as intuitionistic fuzzy optimization in case of structural engineering not only helps the engineers in their design and analysis of systems but also leads to significant advances and new discoveries in fuzzy optimization theory and technique. This fuzzy set theory was first introduced by Zadeh (1965). As an extension Intuitionistic fuzzy set theory was first introduced by Atanassov (1986) .When an imprecise information can not be expressed by means of conventional fuzzy set Intuitionistic Fuzzy set play an important role. In intuitionistic fuzzy (IF) set we usually consider degree of acceptance, degree of non membership and a hesitancy function whereas we consider only membership function in fuzzy set. A few research work has been done on intuitionistic fuzzy optimization in the field of structural optimization. Dey *et al.* (2014) used intuitionistic fuzzy technique to optimize single objective two bar truss structural model. Dey *et.al.* (2015) multi-objective intuitionistic optimization technique in their paper on three bar truss structural model. This is the first time a parameterized intuitionistic multi-objective nonlinear programming is introduced in this paper with an application in structural design.

In this paper we have considered three-bar planar truss subjected to a single load condition where the objective functions are weight of the truss and deflection of loaded joint in test problem and the design variables are the cross-sections of bars with the constraints as stresses in members. We have developed an approach to solve multi-objective structural design using probabilistic operator. Here total integral values of triangular intuitionistic fuzzy number has been considered for intuitionistic fuzzy applied load and stress.

The remainder of this paper is organized in the following way. In section 2 structural optimization model is discussed . In section 3, mathematics Prerequisites i.e fuzzy Set, intuitionistic fuzzy set, generalized triangular intuitionistic fuzzy number, total integral value of triangular fuzzy number are discussed. In section 4, we proposed the technique to solve a multi-objective non-linear programming problem using intuitionistic Programming technique. In section 5,we discussed the solution of crisp multi-objective structural model by intuitionistic programming technique. Numerical illustration of structural model of three bar truss are discussed in section 6.Finally we draw conclusions in section 7.

2. Multi-objective structural model :

In the design problem of the structure *i.e* lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure .In truss structure system, the basic parameters (including allowable stress, e.t.c) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions.

The multi-objective structural model can be expressed as

$$\begin{aligned} &Minimize\ \ WT(A) \qquad (1)\\ &Minimize\ \ \delta(A)\\ &subject\ to\ \ \sigma(A)\le[\sigma]\\ &A^{\min}\le A\le A^{\max}\end{aligned}$$

Where $A=\left[A_1,A_2,...A_n\right]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar , $WT(A)=\sum_{i=1}^{n}\rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length ,cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and $A^{\max}$ are the lower and upper bounds of cross section area A respectively.

3. Mathematical preliminaries

3.1. Fuzzy Set :

Let X denotes a universal set. Then the fuzzy subset A in X is a subset of order pairs $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right):x\in X\right\}$ **where** $\mu_{\tilde{A}}:X\to[0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x\in X$. A is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set.It is clear that the range of membership function is a subset of non-negative real numbers.

3.2. Intuitionistic Fuzzy Set :

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set A^i in the sense of Atanassov [14] is given by equation $\tilde{A}^i = \{< X, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x) > | x_i \in X\}$ where the function $\mu_{\tilde{A}^i}(x^i): X \to [0,1]$; $x_i \in X \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$ and $\upsilon_{\tilde{A}^i}(x^i): X \to [0,1]$; $x_i \in X \to \upsilon_{A^i}(x_i) \in [0,1]$ define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $\tilde{A}^i \subseteq X$, such that they satisfy the condition $0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1$, $\forall \ x_i \in X$. For each IFS $\tilde{A}^i$ in X the amount $\Pi_{\tilde{A}^i}(x_i) = 1 - \left(\mu_{\tilde{A}^i}(x^i) + \upsilon_{\tilde{A}^i}(x^i)\right)$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in $\tilde{A}^i$ we call it intuitionistic fuzzy index of $\tilde{A}^i$ with respect of an element $x_i \in X$.

3.4. Generalized Intuitionistic Fuzzy Number :

A generalised intuitionistic fuzzy number $\tilde{A}^i$ can be defined as with the following properties

i) It is an intuitionistic fuzzy subset of real line.

ii) It is normal *i.e* there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = w(\in R)$ and $\upsilon_{\tilde{A}^i}(x_0) = \tau(\in R)$ for $w + \tau \le 1;$.

iii) It is a convex set for membership function $\mu_{\tilde{A}^i}(x)$ i.e $\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \min\left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right)$ for all .

iv) It is a concave set for membership function $\mu_{\tilde{A}^i}(x)$ i.e $\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \ge \max\left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right)$ for all $x_1, x_2 \in R, \lambda \in [\tau, 1]$.

v) $\mu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, w]$ and $\upsilon_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[\tau, 1]$ and for $x_0 \in R$ the relation $\mu_{\tilde{A}^i} + \upsilon_{\tilde{A}^i} \le 1$ holds.

3.5. Generalized Triangular Intuitionistic Fuzzy Number:

A generalized triangular intuitionistic fuzzy number $\tilde{A}^i = \left(\left(a_1^\mu, a_2, a_3^\mu; w_a\right)\left(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a\right)\right)$ is a IFN in R and can be defined with the following membership function and non-membership function as follows

$$\mu_{\tilde{A}^i} = \begin{cases} w_a \dfrac{x - a_1^\mu}{a_2 - a_1^\mu} & for \ a_1^\mu \le x \le a_2 \\ w_a & for \ x = a_2 \\ w_a \dfrac{a_3^\mu - x}{a_3^\mu - a_2} & for \ a_2 \le x \le a_3^\mu \\ 0 & otherwise \end{cases}$$

$$\upsilon_{\tilde{A}^i} = \begin{cases} \tau_a \dfrac{x - a_1^\upsilon}{a_2 - a_1^\upsilon} & for \ a_1^\upsilon \le x \le a_2 \\ \tau_a & for \ x = a_2 \\ \tau_a \dfrac{x - a_2}{a_3^\upsilon - a_2} & for \ a_2 \le x \le a_3^\upsilon \\ 1 & otherwise \end{cases}$$

where $a_1^\upsilon \le a_1^\mu \le a_2 \le a_3^\mu \le a_3^\upsilon$.

3.6.Property

Property:3.6.1.

Let $\tilde{A}^i = \left(\left(a_1^\mu, a_2, a_3^\mu; w_a\right)\left(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a\right)\right)$ and $\tilde{B}^i = \left(\left(b_1^\mu, b_2, b_3^\mu; w_b\right)\left(b_1^\upsilon, b_2, b_3^\upsilon; \tau_b\right)\right)$ be two triangular intuitionistic fuzzy number then the arithmetic operations on these numbers can be defined as follows

$$\tilde{A}^i + \tilde{B}^i = \left(\left(a_1^\mu + b_1^\mu, a_2 + b_2, a_3^\mu + b_3^\mu; \min(w_a, w_b)\right)\right.$$
$$\left.\left(a_1^\upsilon + b_1^\upsilon, a_2 + b_2, a_3^\upsilon + b_3^\upsilon; \max(\tau_a, \tau_b)\right)\right)$$

Proof:

With the transformation $z = x + y$ we can find the membership function of acceptance (membership) function of IFS $\tilde{C}^i = \tilde{A}^i + \tilde{B}^i$ by $\alpha -$ cut method.

