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A Note on Transformations of Generalized Burgers Equations

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Abstract

Lie Classical Method is applied to find general transformations for two Generalized Burgers Equations. Also transformations of Generalized Burgers Equation with time-dependent viscosity to another generalized Burgers equation with constant viscosity has been done.

Key wors and Phrases : GBE, Time-dependent viscocity, Lie classical method.

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Introduction

The Burgers equation for $u(x,t)$ is [1]

$$u_t + uu_x = \frac{1}{2}u_{xx}. \tag{1}$$

Hopf [2] and Cole[3] have shown that the second order nonlinear "partial differential equation (PDE)" (1) may be directly mapped to the second order linear PDE

$$\phi_t = \frac{\delta}{2}\phi_{xx}, \tag{2}$$

through the Cole-Hopf transformation

$$u(x,t) = \frac{1}{\phi(x,t)}\frac{\partial\phi(x,t)}{\partial x}. \tag{3}$$

In 1983, Kawamota[4] related a new PDE to the original PDE to which the Lie's Classical Method[5,6,7] is applied by the similarity transformations. After Clarkson and Kruskal have introduced "the direct method" to determine similarity transformations for the NLPDEs, Sachdev and Mayil Vaganan[8] generalized the direct method to relate the solutions of two nonlinear PDEs. Mayil Vaganan and Jeyalakshmi[9] transformed Generalized Burgers equations to the Burgers equation.

Mayil Vaganan and Senthil Kumaran[10] derived the Exact linearization and invariant solutions of the generalized Burgers equation with linear damping and variable viscosity. Mayil Vaganan and Senthil Kumaran[11] also derived Exact linearization and invariant solutions of a generalized Burgers equation with variable viscosity.

Transformations of Generalized Burgers Equations:

We shall demonstrate the successive applications of Lie classical Method results in the determination of general similarity transformation. We shall explain this new concept now. Suppose that we apply Lie Classical Method to a PDE L([u(,t)] = 0 to replace it by ODE $P[f(z)]=0$, where $z=z(x,t)$ is the similarity variable. The similarity transformation may be taken in the form $u(x,t)=U(x,t,f(z))$ Then the ODE $P[f(z)]=0$ is transformed to a PDE $M[v(z,\tau)]=0$ through a generalization of the similarity transformation, namely, $u(x,t)=U(x,t,v(z,\tau))$ where $\tau=\tau u(t)$. The key is that the newly introduced function $\tau(t)$ is to be determined in such a way that the ODE $P[f(z)$] =0 is replaced by a PDE $M[v(z,\tau)]=0$, Note here that we simply replaced $f(z)$ by $v(z,\tau)$.

The crucial step is an another application of Lie Classical Method now to the PDE $M[v(z,\tau)]=0$, and this may yield a more general similarity transformation $v(z,\tau)=V(z,\tau,g(\xi))$, where $\xi=\xi(z,\tau)$ is the similarity variable. To be precise, if either V is more general that U or if ξ is more general than z, then $v(z,\tau)=V(z,\tau,g(\xi))$ is certainly a more general similarity transformation. We study here two GBEs

$$u_t-\delta u_{xx}+(\alpha+1)u^{\alpha}u_x+\frac{ju}{2t}=0,\quad j=1,2,\alpha\in Z^{+} \tag{4}$$

$$u_t-u_{xx}+nu^{n-1}u_x+cu^{p}=0,\quad n\in Z^{+},c>0 \tag{5}$$

Now seek the following group of infinitesimal transformations

$$\begin{aligned} u^{*}&=u+\varepsilon U(t,x,u)+O(\varepsilon^{2})\\ t^{*}&=t+\varepsilon T(t,x,u)+O(\varepsilon^{2})\\ x^{*}&=x+\varepsilon X(t,x,u)+O(\varepsilon^{2}) \end{aligned} \tag{6}$$

under which (4) is invariant. We then have

$$\begin{aligned} &\left[\alpha(\alpha+1)u^{\alpha-1}u_x+\frac{j}{2t}\right]U-\frac{ju}{2t^{2}}T+U_t+(U_u-T_t)u_t\\ &-X_tu_x-X_uu_xu_t+(\alpha+1)u^{\alpha}[U_x+(U_u-X_x)u_x-T_xu_t-X_uu_x^{2}-T_uu_tu_x]\\ &-\delta[U_{xx}+(2U_{xu}-X_{xx})u_x-T_{xx}u_t+(U_{uu}-2X_{xu})u_x^{2}-2T_{xu}u_xu_t\\ &-X_{uu}u_x^{3}-T_{uu}u_x^{2}u_t-2T_xu_{xt}-2T_uu_{xt}u_x]-T_uu_t^{2}\\ &-[U_u-2X_x-3X_uu_x-T_uu_t]\left(u_t+(\alpha+1)u^{\alpha}u_x+\frac{ju}{2t}\right)=0 \end{aligned} \tag{7}$$

