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# The Batch Service Vacation Model $\left(M/G^{(a,b)}/1\right):(E,SV)$:

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**Abstract**

In this chapter we discuss the batch service vacation model. In such system, customers arrive according to a Poisson process and are served in batches of maximum size b and minimum thre shold a. The server takes a single vacation when it finds less than a customers after the service completion.

*Key words:* Batch Service, Single Vacation, Poisson Process.

## Introduction

**C**onsider the $\left(M/G^{(a,b)}/1\right)$ queue with batch service vacation model. The steady state of the system can be described by the following random variable.

$$\xi = \begin{cases} 0 & \text{If the server is dormant and ready to serves} \\ 1 & \text{If the server is on vacation,} \\ 2 & \text{If the server is busy} \end{cases}$$

$L_s$ = The number of customers Present in the system,

T = The residual service time of the batch in service,

V = The residual vacation time for the server on vacation.

There are difference in the definitions of $\xi$ and T between the Batch service Model and the batch arrival Model. We define

$$\pi_n(x)\ dx = P(L_S = n,\ x < T \le x + dx,\ \xi = 2), \qquad n = 0,1,2...$$

$$w_n(x)\ dx = P(L_S = n,\ x < V \le x + dx,\ \xi = 1), \qquad n = 0,1,2,...$$

$$R_n = P(L_S = n, \xi = 0) \qquad n = 0,1,\ 2......a-1$$

And the LST

$$\pi_n^*(s) = \int_0^\infty e^{-s\chi}\pi_n(x)\ dx,$$

$$w_n^*(s) = \int_0^\infty e^{-s\chi} w_n(x)\ dx$$

It follows from the above that

$$\pi_n^*(0) = \pi_n = \int_0^\infty \pi_n(\chi)\, d\chi \text{ and } w_n^*(o) = w_n = \int_0^\infty w_n(x)\ dx$$

It is clear that $\pi_n(w_n), n \ge 0,$ represent the Prob. of n customers in the queue when the server is busy at arbitrary time instants.

By considering the steady-state transitions, we obtain the following system of differential difference equations.

$$0 = -\lambda R_0 + W_0(0)$$

$$0 = -\lambda R_n + \lambda R_{n-1} + w_n(o), 1 \le n \le a-1 \qquad \text{(A)}$$

$$-\frac{d\pi_0,(x)}{dx} = -\lambda\pi_0(x) + b(x)\sum_{n=a}^{b}\left[\pi_n(o) + w_n(o)\right] + \lambda R_{a-1} b(x), \qquad (1)$$

$$-\frac{d\pi_n(x)}{dx} = -\lambda\pi_n(x) + \lambda_{n-1}(x) + b(x)\left[\pi_{n+b}(o) + w_{n+b}(o)\right], \qquad n \ge 1 \quad (2)$$

$$-\frac{dw_0(x)}{dx} = -\lambda w_0(x) + \pi_0(o)\,\upsilon(x), \quad (3)$$

$$-\frac{dw_n(x)}{dx} = -\lambda w_n(x) + \lambda w_n(x) + \lambda w_{n-1}(x) + \pi_n(o)\,\nu(x),$$

$$1 \le n \le a-1, \quad (4)$$

$$-\frac{dw_n(x)}{dx} = -\lambda w_n(x) + \lambda w_{n-1}(x), \qquad n \ge a \quad (5)$$

Taking the LST on both sides of the above equation (1) to (5).

$$(\lambda - S)\pi_0^*(s) = T^*(s)\sum_{n=a}^{b}\left[\pi_n(o) + w_n(o)\right] + \lambda R_{a-1}T^*(s) - \pi_0(o) \quad (6)$$

$$(\lambda - S)\pi_n^*(s) = \lambda\pi_{n-1}^*(s) + T^*(s)[\pi_{n+b}(o) + w_{n+b}(o)] - \pi_n(o), \quad n \ge 1 \quad (7)$$

$$(\lambda - S)\,w_0^*(s) = v^*(s)\,\pi_0(o) - w_0(o) \quad (8)$$

$$(\lambda - S)w_n^*(s) = \lambda\, w_{n-1}^*(s) + V^*(s)\pi_n(o) - w_n(o), \quad 1 \le n \le a-1 \quad (9)$$

$$(\lambda - S)w_n^*(s) = \lambda w_{n-1}^*(s) - w_n(o), \qquad n \ge a \quad (10)$$

Now using equation (1) & (2) and equation (6) to (10) we obtain a set of Results that the later lead to queue length distribution at various epochs.