$\alpha -$cut of $\tilde{A}^i$ is $\left[a_1^\mu + \frac{\alpha}{w_a}\left(a_2 - a_1^\mu\right), a_3^\mu + \frac{\alpha}{w_a}\left(a_1^\mu - a_2\right)\right] \forall \alpha \in [0,1]$ i.e

$$x \in \left[a_1^\mu + \frac{\alpha}{w_a}\left(a_2 - a_1^\mu\right), a_3^\mu + \frac{\alpha}{w_a}\left(a_1^\mu - a_2\right)\right]$$

$\alpha -$cut of $\tilde{B}^i$ is $\left[b_1^\mu + \frac{\alpha}{\tau_a}\left(b_2 - b_1^\mu\right), b_3^\mu + \frac{\alpha}{\tau_a}\left(b_1^\mu - b_2\right)\right] \forall \alpha \in [0,1]$

i.e $y \in \left[b_1^\mu + \frac{\alpha}{\tau_a}\left(b_2 - b_1^\mu\right), b_3^\mu + \frac{\alpha}{\tau_a}\left(b_1^\mu - b_2\right)\right]$

so i.e $z(=x+y) \in \Big[a_1^\mu + b_1^\mu + \frac{\alpha}{w}\left(\left(a_2 - a_1^\mu\right) + \left(b_2 - b_1^\mu\right)\right),$

$$a_3^\mu + b_3^\mu - \frac{\alpha}{w}\left(\left(a_3^\mu - a_2\right) + \left(b_3^\mu - b_2\right)\right)\Big]$$

where $w = \min\left\{w_a, w_b\right\}$.

Thus we get the membership (acceptance) function of $\tilde{C}^i = \tilde{A}^i + \tilde{B}^i$ as

$$\mu_{\tilde{C}^i}(z) = \begin{cases} w\left(\dfrac{z - a_1^\mu - b_1^\mu}{\left(a_2 - a_1^\mu\right) + \left(b_2 - b_2^\mu\right)}\right) & for\ a_1^\mu + b_1^\mu \le z \le a_2 + b_2 \\ w & for\ z = a_2 + b_2 \\ w\left(\dfrac{a_3^\mu + b_3^\mu - z}{\left(a_3^\mu - a_2\right) + \left(b_3^\mu - b_2\right)}\right) & for\ a_2 + b_2 \le z \le a_3^\mu + b_3^\mu \\ 0 & otherwise \end{cases}$$

Hence the addition rule is proved for membership function. For the non-membership function, β – cut of $\tilde{A}^i$ is $\upsilon_{\tilde{A}^i}(x) \le \beta$ i.e $x \le a_2 + \frac{\beta}{\tau_a}\left(a_3^\upsilon - a_2\right)$ and $x \ge a_2 - \frac{\beta}{\tau_a}\left(a_2 - a_1^\upsilon\right)$

β – cut for the non-membership function of $\tilde{B}^i$ is $\upsilon_{\tilde{B}^i}(y) \le \beta$ i.e $y \le b_2 + \frac{\beta}{\tau_b}\left(b_3^\upsilon - b_2\right)$ and $y \ge b_2 - \frac{\beta}{\tau_b}\left(b_2 - b_1^\upsilon\right)$

So $z(=x+y) \le a_2 + b_2 + \frac{\beta}{\tau}\left(a_3^\upsilon + b_3^\upsilon - a_2 - b_2\right)$ and

$$z(=x+y) \ge a_2 + b_2 - \frac{\beta}{\tau}\left(a_2 + b_2 - a_1^\upsilon - b_1^\upsilon\right)$$

Where $\tau = \max\left\{\tau_a, \tau_b\right\}$. Thus we have the non-membership function of $\tilde{C}^i = \tilde{A}^i + \tilde{B}^i$ as

$$\upsilon_{\tilde{C}^i}(z) = \begin{cases} \tau\left(\dfrac{a_2 + b_2 - z}{\left(a_2 - a_1^\upsilon\right) + \left(b_2 - b_2^\upsilon\right)}\right) & for\ a_1^\upsilon + b_1^\upsilon \le z \le a_2 + b_2 \\ 0 & for\ z = a_2 + b_2 \\ \tau\left(\dfrac{z - a_2 + b_2}{\left(a_3^\upsilon - a_2\right) + \left(b_3^\upsilon - b_2\right)}\right) & for\ a_2 + b_2 \le z \le a_3^\upsilon + b_3^\upsilon \\ \tau & otherwise \end{cases}$$

Hence the addition rule is proved.

Thus we have

$$\tilde{A}^i + \tilde{B}^i = \left(\left(a_1^\mu + b_1^\mu, a_2 + b_2, a_3^\mu + b_3^\mu; \min\left(w_a, w_b\right)\right)\right.$$
$$\left.\left(a_1^\upsilon + b_1^\upsilon, a_2 + b_2, a_3^\upsilon + b_3^\upsilon; \max\left(\tau_a, \tau_b\right)\right)\right)$$

Property : 3.6.2.

Let $\tilde{A}^i = \left(\left(a_1^\mu, a_2, a_3^\mu; w_a\right)\left(a_1^\upsilon, a_2, a_3^\upsilon; \tau_a\right)\right)$ be a triangular intuitionistic fuzzy number then

$$k\tilde{A}^i = \begin{cases} \left(\left(ka_1^\mu, ka_2, ka_3^\mu; w_a\right)\left(ka_1^\upsilon, ka_2, ka_3^\upsilon; \tau_a\right)\right) & for\ k > 0 \\ \left(\left(ka_3^\mu, ka_2, ka_1^\mu; w_a\right)\left(ka_3^\upsilon, ka_2, ka_1^\upsilon; \tau_a\right)\right) & for\ k < 0 \end{cases}$$

Proof:

When $k > 0$, with transformation $y = ka$ we can find the membership function for membership or acceptance function of TrIFN $\tilde{Y}^i = k\tilde{A}^i$ by α – cut method. The α – cut of $\tilde{A}^i$ is $\mu_{\tilde{A}^i}(x) \ge \alpha$ i.e $x \in \left[a_1^\mu + \frac{\alpha}{w_a}\left(a_2 - a_1^\mu\right), a_3^\mu + \frac{\alpha}{w_a}\left(a_3^\mu - a_2\right)\right]$

So $y(=ka) \in \left[ka_1^\mu + \frac{\alpha}{w_a}\left(ka_2 - ka_1^\mu\right), ka_3^\mu + \frac{\alpha}{w_a}\left(ka_3^\mu - ka_2\right)\right]$

Thus we get membership function of $\tilde{Y}^i = k\tilde{A}^i$ as

$$_{\tilde{Y}^i}(y) = \begin{cases} w_a\left(\dfrac{y - ka_1^\mu}{\left(ka_2 - ka_1^\mu\right)}\right) & for\ ka_1^\mu \le y \le ka_2 \\ w_a & for\ y = ka_2 \\ w_a\left(\dfrac{ka_3^\mu - y}{\left(ka_3^\mu - ka_2\right)}\right) & for\ ka_2 \le y \le ka_1^\mu \\ 0 & otherwise \end{cases}$$

Hence the rule is proved for membership function .