where we have replaced the highest derivative term u_{xxx} using (4). Now equating the different powers u_x,u^{0},u_x^{2} and $u_{xt},u_{xt}u_x,u_xu_t$ to zero, we get

$$\alpha(\alpha+1)u^{\alpha-1}U - X_t + (\alpha+1)u^a X_x - \delta(2U_{xu} - X_{xx}) = 0 \tag{8}$$

$$\frac{j}{2t}U - \frac{ju}{2t^2}T + U_t + (\alpha+1)u^\alpha U_x - \delta U_{xx} - \frac{ju}{2t}(U_u - 2X_x) = 0 \tag{9}$$

$$T_t - 2X_x = 0, U_{xx} = T_x = T_u = X_u = 0 \tag{10}$$

Solving equations $(8)-(10)$ we get the infinitesimals as

$$U = A_u, \quad X = -\alpha A_x + b_1, \quad T = -\alpha At \tag{11}$$

where A, b_1 are arbitrary constants and $j\alpha = 1$. The condition $j\alpha$ =1 requires that

$$\alpha = j = 1 \tag{12}$$

In view of (12) the PDE under investigation (4) becomes

$$u_t - \delta u_{xx} + 2uu_x + \frac{u}{2t} = 0 \tag{13}$$

The variant surface condition $dx/X = dt/T = du/U$ to determine the similarity variable and the similarity form of the solution is

$$\frac{du}{Au} = \frac{dx}{\alpha Ax} = \frac{dt}{-2} \tag{14}$$

Integration of (14) yield

$$u = t^{-1/2} f(z), \quad z = xt^{1/2} \tag{15}$$

where z is the similarity variable. Putting (15) in (13) we get the ODE

$$\delta f'' + \frac{1}{2}zf' - 2ff' = 0 \tag{16}$$

$$\delta f_{zz} + \frac{1}{2}zf_z - tf_\tau \tau_t - 2ff_z = 0 \tag{17}$$

Case 1: If $\tau = t$, then equation (16) becomes

$$\delta f_{zz} + \frac{1}{2}zf_z - tf_t - (\alpha+1)f^\alpha f_z = 0 \tag{18}$$

To apply the Lie Classical again to (18) we assume that (18) is invariant under the group of infinitesimal transformations

$$f^* = f + \varepsilon F(t,z,f), \quad t^* = t + \varepsilon T(t,z,f), \quad Z^* = z + \varepsilon Z(t,z,f) \tag{19}$$

The the determining equations are

$$\frac{Z}{2} - \alpha(\alpha+1)f^{\alpha-1}F + tZ_t + \left[\frac{z}{2} - (\alpha+1)f^\alpha\right]Z_z + \delta(2F_{zf} - Z_{zz}) = 0 \tag{20}$$

$$-T + tT_t - 2tZ_z = 0 \tag{21}$$

$$F_{zz} - tF_t + \left(\frac{z}{2} - (\alpha+1)f^\alpha\right)F_z = 0, \quad F_{ff} = 0 \tag{22}$$

$$T_z = T_f = Z_f = 0 \tag{23}$$

Solving equations (20)-(23) we get the infinitesimal as

$$F = 0, \quad Z = b_0 t^{-1/2} \ , \quad T = b_3 t \tag{24}$$

The invariant surface conditions to determine the similarity variables and the similarity form of the solution are

$$\frac{df}{0} = \frac{dz}{b_0 t^{-1/2}} = \frac{dt}{b_3 t} \tag{25}$$

Integration of equations in (25) yields

$$f = G(\xi), \quad \xi = z + \frac{2b_0}{b_3} t^{-1/2}, \tag{26}$$

where ξ is the similarity variable. Putting (26) in (28) we get the following PDE for the similarity function $G(\xi)$

$$\delta G'' + \frac{1}{2}\xi G' - (\alpha+1) G^{\alpha} G' = 0 \tag{27}$$

Case 2: If $\tau = \log t$, then equation (16) becomes

$$\delta f_{zz} + \frac{1}{2} z f_z - f_t (\alpha+1) f^{\alpha} f_z = 0 \tag{28}$$

