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Partial Derivatives Involving Generalized H–Function of two Variables

MEHPHOOJ BEG[1] and S. S. SHRIVASTAVA[2*]

[1]VITS, Satna (M. P.) India

[2]Department of Mathematics Institute for Excellence in Higher Education Bhopal (M.P.) India

Corresponding Author Email:-sss.math@yahoo.co.in

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Abstract

The aim of this paper is to derive partial derivatives involving generalized H–function of two variables.

Key words: H- Function, partial derivatives.

1. Introduction

The generalized H–function of two variables is given by Shrivastava, H. S. P.[3] and defined as follows:

$$H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$

$$=\frac{-1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}d\xi d\eta \tag{1}$$

where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)\prod_{j=1}^{m_1}\Gamma(b_j-\beta_j\xi-B_j\eta)}{\prod_{j=n_1+1}^{p_1}\Gamma(a_j-\alpha_j\xi-A_j\eta)\prod_{j=1}^{q_1}\Gamma(1-b_j+\beta_j\xi+B_j\eta)},$$

$$\theta_2(\xi)=\frac{\prod_{j=1}^{m_2}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_2}\Gamma(1-c_j+\gamma_j\xi)}{\prod_{j=m_2+1}^{q_2}\Gamma(1-d_j+\delta_j\xi)\prod_{j=n_2+1}^{p_2}\Gamma(c_j-\gamma_j\xi)}$$

$$\theta_3(\eta)=\frac{\prod_{j=1}^{m_3}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_3}\Gamma(1-e_j+E_j\eta)}{\prod_{j=m_3+1}^{q_3}\Gamma(1-f_j+F_j\eta)\prod_{j=n_3+1}^{p_3}\Gamma(e_j-E_j\eta)}$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, (i = 1, 2, 3; j = 2, 3). Also, all the A's, α's, B's, β's, γ's, δ's, E's, and F's are assumed to the positive quantities for standardization purpose.

The contour L_1 is in the ξ-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \delta_j\xi)$ (j = 1, ..., m_2) lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ (j = 1, ..., n_2), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The contour L_2 is in the η-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ (j = 1, ..., m_3) lie to the right, and the poles of $(1 - e_j + E_j\eta)$ (j = 1, ..., n_3), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The generalized H-function of two variables given by (1) is convergent if

$$U = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_2}\gamma_j + \sum_{j=1}^{m_2}\delta_j - \sum_{j=n_1+1}^{p_1}\alpha_j - \sum_{j=m_1+1}^{q_1}\beta_j - \sum_{j=n_2+1}^{p_2}\gamma_j - \sum_{j=m_2+1}^{q_2}\delta_j; \tag{2}$$

$$V = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{n_3}E_j + \sum_{j=1}^{m_3}F_j - \sum_{j=n_1+1}^{p_1}A_j - \sum_{j=m_1+1}^{q_1}B_j - \sum_{j=n_3+1}^{p_3}E_j - \sum_{j=m_3+1}^{q_3}F_j, \tag{3}$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ.

2. Result Required :

The following result are required in our present investigation:

From Rainville[1] :

$$z\Gamma(z) = \Gamma(z+1). \tag{4}$$

3. Main Result:

In this paper we will establish the following partial derivatives:

$$\frac{\partial}{\partial x}\left\{H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= x^{-1}H^{m_1,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2+1;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(0,1),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1,1):(f_j,F_j)_{1,q_3}\end{matrix}\right], \tag{5}$$

where| arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

$$\frac{\partial}{\partial y}\left\{H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= y^{-1}H^{m_1,n_1;m_2,n_2;m_3,n_3+1}_{p_1,q_1;p_2,q_2;p_3+1,q_3+1}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(0,1),(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3},(1,1)\end{matrix}\right], \tag{6}$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

$$\frac{\partial^2}{\partial x\partial y}\left\{H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= (xy)^{-1}H^{m_1,n_1;m_2,n_2+1;m_3,n_3+1}_{p_1,q_1;p_2+1,q_2+1;p_3+1,q_3+1}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(0,1),(c_j,\gamma_j)_{1,p_2}:(0,1),(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1,1):(f_j,F_j)_{1,q_3},(1,1)\end{matrix}\right], \tag{7}$$

where| arg x | < ½ Uπ, | arg y | < ½ Vπ, where U and V are given in (2) and (3) respectively.

Proof:

To establish (5), we use for the generalized H-function of two variables Mellin-Barnes types of contour integral as given in (1), on the left-hand side of (5), change the order of integration and derivative (which is justified under the conditions given with (5)), we then obtain

Left-hand side of (5) = $\frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\left[\frac{\partial}{\partial x}x^{\xi}\right]y^{\eta}d\xi d\eta$

$$= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\left[\xi x^{\xi-1}\right]y^{\eta}d\xi d\eta$$

$$= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\left[\frac{\xi\Gamma(\xi)}{\Gamma(\xi)}x^{\xi-1}\right]y^{\eta}d\xi d\eta$$

Now using the result (4) and interpreting the resulting contour integral as the generalized H-function of two variables, we once get the right-hand side of (5). Proceeding on the similar way, the results (6) and (7) can be obtained.

4. Special Cases:

On choosing m_1 = 0main results, we get following partial derivatives in terms of H-function of two variables given in [2, (6.5.9) to (6.5.11), pp. 95]:

$$\frac{\partial}{\partial x}\left\{H^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= x^{-1}H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2+1;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(0,1),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1,1):(f_j,F_j)_{1,q_3}\end{matrix}\right]; \tag{5}$$

$$\frac{\partial}{\partial y}\left\{H^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= y^{-1}H^{0,n_1;m_2,n_2;m_3,n_3+1}_{p_1,q_1;p_2,q_2;p_3+1,q_3+1}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(0,1),(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3},(1,1)\end{matrix}\right]; \tag{6}$$

$$\frac{\partial^2}{\partial x\partial y}\left\{H^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]\right\}$$

$$= (xy)^{-1}H^{0,n_1;m_2,n_2+1;m_3,n_3+1}_{p_1,q_1;p_2+1,q_2+1;p_3+1,q_3+1}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(0,1),(c_j,\gamma_j)_{1,p_2}:(0,1),(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1,1):(f_j,F_j)_{1,q_3},(1,1)\end{matrix}\right], \tag{7}$$

where $|\arg x| < \frac{1}{2}U\pi$, $|\arg y| < \frac{1}{2}V\pi$, where U and V are given as folows respectively:

$$U = -\sum_{j=n_1+1}^{p_1}\alpha_j - \sum_{j=1}^{q_1}\beta_j + \sum_{j=1}^{m_2}\delta_j - \sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_2}\gamma_j - \sum_{j=n_2+1}^{p_2}\gamma_j > 0,$$

$$V = -\sum_{j=n_1+1}^{p_1}A_j - \sum_{j=1}^{q_1}B_j + \sum_{j=1}^{m_3}F_j - \sum_{j=m_3+1}^{q_3}F_j + \sum_{j=1}^{n_3}E_j - \sum_{j=n_3+1}^{p_3}E_j > 0,$$

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