The β – cut of $\tilde{A}^i$ is $\mu_{\tilde{A}^i}(x) \le \beta$ i.e $x \le a_2 + \frac{\beta}{\tau_a}\left(a_3^\upsilon - a_2\right)$ and $x \ge a_2 - \frac{\beta}{\tau_a}\left(a_2 - a_1^\upsilon\right)$ So $y(=ka) \le ka_2 + \frac{\beta}{\tau_a}\left(ka_3^\upsilon - ka_2\right)$ and $y(=ka) \ge ka_2 - \frac{\beta}{\tau_a}\left(ka_2 - ka_1^\upsilon\right)$

Thus we get non-membership function of $\tilde{Y}^i = k\tilde{A}^i$ as

$$\upsilon_{\tilde{Y}^i}(y)=\begin{cases}\tau_a\left(\dfrac{ka_2-y}{\left(ka_2-ka_1^{\upsilon}\right)}\right) & for\ ka_1^{\upsilon}\le y\le ka_2\\ 0 & for\ \ y=ka_2\\ \tau_a\left(\dfrac{y-ka_2}{\left(ka_3^{\upsilon}-ka_2\right)}\right) & for\ ka_2\le y\le ka_3^{\upsilon}\\ \tau_a & otherwise\end{cases}$$

Hence the result is proved for non-membership function . Thus we have $\tilde{Y}^i=k\tilde{A}^i=\left(\left(ka_1^{\mu},ka_2,ka_3^{\mu};w_a\right)\left(ka_1^{\upsilon},\right.\right.$ $\left.\left.ka_2,ka_3^{\upsilon};\tau_a\right)\right)$ $\quad for\, k>0$

Similarly we can prove that if $y=ka\ (k<0)$ then

$$\mu_{\tilde{Y}^i}(y)=\begin{cases}w_a\left(\dfrac{y-ka_3^{\mu}}{\left(ka_2-ka_3^{\mu}\right)}\right) & for\ ka_3^{\mu}\le y\le ka_2\\ w_a & for\ \ y=ka_2\\ w_a\left(\dfrac{ka_1^{\mu}-y}{\left(ka_1^{\mu}-ka_2\right)}\right) & for\ ka_2\le y\le ka_1^{\mu}\\ 0 & otherwise\end{cases}$$

and $$\upsilon_{\tilde{Y}^i}(y)=\begin{cases}\tau_a\left(\dfrac{ka_2-y}{\left(ka_2-ka_3^{\upsilon}\right)}\right) & for\ ka_3^{\upsilon}\le y\le ka_2\\ 0 & for\ \ y=ka_2\\ \tau_a\left(\dfrac{y-ka_2}{\left(ka_1^{\upsilon}-ka_2\right)}\right) & for\ ka_2\le y\le ka_1^{\upsilon}\\ \tau_a & otherwise\end{cases}$$

Thus we have $$\tilde{Y}^i=k\tilde{A}^i=\begin{cases}\left(\left(ka_1^{\mu},ka_2,ka_3^{\mu};w_a\right)\left(ka_1^{\upsilon},ka_2,ka_3^{\upsilon};\tau_a\right)\right) & for\, k>0\\ \left(\left(ka_3^{\mu},ka_2,ka_1^{\mu};w_a\right)\left(ka_3^{\upsilon},ka_2,ka_1^{\upsilon};\tau_a\right)\right) & for\, k<0\end{cases}$$

3.7. Ranking of Triangular Intuitionistic Fuzzy Number :

A triangular intuitionistic fuzzy number $\tilde{A}^i=\left(\left(a_1^{\mu},a_2,a_3^{\mu};w_a\right)\left(a_1^{\upsilon},a_2,a_3^{\upsilon};\tau_a\right)\right)$ is completely defined by $L_{\mu}(x)=w_a\dfrac{x-a_1^{\mu}}{a_2-a_1^{\mu}}$ $for\ a_1^{\mu}\le x\le a_2$ and $R_{\mu}(x)=w_a\dfrac{a_3^{\mu}-x}{a_3^{\mu}-a_2}$ $for\ a_2\le x\le a_3^{\mu}$; $L_{\upsilon}(x)=\tau_a\dfrac{a_2-x}{a_2-a_1^{\upsilon}}$ $for\ a_1^{\upsilon}\le x\le a_2$ and $\tau_a\dfrac{x-a_2}{a_3^{\upsilon}-a_2}$ $for\ a_2\le x\le a_3^{\upsilon}$.

The inverse functions can be analytically express as

$$L_{\mu}^{-1}(h)=a_1^{\mu}+\frac{h}{w_a}\left(a_2-a_1^{\mu}\right);\ R_{\mu}^{-1}(h)=a_3^{\mu}-\frac{h}{w_a}\left(a_3^{\mu}-a_2\right);$$

$$L_{\upsilon}^{-1}(h)=a_2+\frac{h}{\tau_a}\left(a_2-a_1^{\upsilon}\right);\ R_{\upsilon}^{-1}(h)=a_2-\frac{h}{\tau_a}\left(a_3^{\upsilon}-a_2\right);$$

Now left integral value of membership and non-membership functions of $\tilde{A}^i$ are

$I_L\left(\tilde{A}^i\right)=\int_0^1 L_{\mu}^{-1}(h)=\dfrac{(2w_a-1)a_1^{\mu}+a_2}{2w_a}$ and $I_L\left(\tilde{A}^i\right)=\int_0^1 L_{\upsilon}^{-1}(h)=\dfrac{(2\tau_a-1)a_1^{\upsilon}+a_2}{2\tau_a}$

respectively

And right integral value of membership and non-membership functions are

$$I_R\left(\tilde{A}^i\right)=\int_0^1 L_{\mu}^{-1}(h)=\frac{(2w_a-1)a_3^{\mu}+a_2}{2w_a}$$ and

$$I_R\left(\tilde{A}^i\right)=\int_0^1 L_{\upsilon}^{-1}(h)=\frac{(2\tau_a-1)a_3^{\upsilon}+a_2}{2\tau_a}$$ respectively.

The total integral value of the membership functions is

$$I_T^{\alpha}\left(\tilde{A}^i\right)=\frac{(2w_a-1)a_3^{\mu}+a_2}{2w_a}\alpha+(1-\alpha)\frac{(2w_a-1)a_1^{\mu}+a_2}{2w_a}$$
$$=\frac{a_2+(2w_a-1)\left\{\alpha a_3^{\mu}+(1-\alpha)a_1^{\mu}\right\}}{2w_a}$$

The total integral value of the non membership functions is

$$I_T^{\beta}\left(\tilde{A}^i\right)=\frac{(2\tau_a-1)a_3^{\upsilon}+a_2}{2\tau_a}\beta+(1-\beta)\frac{(2\tau_a-1)a_1^{\upsilon}+a_2}{2\tau_a}$$
$$=\frac{a_2+(2\tau_a-1)\left\{\beta a_3^{\upsilon}+(1-\beta)a_1^{\upsilon}\right\}}{2\tau_a}$$

Now if $\tilde{A}^i=\left(\left(a_1^{\mu},a_2,a_3^{\mu};w_a\right)\left(a_1^{\upsilon},a_2,a_3^{\upsilon};\tau_a\right)\right)$ and $\tilde{B}^i=\left(\left(b_1^{\mu},b_2,b_3^{\mu};w_b\right)\left(b_1^{\upsilon},b_2,b_3^{\upsilon};\tau_b\right)\right)$ be two triangular intuitionistic fuzzy number then the following relations hold good

i) If $I_T^{\alpha}\left(\tilde{A}^i\right) < I_T^{\alpha}\left(\tilde{B}^i\right)$ and $I_T^{\beta}\left(\tilde{A}^i\right) < I_T^{\beta}\left(\tilde{B}^i\right)$ for $\alpha,\beta \in [0,1]$ then $\tilde{A}^i < \tilde{B}^i$

ii) If $I_T^{\alpha}\left(\tilde{A}^i\right) > I_T^{\alpha}\left(\tilde{B}^i\right)$ and $I_T^{\beta}\left(\tilde{A}^i\right) > I_T^{\beta}\left(\tilde{B}^i\right)$ for $\alpha,\beta \in [0,1]$ then $\tilde{A}^i > \tilde{B}^i$

iii) If $I_T^{\alpha}\left(\tilde{A}^i\right) = I_T^{\alpha}\left(\tilde{B}^i\right)$ and $I_T^{\beta}\left(\tilde{A}^i\right) = I_T^{\beta}\left(\tilde{B}^i\right)$ for $\alpha,\beta \in [0,1]$ then $\tilde{A}^i = \tilde{B}^i$

4. Mathematical Analysis :

4.1 Formulation of Intuitionistic Programming with imprecise coefficient : A multi-objective intuitionistic fuzzy non-linear programming problem with imprecise co-efficient can be formulated as

$$\text{Minimize } \tilde{f}_{k_0}(x) = \sum_{t=1}^{T_{k_0}} \xi_{k_0 t}\tilde{c}_{k_0 t}\prod_{j=1}^{n} x_j^{a_{k_0 tj}} \text{ for } k_0 = 1,2,....,p$$

$$\text{Such that } \tilde{f}_i(x) = \sum_{t=1}^{T_i} \xi_{it}\tilde{c}_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le \xi_i\tilde{b}_i \text{ for } i = 1,2,....,m$$

$$x_j > 0 \quad j = 1,2,....,n$$

Here $\xi_{k_0 t}$, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\tilde{c}_{k_0 t} > 0$, $\tilde{c}_{it} > 0$. $a_{k_0 tj}$, a_{itj} are real numbers for all i,t,k_0,j.

