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Properties of k - Centrohermitian and k – Skew Centrohermitian Matrices

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Abstract

The basic concepts and Theorems of k- Centrohermitian, k- Skew Centrohermitian matrices are introduced with examples.

Key words : Hermitian matrix, Centrohermitian, k- Centrohermitian matrix, Skewhermitian matrix Skew Centrohermitian matrix and k-Skew centrohermitian matrix.

AMS CLASSIFICATIONS: 15A09, 15B05, 15B99.

Introduction

The concept of k-Hermitian matrices and was introduced in[1,2,3] Some properties of hermitian matrices given in[5,6,8]. In this paper, our intention is to define k- Centrohermitian matrix, k-Skew Centrohermitian matrix and also we discussed some results on Centrohermitian matrices.

Preliminaries and Notations :

A is centrosymmetric matrix, A^* is called conjugate Transpose of A. Let k be a fixed product of disjoint transposition in S_n and 'K' be the permutation matrix associated with KI. Clearly K satisfies the following properties. $K^2 = I$, $K^T = K$.

Definitions and Theorems :

Definition:1

A Square matrix $A \in C^{nxn}$ is said to be hermitian if $A = A^*$ (ie) $a_{ij} = \overline{a_{ji}}$ $\forall$ ij

Definition:2

A Square matrix $A \in C^{nxn}$ is said to be centrohermitian matrix if the elements of A satisfy the relation $a_{ij} = \overline{a_{n-i+1,n-j+1}}$

Theorem: 1

Let $A \in C^{n \times n}$ is k-centrohermitian matrix then $A = K A^* K$.

Proof:

$K A^* K = K A K$ where $A^* = A$

$= A K K$ where $AK = A^* K$

$= A K^2 = A$

Theorem: 2

Let $A \in C^{n \times n}$ is k-centrohermitian matrix then $A^* = K A K$.

Proof :

$K A K = K A^* K$ where $A = A^*$

$= A^* K K$ where $K A^* = A^* K$

$= A^* K^2 = A^*$

Theorem: 3

If $A \in C^{n \times n}$ is k-centrohermitian matrix then $A + A^*$ is also k- centrohermitian.

Proof :

A matrix $A \in C^{n \times n}$ is k-centrohermitian matrix if $A = K A^* K$.

Since A^* is also is k-centrohermitian matrix then $A^* = K A K$.

To prove $A + A^*$ is also K- centrohermitian matrix

We will show that $A + A^* = K (A + A^*)K$

Now $K (A + A^*)K = K (A + A^*)^* K$

$= K (A^* + (A^*)^*) k$

$= K A^* K + K A K$

$= A + A^*$

Theorem: 4

If $A \in C^{n \times n}$ is k-centrohermitian matrix then AA^* and A^*A is also k- centrohermitian matrix.

Proof :

A matrix $A \in C^{n \times n}$ is k-centrohermitian matrix if $A = K A^* K$.

Since A^* is also is k-centrohermitian matrix then $A^* = K A K$.

We will show that, $AA^* = K (AA^*)^* K$

Now $K (AA^*)^* K = K [A^* (A^*)^*] K$

$= K (A^*A) K$

$= K A^* K \quad K A K.$

$= AA^*$

Similarly $K (A^*A)^* K = A^*A$

Theorem :5

If $A \in C^{n \times n}$ is k-centrohermitian matrix then KA is also k- centrohermitian matrix

Proof :

A matrix $A \in C^{n \times n}$ is k-centrohermitian matrix if $A = K A^* K$

Since A^* is also is k-centrohermitian matrix then $A^* = K A K$.

We will show that, $KA = K (KA)^* K$

Now, $K (KA)^* K = K [K^* A^*] K$

$= K A^*(KK^*)$

$= KA$ where $KA^* = KA$

Theorem:6

If A and B are k-centrohermitian matrices then AB is also k-centrohermitian matrix.

Proof :

Let A and B are k-centrohermitian matrices if $A = K A^* K$ and $B = K B^* K$

Since A^* and B^* are also k-centrohermitian matrices then $A^* = K A K$. and $B^* = K B K$.

To prove AB is k-centrohermitian matrix

We will show that $AB = K (AB)^* K$

Now $K (AB)^* K = K(B^* A^*) K$

$= K [(K B K) (K A K)] K$

$= K^2 B K^2 A K^2$
$= BA.$
$= AB.$

Theorem: 7

If A and B are k-centrohermitian matrices of same order then $\lambda_1 A + \lambda_2 B$ is also k-centrohermitian matrix.

Proof :

Let A and B are k-centrohermitian matrices if $A = K A^* K$ and $B = K B^* K$

Since A^* and B^* are also k-centrohermitian matrices then $A^* = K A K$. and $B^* = K B K$.

To prove $\lambda_1 A + \lambda_2 B$ is k-centrohermitian matrix

We will show that $\lambda_1 A + \lambda_2 B = K(\lambda_1 A + \lambda_2 B)^* K$

Now $K(\lambda_1 A + \lambda_2 B)^* K = K(\lambda_1 A^* + \lambda_2 B^*) K$
$= \lambda_1 (K A^* K) + \lambda_2 (K B^* K)$
$= \lambda_1 A + \lambda_2 B.$

Theorem:8

If A and B are k-centrohermitian matrices of same order then AB + BA is also k-centrohermitian matrix.