*Lemma 1:*- There exist two relations

$$\sum_{n=0}^{j} w_n(o) = \lambda R_j \qquad o \le j \le a-1 \quad (11)$$

$$\sum_{n=0}^{a-1}\pi_n(o) = \sum_{n=0}^{\infty} w_n(o) \quad (12)$$

*Proof:* From equation (A) and letting n=1, we obtain

$$\sum_{n=0}^{1} w_n(o) = \lambda R_1 \qquad n = 0, 1, 2 \ldots\ldots a-1$$

We get equation (11) of lemma 1.

Setting S=0 from equation (6) & (7) we get

$$\pi_0(o) = \sum_{n=a}^{b}\left[\pi_n(o) + w_n(o)\right] + \lambda R_{a-1} - \lambda\pi_0 \quad (13)$$

$$\pi_0(o) = \pi_{n+b}(o) + w_{n+b}(o) + \lambda(\pi_{n-1} - \pi_n) \quad n \ge 1 \quad (14)$$

Summing over n equation (14) adding equation (13) and using equation (11) we obtain equation (12).

Define the non-serving period $D_v^c$ as the sum of vacation V and on Idle time $I_v$, we have the following Lemma.

*Lemma 2:* The Expected value of $D_v^c$ is given by

$$E\,(D_v^c) = E(v) + \frac{1}{\sum_{n=0}^{a-1} P_n^+}\left[\sum_{i=0}^{a-1} P_i^+ \sum_{j=0}^{a-1-i} h_j \frac{(a-i-j)}{\lambda}\right] \quad (15)$$

Where $P_i^+$ is the stationary probability that i customers are left at a departure instant of a batch, and

$$h_j = \int_0^\infty \frac{(\lambda x)^j}{j!} e^{-\lambda x} dv(x)$$

*Proof :* Let N (t) [The number of customers in the system at time (t)] be the state of system at time t. Thus, at the end of busy period, N (t) enters the set of vacation states $S = \{0,1,2,\ldots..a-1\}$. The conditional probability that N (t) enters state $i \in S$, given that N (t) enters, then

$$S = P_i^+ / \sum_{n=0}^{a-1} P_n^+$$

For fixed $i \in S$, if $j \le (a-1-i)$ customers arrive during a vacation with probability $h_j$, then at the vacation completion instant, N(t) enters the let of idle states $U = \{K : k = a - i - j\}$. That N (t) leaves the Set U when a - (i+j) customers arrive. Thus the expected time of N (t) in is in

$$(a - i - j)/\lambda$$

Using the conditional Argument, Then

$$E\,(D_V^C) = E(V) + E\,(I_V)$$

## References


1. Chaudhary, M. and Templeton, J., The queuing system (M/G*/1) and its ramifications.Eur. J. oper. Res. *6,* 56-60 (1981).
2. Chaudhary, M. Madill, B. and Briere, G., Computational analysis of steady-stateprobabilities of (M/G*/1) and related non-bulk queues. Queuing Sys. *2,* 93-114 (1987).
3. Ke, J.C., The Control Policy of an (M/G/1) queuing system with server startup and two vacation types. Math. Method. oper. Res. *54,* 471-490 (2001).
4. Zhang, Z.G. and Tianm N., The N-Threshold for the (GI/M/1) queen, oper. Res. Lelt. *32*, 77-84 (2004).
5. Sikdar, K. and Gupta, U., Analysis and Numerical aspect of batch service queue with single vacation. Comput.Oper. Res. *32,* 943-966 (2005).
6. Zhang, Z.G., On the convexity of the two-threshold Policy for an M/G/1) queue with Vacation. To appear in Oper. Res. Lett (2005).
7. Zhang, Z.G., On the three threshold policy in the Multiserver queuing system with vacation.To appear in Queuing Sys (2006).