Here $\tilde{c}_{k_0 t} = \left(\left(c_{k_0 t}^{1\mu}, c_{k_0 t}^{2}, c_{k_0 t}^{3\mu}; w_{k_0 t}\right)\left(c_{k_0 t}^{1\upsilon}, c_{k_0 t}^{2}, c_{k_0 t}^{3\upsilon}; \tau_{k_0 t}\right)\right)$;

$$\tilde{c}_{it} = \left(\left(c_{it}^{1\mu}, c_{it}^{2}, c_{it}^{3\mu}; w_{it}\right)\left(c_{it}^{1\upsilon}, c_{it}^{2}, c_{it}^{3\upsilon}; \tau_{it}\right)\right);$$

$$\tilde{b}_i = \left(\left(b_i^{1\mu}, b_i^{2}, b_i^{3\mu}; w_i\right)\left(b_i^{1\upsilon}, b_i^{2}, b_i^{3\upsilon}; \tau_i\right)\right).$$

Using total integral value of membership and non-membership function, we transform above intuitionistic multi-objective programming with imprecise parameter as

$$\text{Minimize } \hat{f}_{1k_0}(x;\alpha) = \sum_{t=1}^{T_{k_0}} \xi_{k_0 t}\hat{c}_{1k_0 t}\prod_{j=1}^{n} x_j^{a_{k_0 tj}} \text{ for } k_0 = 1,2,....,p$$

$$\text{Minimize } \hat{f}_{2k_0}(x;\beta) = \sum_{t=1}^{T_{k_0}} \xi_{k_0 t}\hat{c}_{2k_0 t}\prod_{j=1}^{n} x_j^{a_{k_0 tj}} \text{ for } k_0 = 1,2,....,p$$

$$\text{Such that } \hat{f}_{1i}(x;\alpha) = \sum_{t=1}^{T_i} \xi_{it}\hat{c}_{1it}\prod_{j=1}^{n} x_j^{a_{itj}} \le \xi_i\hat{b}_{1i} \text{ for } i = 1,2,....,m$$

$$\hat{f}_{2i}(x;\beta) = \sum_{t=1}^{T_i} \xi_{it}\hat{c}_{2it}\prod_{j=1}^{n} x_j^{a_{itj}} \le \xi_i\hat{b}_{2i} \text{ for } i = 1,2,....,m$$

$$x_j > 0; \alpha,\beta \in [0,1] \quad j = 1,2,....,n$$

Here $\xi_{k_0 t}$, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\hat{c}_{1k_0 t} > 0$, $\hat{c}_{1it} > 0$; $\hat{b}_{1i} > 0$ denote the total integral value of membership function i.e

$$\hat{c}_{1k_0 t} = \frac{c_{k_0 t}^2 + \left(2w_{k_0 t} - 1\right)\left\{\alpha c_{k_0 t}^{3\mu} + (1-\alpha)c_{k_0 t}^{1\mu}\right\}}{2w_{k_0 t}},$$

$$\hat{c}_{1it} = \frac{c_{it}^2 + \left(2w_{it} - 1\right)\left\{\alpha c_{it}^{3\mu} + (1-\alpha)c_{it}^{1\mu}\right\}}{2w_{it}} \text{ and}$$

$$\hat{b}_{1i} = \frac{b_i^2 + \left(2w_i - 1\right)\left\{\alpha b_i^{3\mu} + (1-\alpha)b_i^{1\mu}\right\}}{2w_i} \text{ and } \hat{c}_{1k_0 t} > 0,$$

$\hat{c}_{1it} > 0$; $\hat{b}_{1i} > 0$ denote the total integral value of non-membership function ie $\hat{c}_{2k_0 t} = \dfrac{c_{k_0 t}^2 + \left(2\tau_{k_0 t} - 1\right)\left\{\beta c_{k_0 t}^{3\upsilon} + (1-\beta)c_{k_0 t}^{1\upsilon}\right\}}{2\tau_{k_0 t}}$,

$$\hat{c}_{2it} = \frac{c_{it}^2 + \left(2\tau_{it} - 1\right)\left\{\beta c_{it}^{3\upsilon} + (1-\beta)c_{it}^{1\upsilon}\right\}}{2\tau_{it}} \text{ and}$$

$$\hat{b}_{2i} = \frac{b_i^2 + \left(2\tau_i - 1\right)\left\{\beta b_i^{3\upsilon} + (1-\beta)b_i^{1\upsilon}\right\}}{2\tau_i}$$

4.2. Intuitionistic Fuzzy Non-linear Programming (IFNLP) Optimization to solve Parametric Multi-Objective Non-linear Programming Problem (PMONLP) :

A multi-objective non-linear parametric intuitionistic programming (MONLP)Problem can be formulated as

$$\text{Minimize } \left\{ f_1(x;\alpha), f_2(x;\alpha),...., f_p(x;\alpha), f_1(x;\beta), f_2(x;\beta),...., f_p(x;\beta)\right\}^T \tag{2}$$

$$\text{Subject to } g_j(x;\alpha) \le b_j; \quad j = 1,2,.....,m$$

$$g_j(x;\beta) \le b_j; \quad j = 1,2,.....,m$$

$$x > 0 \ \alpha,\beta \in [0,1]$$

Following Zimmermann (1978), we have presented a solution algorithm to solve the MONLP Problem by fuzzy optimization technique.

Step-1: Solve the MONLP (2) as a single objective non-linear programming problem p tiby taking one of the objective at a time and ignoring the others .These solutions areknown as ideal solutions. Let x^i be the respective optimal solution for the i^{th} different objectives with same constraints and evaluate each objective values for all these i^{th} optimal solutions.

Step-2: From the result of step -1 determine the corresponding values for every objective for each derived solutions. With the values of all objectives at each ideal solutions, pay-off matrix can be formulated as follows

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \vdots \\ x^{2p} \end{array}
\begin{array}{c}
\begin{array}{cccccc} f_1(x;\alpha) & \dots & f_p(x;\alpha) & f_1(x;\beta) & \dots & f_p(x;\beta) \end{array} \\
\begin{bmatrix}
f_1^*(x^1;\alpha) & \dots & f_p^*(x^1;\alpha) & f_1^*(x^1;\beta) & \dots & f_p^*(x^1;\beta) \\
f_1^*(x^2;\alpha) & \dots & f_p^*(x^2;\alpha) & f_1^*(x^2;\beta) & \dots & f_p^*(x^2;\beta) \\
\dots & \dots & \dots & \dots & \dots & \dots \\
f_1^*(x^{2p};\alpha) & \dots & f_p^*(x^{2p};\alpha) & f_1^*(x^{2p};\beta) & \dots & f_p^*(x^{2p};\beta)
\end{bmatrix}
\end{array}$$

Here $x^1, x^2, \dots\dots, x^p$ are the ideal solution of the objectives $f_1(x;\alpha), f_2(x;\alpha), \dots., f_p(x;\alpha), f_1(x;\beta), f_2(x;\beta), \dots., f_p(x;\beta)$ respectively.