The determining equations are

$$\frac{Z}{2} - \alpha(\alpha+1) f^{\alpha-1} F + Z_t + \left(\frac{z}{2} - (\alpha+1) f^{\alpha}\right) Z_z + \delta(2F_{zf} - Z_{zz}) = 0 \tag{29}$$

$$T_t - 2Z_z = 0 \tag{30}$$

$$\delta F_{zz} - F + \left(\frac{z}{2} - (\alpha+1) f^{\alpha}\right) f_z = 0, \quad F_{zz} = 0 \tag{31}$$

$$T_z = T_f = Z_f = 0 \tag{32}$$

Solving equations (29) to (35) we get the infinitesimals as

$$F = 0, \quad Z = b_4 e^{-t/2}, \quad T = b_5 \tag{33}$$

The invariant surface conditions to determine the similarity variable and the similarity form of the solution are

$$\frac{df}{0} = \frac{dz}{b_4 e^{-t/2}} = \frac{dt}{b_5} \tag{34}$$

Integration of equations in (25) yields

$$f = G(\xi), \quad \xi = z + \frac{2b_4}{b_5} e^{-t/2} \tag{35}$$

Where ξ is the similarity variable. Putting (35) in (28) we get the following PDE for the similarity function $G(\xi)$:

$$\delta G'' + \frac{1}{2}\xi G' - (\alpha+1) G^{\alpha} G' = 0 \tag{36}$$

It is remarkable that we are able to generalize the similarity variable from (15) to (35) . In fact the similarity transformation of (26) now takes a more general form

$$f = G(\xi), \quad \xi = xt^{-1/2} + \frac{2b_4}{b_5} e^{-t/2} \tag{37}$$

Equations (27) and (36) are same. Now putting the transformation $H = kG^{-\alpha}$ into equation (36) we arrive at the Euler-Painlevé equation

$$HH'' - (1+1/\alpha)H'^2 + \frac{\alpha\xi}{4\delta} HH' - \frac{(\alpha+1)}{\delta} k^{\alpha} H' = 0 \tag{38}$$

Now we apply Lie Classical Method to the Equation

$$u_t - u_{xx} + nu^{n-1}u_x + cu^p = 0 \tag{39}$$

Now we seek the following group of infinitesimal transformations

$$u^* = u + \varepsilon U(t,x,u) + O(\varepsilon^2)$$

$$t^* = t + \varepsilon T(t,x,u) + O(\varepsilon^2)$$

$$x^* = x + \varepsilon X(t,x,u) + O(\varepsilon^2) \tag{40}$$

under which (2.39) is invariant, Then

$$\begin{aligned}
&[n(n-1)u^{n-2}u_x + cpu^{p-1}]U + U_t + (U_u - T_t)u_t - X_t u_x \\
&+ nu^{n-1}[U_x + (U_u - X_x)u_x - T_x u_t - X_u u_x^2 - T_u u_t u_x] - [U_{xx} \\
&+ (2U_{xu} - X_{xx})u_x - T_{xx}u_t + (U_{uu} - 2X_{xu})u_x^2 - 2T_{xu}u_x u_t \\
&- X_{uu}u_x^3 - T_{uu}u_x^2 u_t - 2T_x u_{xt} - 2Tu_{xt}u_x] - X_u u_x u_t - T_u u_t^2 \\
&- [U_u - 2X_x - 3X_u u_x - T_u u_t](u_t + nu^{n^{-1}} u_x + cu^p) = 0
\end{aligned} \tag{41}$$

where we have replaced the highest derivative term u_{xx} using (39) . Now equating the different powers of u_x, u^0, u_t, u_x^2 and $u_{xt}, u_{xt}u_x, u_x u_t$ to zero we get

$$n(n-1)u^{n-2}U - X_t + nu^{n-1}X_x - \alpha(2U_{xu} - X_{xx}) = 0 \tag{42}$$

$$cpu^{n-1}U + U_t + nu^{n-1}U_x - \alpha U_{xx} - cu^p(U_x - 2X_x) = 0 \tag{43}$$

$$T_t - 2X_x = 0, \quad U_{uu} = 0 \tag{44}$$

$$T_x = T_u = X_u = 0 \tag{45}$$

Solving equations $(42) - (45)$ we get the infinitesimal as

$$U = \frac{\alpha u}{\delta}, \quad X = x, \quad T = \frac{t}{\delta} \tag{46}$$

provided that $\alpha = 1/(1-p)$ and $p = 2n-1$. The invariant surface conditions to determine the similarity variables and the similarity form the solutions are

$$\frac{du}{\alpha u/\delta} = \frac{dx}{x} = \frac{dt}{t/\delta} \tag{47}$$

Integration of equation in (47) yields

$$u = t^{\alpha} f(z, \tau(t)), \quad z = xt^{-1/2}, \tag{48}$$

where z and τ are similarity variables. Putting (48) in (39) we get the following PDE for the similarity function $f(z, \tau)$

$$f_{zz} + \frac{1}{2} z f_z - t f_\tau \tau_t - n f^{n-1} f_z - c f^p - \alpha f = 0 \tag{49}$$