Proof :

Let A and B are k-centrohermitian matrices if $A = K A^* K$ and $B = K B^* K$

Since A^* and B^* are also k-centrohermitian matrices then $A^* = K A K$. and $B^* = K B K$.

To prove AB + BA is k-centrohermitian matrix

We will show that $AB + BA = K(AB + BA)^* K$

Now $K(AB + BA)^* K = K[(AB)^* + (BA)^*] K$
$= K(AB)^* K + K(BA)^* K$
$= AB + BA.$

Result

Let A_1 and A_2 are k-centrohermitian matrices for the following conditions are holds

[i] $(A_1{}^* A_2 A_1)$ and $(A_2{}^* A_1 A_2)$ are also k- centrohermitian matrices.

[ii] Adj A_1 also k-centrohermitian matrix.

[iii] $A_1(\text{Adj } A_1)$ is also k-centrohermitian matrix.

Example: 1

Let $A=\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$ and $A^* = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$; $K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(i) $K(A)^* K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix} = A$.

(ii) $K(AA^*) K == \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix} \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = AA^*$.

Definition : 4

A Square matrix $A \in C_{nxn}$ is said to be skew hermitian if $A^* = -A$ (ie) $a_{ij} = -\overline{a_{ji}}$ $\forall i, j$

Definition : 5

A Square matrix $A \in C_{nxn}$ is said to be skew centrohermitian matrix if the elements of A satisfy the relation $a_{ij} = -\overline{a_{n-i+1,n-j+1}}$. (ie) $A^* = -A$

Theorem : 9

Let $A \in C^{n \times n}$ is k- skew centrohermitian matrix then $K A^* K = -A$.

Proof :

$K A^* K = K(-A) K$ where $A^* = -A$
$= -A K^2 = -A$

Theorem : 10

Let $A \in C^{n \times n}$ is k-skew centrohermitian matrix then $K A K = -A^*$

Proof :

$$K A K = K (-A^{*}) K$$
$$= -A^{*} K^{2} = -A^{*}$$

Theorem : 11

If $A \in C^{n \times n}$ is k-skew centrohermitian matrix then KA is also k- skew centrohermitian matrix

Proof :

A matrix $A \in C^{n \times n}$ is k-skew centrohermitian matrix if $K A^{*} K = -A$

Since A^{*} is also is k-skew centrohermitian matrix then $K A K = -A^{*}$

We will show that , $KA = K (KA)^{*} K$

Now, $K (KA)^{*} K = -K [K^{*} A^{*}] K$
$= -K A^{*}(KK^{*})$
$= -K(-A)$
$= KA.$

Theorem : 12

If A and B are k-skew centrohermitian matrices then AB is also k- skew centrohermitian matrix.

Proof :

Let A and B are k- skew centrohermitian matrices if $K A^{*} K = -A$ and $K B^{*} K = -B$

Since A^{*} and B^{*} are also k-skew centrohermitian matrices then $K A K = -A^{*}$. and $K B K = -B^{*}$.

To prove AB is k-skew centrohermitian matrix

We will show that $AB = K (AB)^{*} K$

Now $K (AB)^{*} K = K(B^{*} A^{*}) K$
$= K [(-K B K) (-K A K)] K$
$= K^{2} B K^{2} A K^{2}$
$= BA = AB.$

Theorem : 13

If A and B are k-skew centrohermitian matrices of same order then AB - BA is also k-skew centrohermitian matrix.

Proof :

Let A and B are k-skew centrohermitian matrices if $K A^{*} K = -A$ and $K B^{*} K = -B$

Since A^{*} and B^{*} are also k-skew centrohermitian matrices then $K A K = -A^{*}$. and $K B K = -B^{*}$.

To prove AB - BA is k-skew centrohermitian matrix

We will show that $AB - BA = K (AB - BA)^{*} K$

Now $K (AB - BA)^{*} K = K(AB)^{*} K - K (BA)^{*} K$
$= K (A^{*}B^{*}) K - K (B^{*}A^{*})K$
$= K [(-KAK)(-KBK)] K – K[(-KBK)(-KAK)] K$
$= K^{2} A K^{2} B K^{2} - K^{2} B K^{2} A K^{2}$
$= AB- BA.$

Theorem : 14

If $A \in C^{n \times n}$ is k- skew centrohermitian matrix then $A - A^{*}$ is also k- skew centrohermitian matrix.

Proof :

A matrix $A \in C^{n \times n}$ is k-skew centrohermitian matrix if $K A^{*} K = -A$.

Since A^{*} is also is k-skew centrohermitian matrix then $K A K = -A^{*}$.

To prove $A - A^{*}$ is also K- skew centrohermitian matrix

We will show that $A - A^{*} = K (A - A^{*})K$

Now $K (A - A^{*})K = -K (A + A^{*})^{*} K$
$= -[K A^{*}K - K(A^{*})^{*} K]$
$= -[-A –(-A^{*})]$
$= A - A^{*}$

Result

Let A and B are k-skew centrohermitian matrices for the following conditions are holds

[i] A is k- skew centrohermitian A is also k- skew centrohermitian matrices.

[ii] Adj(A') = (Adj A)' also k- skew centrohermitian matrix.
[iii] Adj (AB) = (Adj B) (Adj A) is also k-skew centrohermitian matrix.

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