Step-3: From the result of step 2 now we find lower bound (minimum) L_i^{ACC} and upper bound (maximum) U_i^{ACC} by using following rule $U_i^{ACC} = \max\left\{f_i(x^p;\alpha), f_i(x^p;\beta)\right\}$, $L_i^{ACC} = \min\left\{f_i(x^p;\alpha), f_i(x^p;\beta)\right\}$ where $1 \le i \le p$. But in IFO The degree of non-membership (rejection) and the degree of membership (acceptance) are considered so that the sum of both value is less than one. To define the non -membership of NLP problem let $U_i^{\mathrm{Re}\,j}$ and $L_i^{\mathrm{Re}\,j}$ be the upper bound and lower bound of objective function $f_i(x,\alpha), f_i(x,\beta)$ where $L_i^{ACC} \le L_i^{\mathrm{Re}\,j} \le U_i^{\mathrm{Re}\,j} \le U_i^{ACC}$. For objective function of minimization problem, the upper bound for non-membership function (rejection) is always equals to that the upper bound of membership function (acceptance). One can take lower bound for non-membership function as follows $L_i^{\mathrm{Re}\,j} = L_i^{Acc} + \varepsilon_i$ where $0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ based on the decision maker choice.

The initial intuitionistic fuzzy model with aspiration level of objectives becomes

Find

$$\{x_i, i = 1,2,\dots., p\}$$

so as to satisfy $f_i(x) \le^i L_i^{Acc}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for the degree of acceptance for $i = 1,2,\dots\dots, p$. $f_i(x;s) \ge^i U_i^{\mathrm{Re}\,j}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for degree of rejection for $i = 1,2,\dots\dots, p$. Define the membership (acceptance) and non-membership (rejection) functions of above uncertain objectives as follows. For the $i^{th}, i = 1,2,\dots., p$ objectives functions the linear membership functions $\mu_i\left(f_i(x;\alpha)\right)$ and $\mu_i\left(f_i(x;\beta)\right)$ and linear non-membership functions $\upsilon_i\left(f_i(x;\alpha)\right)$ and $\upsilon_i\left(f_i(x;\beta)\right)$ are defined as follows

$$\mu_i\left(f_i(x;\alpha)\right) = \begin{cases} 1 & if \quad f_i(x;\alpha) \le L_i^{Acc} \\ \dfrac{e^{-T\left(\frac{f_i(x;\alpha) - L_i^{Acc}}{U_i^{Acc} - L_i^{Acc}}\right)} - e^{-T}}{1 - e^{-T}} & if \quad L_i^{Acc} \le f_i(x;\alpha) \le U_i^{Acc} \\ 0 & if \quad f_i(x;\alpha) \ge U_i^{Acc} \end{cases}$$

$$\mu_i\left(f_i(x;\beta)\right) = \begin{cases} 1 & if \quad f_i(x;\beta) \le L_i^{Acc} \\ \dfrac{e^{-T\left(\frac{f_i(x;\beta) - L_i^{Acc}}{U_i^{Acc} - L_i^{Acc}}\right)} - e^{-T}}{1 - e^{-T}} & if \quad L_i^{Acc} \le f_i(x;\beta) \le U_i^{Acc} \\ 0 & if \quad f_i(x;\beta) \ge U_i^{Acc} \end{cases}$$

$$\upsilon_i\left(f_i(x;\alpha)\right) = \begin{cases} 0 & if \quad f_i(x;\alpha) \le L_i^{\mathrm{Re}\,j} \\ \left(\dfrac{f_i(x;\alpha) - L_i^{\mathrm{Re}\,j}}{U_i^{\mathrm{Re}\,j} - L_i^{\mathrm{Re}\,j}}\right)^2 & if \quad L_i^{\mathrm{Re}\,j} \le f_i(x;\alpha) \le U_i^{\mathrm{Re}\,j} \\ 1 & if \quad f_i(x;\alpha) \ge U_i^{\mathrm{Re}\,j} \end{cases}$$

$$\upsilon_i\left(f_i(x;\beta)\right) = \begin{cases} 0 & if \quad f_i(x;\beta) \le L_i^{\mathrm{Re}\,j} \\ \left(\dfrac{f_i(x;\beta) - L_i^{\mathrm{Re}\,j}}{U_i^{\mathrm{Re}\,j} - L_i^{\mathrm{Re}\,j}}\right)^2 & if \quad L_i^{\mathrm{Re}\,j} \le f_i(x;\beta) \le U_i^{\mathrm{Re}\,j} \\ 1 & if \quad f_i(x;\beta) \ge U_i^{\mathrm{Re}\,j} \end{cases}$$

Step-4: Now using Intuitionistic fuzzy probabilistic operator above problem can be written as

$$Maximize \prod_{i=1}^{p}\left(\mu_i\left(f_i(x;\alpha)\right)\right)\left(\mu_i\left(f_i(x;\beta)\right)\right) \quad (3)$$

$$Maximize \prod_{i=1}^{p}\left(1 - \upsilon_i\left(f_i(x;\alpha)\right)\right)\left(1 - \upsilon_i\left(f_i(x;\beta)\right)\right)$$

subject to

$$0 < \mu_i\left(f_i(x;\alpha)\right) < 1;\ 0 < \upsilon_i\left(f_i(x;\alpha)\right) < 1;$$

$$0 \le \mu_i\left(f_i(x;\alpha)\right) + \upsilon_i\left(f_i(x;\alpha)\right) \le 1;$$

$$0 < \mu_i\left(f_i(x;\beta)\right) < 1;\ 0 < \upsilon_i\left(f_i(x;\beta)\right) < 1;$$

$$0 \le \mu_i\left(f_i\left(x;\beta\right)\right) + \upsilon_i\left(f_i\left(x;\beta\right)\right) \le 1;$$
$$g_j\left(x;\alpha\right) \le b_j;$$
$$g_j\left(x;\beta\right) \le b_j;$$
$$x > 0 \ \alpha,\beta \in [0,1]$$
$$i = 1,2,.....,p;\ j = 1,2,.....,m$$

Step-5: Solve the above crisp model (3) by using appropriate mathematical programming algorithm to get optimal solution of objective function.

Step-6: Stop.

5. Solution of Multi-Objective Structural Optimization Problem by Intuitionistic Fuzzy Optimization Technique:

The multi-objective structural model (1) can be expressed as parametric intuitionistic form as

$$\begin{aligned} &Minimize\ \ WT\left(A;\alpha\right) \\ &Minimize\ \ WT\left(A;\beta\right) \\ &Minimize\ \ \delta\left(A;\alpha\right) \\ &Minimize\ \ \delta\left(A;\beta\right) \\ &subject\ to \\ &\sigma\left(A;\alpha\right) \le \left([\sigma];\alpha\right) \\ &\sigma\left(A;\beta\right) \le \left([\sigma];\beta\right) \\ &A^{\min} \le A \le A^{\max}\ \ \alpha,\beta \in [0,1] \end{aligned} \tag{4}$$

