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On Coincidence Points in Pseudocompact Tichonov Spaces

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Abstract

In this paper, we are proving a Certain common fixed point theorem for pairs of selfmaps on a Pseudocompact topological space and pairs as well as families of selfmaps on a compact metric space. We improve the results of Fisher[1], Harinath[2], Jain and Dixit[3].

Key word : Pseudocompact tichonov space,coincidence point.

Introduction

Harinath[2] earlier established some fixed point theorems in Pseudocompact tichonov space. Jain and Dixit[3] established also some fixed point theorems in Pseudocompact tinconov spaces, which generalize the results of Fisher[1], Harinath[2], Liu Zeqing[5] and Stephenson, R.M[6], also established some coincidences point theorems in Pseudocompact tichonov space.

Certain common fixed point theorems for pairs of selfmaps on a Pseudocompact topological space and pairs as well as families of selfmaps on a. compact metric space are obtained by S.V.R. Naidu and K.P.R. Rao[4] and also certain sufficient conditions for the existence of coincidence points for at least one pair of four maps (two of which are multi-functions) on a compact metric space are discussed by them.

We define F to be a non-negative real valued function on X xX
such that $F(x,y) = 0 \Leftrightarrow x = y$

A topological space X is said to Pseudocompact iff every real valued continuous function, on X is bounded. By tichonov space we mean a completely regular Hausdorff space it may be noted that compact space is Pseudocompact. if X is an arbitrary tichonov space. Then X is Pseudocompact iff every real valued continuous function over X is bounded and assumed its bounds.

Throughout this paper unless otherwise stated, X stands for a Pseudocompact tichonov space.

Some coincidence point theorems in Pseudocompact tichonov spaces.

Theorem 1.1

Let f, g and h be three self mapping on X, such that

(I) $f(x) \cup g(x) \subseteq h(x)$,

(II) the functions a and b defined on X by a (x) = F (fx, hx) and b(x) = F (hx, gx) are continuous on X and

(III) $F(fx, gy) < \max\left\{ F(hy, hx), F(fx,hx), F(hy,gy), \tfrac{1}{2}[F(fx,hy)+F(hy,hx)], \frac{F(fx,hx).F(hy.gy)}{f(hy,hx)} \right\}$

for all x,y in X and hx ≠ hy, then f and h or g and h have a coincidence point.

Proof:

Since X is a Pseudocompact tichonov space it follows from (II) that there exist two points u and v in X such that

a (u) = inf {a(x): x ∈ X}

and b (v) = inf {b(x): x ∈ X}

we may assume, without loss of generality

a(u) ≤ b (v).

Since f (x) ⊆ h(x), there exists a point w in X, such that

fu = hw.

We now assert that u is coincidence point of f and h. If not, let us suppose that fu ≠ hu i.e. hu ≠ hw. Then using (III) we have

$$F(hw, gw) < \max. \left\{ f(fu,hu), f(fu,hu), f(hw,gw), \tfrac{1}{2}[F(fu,hw) + F(hw,hu)]. \frac{F(fu,hu).F(hw.gw)}{F(hw,hu)} \right\}$$

i.e.

$$F(hw,gw) < \max. \left\{ F(fu,hu), F(fu,hu), f(hw,gw), \tfrac{1}{2}[F(hw,hw) + F(fu,hu)], \frac{F(fu,hu).F(hw.gw)}{F(fu,hu)} \right\}$$

So that b(w) < max {a(u), a(u), b(w), ½ a(u), b(w) },

which implies that b(w) < max {a(u), b(w)} = a(u).

Since a(u) ≤ b(v) = inf {b(x) : x ∈ X} ≤ b (w),

it follows that a(u) ≤ b(w) < a(u),

which is a contradiction and hence fu=hu.

This comples the proof.

As a consequence of Theorem 1.1 we have the following

Theorem 1.2

Let F be continuous, f, g, and h be three continuous self mapping on X satisfying (I) and (III), then f and h or g and h have a coincidence point.

Remark 1.

Theorem 1.2 extends, Theorem 1 and 3 of Harinath[2] and Theorem 2 of Jain and Dixit[3]

As a particular case of Theorem 1.2, we have following:

Corollary 1.3.

Let (X,d) be a compact metric space f,g and h be three continuous self mapping on X satisfying (I) and (III), then f and h or g and h have a coincidence point

Remark 2.

Corollary 1.3 generalizes fisher's Theorem 1 and 2 of [1]

Corollary 1.4

Let f.g and h be three self mapping on X. if in the Theorem 1.1, the condition (III) is replaced by any one of the following conditions (IV), (V), (VI), (VII), and (VIII), then also we arrive at the same conclusion as in Theorem 1.1.

(IV) $F(fx, gy) < \max\{F(hx,hy), F(hx,fx), F(hy,gy), F(hy,fx), [F(hx,hy) F(hx,fx)]^{1/2}\}$

(V) $[f(fx,gy)]^2 < \max\{F(hx,hy). F(hx,fx), F(hy,fx). F(hx,gx), [F(hx,fx)]^2, [F(hy,gy)^2]\}$

(VI) $[F(fx,gy)]^2 < \max\{F(hx,hy). F(hx,fx), F(hy,gy). F(fx,gy), F(hx,fy). F(hy,fx), F(hy,fx)\}$

(VII) $F(fx,gy) < \max\left\{F(hy,hx), F(fx,hx), F(hy,gy), \tfrac{1}{2}[F(fx,hy) + F(hy,hx)], \frac{[F(hy,hx).F(hy.gy) + F(fx,hx).F(fx,gy)]}{2F(hy,hx)}\right\}$

and

(viii) $F(fx,gy) < \max\left\{F(hy,hx), F(fx,hx), F(hy,gy), \tfrac{1}{2}[F(fx,hy) + F(hy,hx)], \frac{[F(hx,hy).F(hy.gy) + F(fx,hx).F(hy,hx)]}{F(hx,gy) + F(hy,gy)}\right\}$

We define a mapping $\emptyset : (R^+)^5 \to R^+$ such that

(i) ϕ is non-decreasing in each coordinate variable

and (ii) $k(t) = \phi(t,t,t,t,t) < t$, for each $t > 0$.

Theorem 1.4

Let f,g and h be three self mappings on X such that

(IX) $f(x) \cup g(x) \subseteq h(x)$,

(X) the functions a and b defined on X by $a(x) = F(fx,hx)$ and $b(x) = F(hx,gx)$ are continuous on X and

(XI) $F(fx,gy) < [\phi\{F^2(hy,hx), F(hx,hy). F(hx,fx), F(fx,fy). F(hy,gy)], F(hy,gy). F(fx,gy), F^2(fx,gy)\}]^{1/2}$

for all x,y in X and $hx \neq hy$, then f and h or g and h have a coincidence point.

Proof:

Proceeding as in the proof of Theorem 1.1, we have

$$
\begin{aligned}
b(w) &< [\phi\{a^2(u), a(u). a(u), a(w). b(w), b(w). b(w), b^2(w)\}]^{1/2} \\
&\leq [\phi\{b^2(w), b^2(w), b^2(w), b^2(w). b^2(w)\}]^{1/2} \\
&= [k(b^2(w))]^{1/2} \\
&< [b^2(w)]^{1/2} \\
&= b(w), \quad \text{Which is a contradiction}
\end{aligned}
$$

Hence f and g or g and h have a coincidence point.

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