Where $A = \left(A_1, A_2, ..., A_n\right)^T$

To solve the MOSOP (4) step 1 of 4.5 is used. After that according to step 2 pay-off matrix is formulated

$$\begin{array}{c} \\ A^1 \\ A^2 \\ A^3 \\ A^4 \end{array} \begin{array}{c} \begin{array}{cccc} WT\left(A;\alpha\right) & WT\left(A;\beta\right) & \delta\left(A;\alpha\right) & \delta\left(A;\beta\right) \end{array} \\ \begin{bmatrix} WT^*\left(A^1;\alpha\right) & WT^*\left(A^1;\beta\right) & \delta^*\left(A^1;\alpha\right) & \delta^*\left(A^1;\beta\right) \\ WT^*\left(A^2;\alpha\right) & WT^*\left(A^2;\beta\right) & \delta^*\left(A^2;\alpha\right) & \delta^*\left(A^2;\beta\right) \\ WT^*\left(A^3;\alpha\right) & WT^*\left(A^3;\beta\right) & \delta^*\left(A^3;\alpha\right) & \delta^*\left(A^3;\beta\right) \\ WT^*\left(A^4;\alpha\right) & WT^*\left(A^4;\beta\right) & \delta^*\left(A^4;\alpha\right) & \delta^*\left(A^4;\beta\right) \end{bmatrix} \end{array}$$

In next step following step 2 we calculate the bound of the objective $U_1^{Acc}, L_1^{Acc}\ U_2^{Acc}, L_2^{Acc}$ and $U_1^{\mathrm{Re}\,j}, L_1^{\mathrm{Re}\,j}\ U_2^{\mathrm{Re}\,j}, L_2^{\mathrm{Re}\,j}$ for weight function $WT\left(A;\alpha\right)$, $WT\left(A;\beta\right)$ such that $L_1^{Acc} < WT\left(A;\alpha\right) < U_1^{Acc}$; $L_2^{Acc} < WT\left(A;\beta\right) < U_2^{Acc}$ and $L_1^{\mathrm{Re}\,j} < WT\left(A;\alpha\right) < U_1^{\mathrm{Re}\,j}$; $L_2^{\mathrm{Re}\,j} < WT\left(A;\beta\right) < U_2^{\mathrm{Re}\,j}$; and U_3^{Acc}, L_3^{Acc}; U_4^{Acc}, L_4^{Acc}; $U_3^{\mathrm{Re}\,j}, L_3^{\mathrm{Re}\,j}\ U_4^{\mathrm{Re}\,j}, L_4^{\mathrm{Re}\,j}$ for deflection $\delta\left(A;\alpha\right)$, and $\delta\left(A;\beta\right)$, such that $L_3^{Acc} < \delta\left(A;\alpha\right) < U_3^{Acc}$; $L_4^{Acc} < \delta\left(A;\beta\right) < U_4^{Acc}$; and $L_3^{\mathrm{Re}\,j} < \delta\left(A;\beta\right) < U_3^{\mathrm{Re}\,j}$; $L_4^{\mathrm{Re}\,j} < \delta\left(A;\beta\right) < U_4^{\mathrm{Re}\,j}$; with the condition $U_i^{Acc} = U_i^{\mathrm{Re}\,j}$; $L_i^{\mathrm{Re}\,j} = L_i^{Acc} + \varepsilon_i$ $\quad for\ i = 1,2,3,4$ so as $0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ are identified.

According to IFO technique considering membership and non-membership function for MOSOP (4)

$$\mu_{WT(A;\alpha)}\left(WT\left(A;\alpha\right)\right) = \begin{cases} 1 & if\ \ WT\left(A;\alpha\right) \le L_1^{Acc} \\ \dfrac{e^{-T\left(\frac{WT(A;\alpha)-L_1^{Acc}}{U_1^{Acc}-L_1^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_1^{Acc} \le WT\left(A;\alpha\right) \le U_1^{Acc} \\ 0 & if\ \ WT\left(A;\alpha\right) \ge U_1^{Acc} \end{cases} \tag{5}$$

$$\mu_{WT(A;\beta)}\left(WT\left(A;\beta\right)\right) = \begin{cases} 1 & if\ \ WT\left(A;\beta\right) \le L_2^{Acc} \\ \dfrac{e^{-T\left(\frac{WT(A;\beta)-L_2^{Acc}}{U_2^{Acc}-L_2^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_2^{Acc} \le WT\left(A;\beta\right) \le U_2^{Acc} \\ 0 & if\ \ WT\left(A;\beta\right) \ge U_2^{Acc} \end{cases}$$

$$\upsilon_{WT(A;\alpha)}\left(WT\left(A;\alpha\right)\right) = \begin{cases} 0 & if\ \ WT\left(A;\alpha\right) \le L_1^{\mathrm{Re}\,j} \\ \left(\dfrac{WT\left(A;\alpha\right)-L_1^{\mathrm{Re}\,j}}{U_1^{\mathrm{Re}\,j}-L_1^{\mathrm{Re}\,j}}\right)^2 & if\ \ L_1^{\mathrm{Re}\,j} \le WT\left(A;\alpha\right) \le U_1^{\mathrm{Re}\,j} \\ 1 & if\ \ WT\left(A;\alpha\right) \ge U_1^{\mathrm{Re}\,j} \end{cases}$$

$$\upsilon_{WT(A;\beta)}\left(WT\left(A;\beta\right)\right) = \begin{cases} 0 & if\ \ WT\left(A;\beta\right) \le L_2^{\mathrm{Re}\,j} \\ \left(\dfrac{WT\left(A;\beta\right)-L_2^{\mathrm{Re}\,j}}{U_2^{\mathrm{Re}\,j}-L_2^{\mathrm{Re}\,j}}\right)^2 & if\ \ L_2^{\mathrm{Re}\,j} \le WT\left(A;\beta\right) \le U_2^{\mathrm{Re}\,j} \\ 1 & if\ \ WT\left(A;\beta\right) \ge U_2^{\mathrm{Re}\,j} \end{cases}$$

And

$$\mu_{\delta(A;\alpha)}\left(\delta\left(A;\alpha\right)\right) = \begin{cases} 1 & if\ \ \delta\left(A;\alpha\right) \le L_3^{Acc} \\ \dfrac{e^{-T\left(\frac{\delta(A;\alpha)-L_3^{Acc}}{U_3^{Acc}-L_3^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_3^{Acc} \le \delta\left(A;\alpha\right) \le U_3^{Acc} \\ 0 & if\ \ \delta\left(A;\alpha\right) \ge U_3^{Acc} \end{cases}$$

$$\mu_{\delta(A;\beta)}\left(\delta\left(A;\beta\right)\right) = \begin{cases} 1 & if\ \ \delta\left(A;\beta\right) \le L_4^{Acc} \\ \dfrac{e^{-T\left(\frac{\delta(A;\alpha)-L_4^{Acc}}{U_4^{Acc}-L_4^{Acc}}\right)} - e^{-T}}{1-e^{-T}} & if\ \ L_4^{Acc} \le \delta\left(A;\beta\right) \le U_4^{Acc} \\ 0 & if\ \ \delta\left(A;\beta\right) \ge U_4^{Acc} \end{cases}$$

$$\upsilon_{\delta(A;\alpha)}\left(\delta\left(A;\alpha\right)\right) = \begin{cases} 0 & if\ \ \delta\left(A;\alpha\right) \le L_3^{\mathrm{Re}\,j} \\ \left(\dfrac{\delta\left(A;\alpha\right)-L_3^{\mathrm{Re}\,j}}{U_3^{\mathrm{Re}\,j}-L_3^{\mathrm{Re}\,j}}\right)^2 & if\ \ L_3^{\mathrm{Re}\,j} \le \delta\left(A;\alpha\right) \le U_3^{\mathrm{Re}\,j} \\ 1 & if\ \ \delta\left(A;\alpha\right) \ge U_3^{\mathrm{Re}\,j} \end{cases}$$

$$\upsilon_{\delta(A;\beta)}\left(\delta(A;\beta)\right)=\begin{cases} 0 & if \quad \delta(A;\beta)\le L_4^{\mathrm{Re}\,j} \\ \left(\dfrac{\delta(A;\beta)-L_4^{\mathrm{Re}\,j}}{U_4^{\mathrm{Re}\,j}-L_4^{\mathrm{Re}\,j}}\right)^2 & if \quad L_4^{\mathrm{Re}\,j}\le\delta(A;\beta)\le U_4^{\mathrm{Re}\,j} \\ 1 & if \quad \delta(A;\beta)\ge U_4^{\mathrm{Re}\,j} \end{cases}$$

Now using Intuitionistic fuzzy probabilistic operator above problem can be written as

$$Maximize\ \mu_{WT}\left(WT(A;\alpha)\right)\mu_{WT}\left(WT(A;\beta)\right)\mu_{\delta}\left(\delta(A;\alpha)\right)\mu_{\delta}\left(\delta(A;\beta)\right) \qquad (6)$$

$$Minimize\ \left\{1-\upsilon_{WT}\left(WT(A;\alpha)\right)\right\}\left\{1-\upsilon_{WT}\left(WT(A;\beta)\right)\right\}\left\{1-\upsilon_{\delta}\left(\delta(A;\alpha)\right)\right\}\left\{1-\upsilon_{\delta}\left(\delta(A;\beta)\right)\right\}$$

subject to

$$0<\mu_{WT}\left(WT(A;\alpha)\right)<1;\ 0<\mu_{\delta}\left(\delta(A;\alpha)\right)<1;$$

$$0<\upsilon_{WT}\left(WT(A;\alpha)\right)<1;\ 0<\upsilon_{\delta}\left(\delta(A;\alpha)\right)<1;$$

$$0<\mu_{WT}\left(WT(A;\beta)\right)<1;\ 0<\mu_{\delta}\left(\delta(A;\beta)\right)<1;$$

$$0<\upsilon_{WT}\left(WT(A;\beta)\right)<1;\quad 0<\upsilon_{\delta}\left(\delta(A;\beta)\right)<1$$

$$0\le\mu_{WT}\left(WT(A;\alpha)\right)+\upsilon_{WT}\left(WT(A;\alpha)\right)\le1;$$

$$0\le\mu_{\delta}\left(\delta(A;\alpha)\right)+\upsilon_{\delta}\left(\delta(A;\alpha)\right)\le1;$$

$$0\le\mu_{WT}\left(WT(A;\beta)\right)+\upsilon_{WT}\left(WT(A;\beta)\right)\le1;$$

$$0\le\mu_{\delta}\left(\delta(A;\beta)\right)+\upsilon_{\delta}\left(\delta(A;\beta)\right)\le1;$$

$$\sigma(A;\alpha)\le\left([\sigma];\alpha\right)$$

$$\sigma(A;\beta)\le\left([\sigma];\beta\right)$$

$$A^{\min}\le A\le A^{\max}\quad \alpha,\beta\in[0,1]$$

Solve the above crisp model (6) by using appropriate mathematical programming algorithm to get optimal solution of objective function.

6. Numerical Illustration :

If the design objective is to minimize weight of the structure $WT(A_1,A_2)$ and minimize the deflection $\delta(A_1,A_2)$ along x – axis and y – axis at loading point of a statistically loaded three bar planar truss which is subject stress (σ) constraints on each of the truss members,

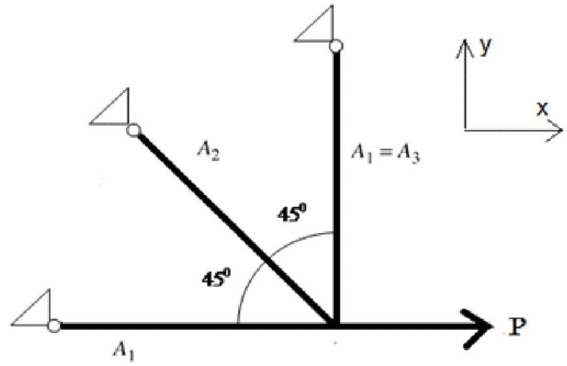

Fig. 1. Design of three bar planar truss

the multi-objective optimization problem can be stated as

$$Minimize\ WT(A_1,A_2)=\rho L(2A_1+A_2) \qquad (7)$$

$$Minimize\ \delta_x(A_1,A_2)=\frac{\rho L(2A_1+A_2)}{E(2A_1^2+2A_1A_2)}$$

$$Minimize\ \delta_y(A_1,A_2)=\frac{\rho LA_2}{E(2A_1^2+2A_1A_2)}$$

such that

$$\sigma_1(A_1,A_2)=\frac{P(2A_1+A_2)}{(2A_1^2+2A_1A_2)}\le\left[\sigma_1^T\right]$$

$$\sigma_2(A_1,A_2)=\frac{P}{\sqrt{2}(A_1+A_2)}\le\left[\sigma_2^T\right]$$

$$\sigma_3(A_1,A_2)=\frac{PA_2}{(2A_1^2+2A_1A_2)}\le\left[\sigma_3^C\right]$$

$$A_i^{\min}\le A_i\le A_i^{\max}\quad i=1,2.$$

Where applied load $\tilde{P}^i=\left((19,20,21;w_p)(18,20,22;\tau_p)\right)$; material density $\rho=100\text{KN/m}^3$; length $L=1m$; Young's modulus $E=2\times10^8$; $A_1=$ Cross section of bar-1 and bar-3; $A_2=$ Cross section of bar-2; δ_x and δ_y are the deflection of loaded joint along x and y axes respectively. $\left[\tilde{\sigma}_1^{Ti}\right]=\left((19.5,20,20.5;w_{\sigma_1^T})(18,20,21;\tau_{\sigma_1^T})\right)$ and $\left[\tilde{\sigma}_2^{Ti}\right]=\left((18.5,20,20.5;w_{\sigma_2^T})(18,20,21;\tau_{\sigma_2^T})\right)$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, $\left[\tilde{\sigma}_3^{Ci}\right]=\left((9,10,11;w_{\sigma_3^C})(8,10,12;\tau_{\sigma_3^C})\right)$ is maximum allowable compressive stress for bar 3 where $w_p=0.8, w_{\sigma_1^T}=0.7, w_{\sigma_2^T}=0.6, w_{\sigma_3^C}=0.9$ are degree of acceptance or aspiration level of applied load, tensile stresses and compressive stress respectively and $\tau_p = 0.2, \tau_{\sigma_1^T} = 0.2, \tau_{\sigma_2^T} = 0.2, \tau_{\sigma_3^C} = 0.1$ are degree of rejection or desperation level of applied load, tensile stresses and compressive stress respectively.

Now total integral value of membership and non-membership function are

$$\hat{P}_1 = 19.625 + 0.75\alpha;\ \hat{P}_2 = 23 - 6\beta;$$
$$\hat{\sigma}_{11}^T = 19.85 + .428\alpha;\ \hat{\sigma}_{21}^T = 23 - 4.5\beta;$$
$$\hat{\sigma}_{12}^T = 19.75 + 0.33\alpha;\ \hat{\sigma}_{22}^T = 23 - 4.5\beta;$$
$$\hat{\sigma}_{13}^C = 9.5 + 0.89\alpha;\ \hat{\sigma}_{23}^C = 18 - 16\beta;$$

Using total integral values of coefficients, problem (7) can be transformed into

$$\text{Minimize } WT(A_1, A_2) = 100(2A_1 + A_2) \tag{8}$$
$$\text{Minimize } \delta_x(A_1, A_2) = \frac{100(2A_1 + A_2)}{2\times 10^8 (2A_1^2 + 2A_1A_2)}$$
$$\text{Minimize } \delta_y(A_1, A_2) = \frac{100A_2}{2\times 10^8 (2A_1^2 + 2A_1A_2)}$$

such that

$$\sigma_{11}(A_1, A_2) = \frac{(19.625 + 0.75\alpha)(2A_1 + A_2)}{(2A_1^2 + 2A_1A_2)} \le 19.85 + .428\alpha;$$
$$\sigma_{21}(A_1, A_2) = \frac{(23 - 6\beta)(2A_1 + A_2)}{(2A_1^2 + 2A_1A_2)} \le 23 - 4.5\beta;$$
$$\sigma_{12}(A_1, A_2) = \frac{(19.625 + 0.75\alpha)}{\sqrt{2}(A_1 + A_2)} \le 19.75 + 0.33\alpha;$$
$$\sigma_{22}(A_1, A_2) = \frac{(23 - 6\beta)}{\sqrt{2}(A_1 + A_2)} \le 23 - 4.5\beta;$$
$$\sigma_{13}(A_1, A_2) = \frac{(19.625 + 0.75\alpha)A_2}{(2A_1^2 + 2A_1A_2)} \le 9.5 + 0.89\alpha;$$
$$\sigma_{23}(A_1, A_2) = \frac{(23 - 6\beta)A_2}{(2A_1^2 + 2A_1A_2)} \le (18 - 16\beta);$$
$$A_i^{\min} \le A_i \le A_i^{\max}\ i = 1, 2.\ \alpha, \beta \in [0, 1]$$

According to step 2 pay-off matrix can be formulated as follows

$$\begin{array}{c} \\ A^1 \\ A^2 \\ A^3 \end{array}
\begin{array}{c}
\begin{array}{ccc} WT(A_1, A_2) & \delta_x(A_1, A_2) & \delta_y(A_1, A_2) \end{array} \\
\begin{bmatrix} 1.983716 & 1.010777 & .0509537 \\ 15 & .15 & .05 \\ 10.1 & .1980392 & .001960784 \end{bmatrix}
\end{array}$$

Here $U_{WT}^{Acc} = 15 = U_{WT}^{\mathrm{Re}j}$, $L_{WT}^{Acc} = 1.983716$, $L_{WT}^{\mathrm{Re}j} = 1.983716 + \varepsilon_{WT}$ with $0 < \varepsilon_{WT} < (15 - 1.983716)$; $U_{\delta_x}^{Acc} = 1.010777 = U_{\delta_x}^{\mathrm{Re}j}$, $L_{\delta_x}^{Acc} = 0.15$, $L_{\delta_x}^{\mathrm{Re}j} = 0.15 + \varepsilon_{\delta_x}$ with $0 < \varepsilon_{\delta_x} < (1.010777 - 0.15)$; and $U_{\delta_y}^{Acc} = .0509537 = U_{\delta_y}^{\mathrm{Re}j}$, $L_{\delta_y}^{Acc} = .001961$, $L_{\delta_y}^{\mathrm{Re}j} = .001961 + \varepsilon_{\delta_y}$ with $0 < \varepsilon_{\delta_y} < (.0509536 - .001961)$;

Here nonlinear membership and non-membership function of objectives $WT(A_1, A_2)$, $\delta_x(A_1, A_2)$ and $\delta_y(A_1, A_2)$ are defined for $T = 2$ as follows

$$\mu_{WT(A_1,A_2)}(WT(A_1, A_2)) = \begin{cases} 1 & \text{if } WT(A_1, A_2) \le 1.983716 \\ \dfrac{e^{-2\left(\frac{WT(A_1,A_2) - 1.983716}{15 - 1.983716}\right)} - e^{-2}}{1 - e^{-2}} & \text{if } 1.983716 \le WT(A_1, A_2) \le 15 \\ 0 & \text{if } WT(A_1, A_2) \ge 15 \end{cases}$$

$$\upsilon_{WT(A_1,A_2)}(WT(A_1, A_2)) = \begin{cases} 0 & \text{if } WT(A_1, A_2) \le 1.983716 + \varepsilon_{WT} \\ \left(\dfrac{WT(A_1, A_2) - (1.983716 + \varepsilon_{WT})}{15 - 1.983716 - \varepsilon_{WT}}\right)^2 & \text{if } 1.983716 + \varepsilon_{WT} \le WT(A_1, A_2) \le 15 \\ 1 & \text{if } WT(A_1, A_2) \ge 15 \end{cases}$$

and

$$\mu_{\delta_x(A_1,A_2)}(\delta_x(A_1, A_2)) = \begin{cases} 1 & \text{if } \delta_x(A_1, A_2) \le 0.15 \\ \dfrac{e^{-2\left(\frac{\delta_x(A_1,A_2) - .15}{1.010777 - 0.15}\right)} - e^{-2}}{1 - e^{-2}} & \text{if } 0.15 \le \delta_x(A_1, A_2) \le 1.010777 \\ 0 & \text{if } \delta_x(A_1, A_2) \ge 1.010777 \end{cases}$$

$$\upsilon_{\delta_x(A_1,A_2)}(\delta_x(A_1, A_2)) = \begin{cases} 0 & \text{if } \delta_x(A_1, A_2) \le 0.15 + \varepsilon_{\delta_x} \\ \left(\dfrac{\delta_x(A_1, A_2) - 0.15 - \varepsilon_{\delta_x}}{1.010777 - 0.15 - \varepsilon_{\delta_x}}\right)^2 & \text{if } 0.15 + \varepsilon_{\delta_x} \le \delta_x(A_1, A_2) \le 1.010777 \\ 1 & \text{if } \delta_x(A_1, A_2) \ge 1.010777 \end{cases}$$

$$\mu_{\delta_y(A_1,A_2)}(\delta_y(A_1, A_2)) = \begin{cases} 1 & \text{if } \delta_y(A_1, A_2) \le 0.001961 \\ \dfrac{e^{-2\left(\frac{\delta_y(A_1,A_2) - .001961}{.0509537 - .001961}\right)} - e^{-2}}{1 - e^{-2}} & \text{if } 0.001961 \le \delta_y(A_1, A_2) \le 0.0509537 \\ 0 & \text{if } \delta_y(A_1, A_2) \ge 0.0509537 \end{cases}$$

$$\upsilon_{\delta_y(A_1,A_2)}(\delta_y(A_1, A_2)) = \begin{cases} 0 & \text{if } \delta_y(A_1, A_2) \le .001961 + \varepsilon_{\delta_y} \\ \left(\dfrac{\delta_y(A_1, A_2) - .001961 - \varepsilon_{\delta_y}}{.0509537 - 0.001961 - \varepsilon_{\delta_y}}\right)^2 & \text{if } .001961 + \varepsilon_{\delta_y} \le \delta_y(A_1, A_2) \le .0509537 \\ 1 & \text{if } \delta_y(A_1, A_2) \ge .0509537 \end{cases}$$

Using Intuitionistic Probabilistic Operator for membership and non-membership function the optimal results of model (7) can be obtained as follows in table 1.

Table 2. Optimal weight and deflection for $\varepsilon_{WT} = 1.3 \; \varepsilon_{\delta_x} = .008 \; \varepsilon_{\delta_y} = .004$

Method	$A_1^* \times 10^{-4} m^2$	$A_2^* \times 10^{-4} m^2$	$WT^* \times 10^2 KN$	$\delta_x^* \times 10^{-7} m$	$\delta_y^* \times 10^{-7} m$
Min-max operator	4.697479	5	14.39496	0.158	.04695370
Probabilistic Operator	4.697474	5	14.39495	0.158	.04502561

From the above table it is clear that the probabilistic operator does not affect too much in results in perspective of structural design optimization in intuitionistic fuzzy environment.

7. Conclusion

In this paper, we have proposed a method to solve multi-objective structural model in intuitionistic fuzzy environment. Here generalized triangular intuitionistic fuzzy number has been considered for applied load and stress parameter. The said model is solved by intuitionistic probabilistic operator and result is compared with max-min operator. A main advantage of the proposed method is that it allows us to overcome the actual limitations in a problem i.e imprecise supplied data during the specification of the flexible objectives. This approximation method can be applied to optimize different models in various fields of engineering and sciences.

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