Case1: If $\gamma = t$, then equation (49) becomes

$$f_{zz} + \frac{1}{2} z f_z - t f_t - n f^{n^{-1}} f_z - c f^p - \alpha f = 0 \tag{50}$$

Again applying the Lie's Classical method to (50) the following group of infinitesimal transformations

$$f^* = f + \varepsilon F(t, z, f), \quad t^* = t + \varepsilon T(t, z, f), \quad z^* = z + \varepsilon X(t, z, f) \tag{51}$$

under which (50) is invariant. Then the determining equations are

$$\frac{Z}{2} - n(n-1) f^{n-1} F + t Z_t + \left(\frac{z}{2} - n f^{n-1} \right) Z_z + 2 F_{zf} - Z_{zz} = 0 \tag{52}$$

$$-T + t T_t - 2t Z_z = 0 \tag{53}$$

$$F_{zz} - t F_t + \left(\frac{z}{2} - n f^{n-1} \right) F_z - [\alpha + cp f^{n-1}] + (\alpha f + c f^p)(F_f - 2 Z_z) = 0 \tag{54}$$

$$F_{ff} = 0, \quad T_z = T_F = Z_f = 0 \tag{55}$$

Solving equations $(55) - (52)$ we get the infinitesimals as

$$F = 0, \quad Z = b_6 t^{-\frac{1}{2}}, \quad T = b_7 t \tag{56}$$

The variant surface conditions to determine the similarity variables and the similarity form of the solution are

$$\frac{df}{0} = \frac{dz}{b_6 t^{-1/2}} = \frac{dt}{b_7 t} \tag{57}$$

Integration of equations in (57) yields

$$f = G(\xi), \quad \xi = z + \frac{2 b_6}{b_7} t^{-1/2} \tag{58}$$

where ξ is the similarity variable. Putting (58) in (50) we get the following PDE for the similarity function $G(\xi)$

$$G'' + \frac{1}{2}\xi G' - nG^{n-1}G' - cG^{p} - \alpha G = 0 \tag{59}$$

Case 2: If $\tau = \log t$, then equation (85) becomes

$$f_{zz} + \frac{1}{2}zf_z - f_t - nf^{n-1} f_z - cf^{p} - \alpha f = 0 \tag{60}$$

The determining equations, when Lie Classical Method is applied for (60) are

$$\frac{Z}{2} - n(n-1) f^{n-2}F + Z_t + \frac{z}{2}Z_z + 2F_{zf} - Z_{zz} = 0 \tag{61}$$

$$T_t - 2Z_z = 0 \tag{62}$$

$$F_{zz} - F_t + \left(\frac{z}{2} - nf^{n-1}\right)F_z - [\alpha + cpf^{p-1}]F + (\alpha f + cf^{p})(F_f - 2Z_z) = 0 \tag{63}$$

$$F_{zz} = T_z + T_f + Z_f = 0 \tag{64}$$

Solving equations (61) to (64) we get the infinitesimals as

$$F = k_3 e^{-t}u, \quad Z = \frac{1-p}{2}k_3 z e^{-t} + c_0 e^{-t/2}, \quad T = -\frac{k_3}{\alpha}e^{-t} + k_4 \tag{65}$$

The invariant surface conditions to determine the similarity variables and the similarity form of the solution are

$$\frac{df}{k_3 e^{-t}} = \frac{dz}{\frac{(1-p)}{2}k_3 z e^{-t}} = \frac{dt}{-\frac{k_3}{\alpha}e^{-t}} \tag{66}$$

where $k_4 = c_0 = 0$. Integration of equations (66) yields

$$f = e^{-\alpha t}G(\xi), \quad \xi = ze^{t/2} \tag{67}$$

where ξ is the similarity variable. Putting (67) in (60) we get the following PDE for the similarity function $G(\xi)$:

$$G'' - \frac{(1+p)}{2}G^{\frac{(p-1)}{2}}G' - cG^{p} = 0 \tag{68}$$

Now putting the transformation $H = G^{(p-1)/2}$ into equation (68) we arrive at the Euler - Painlevé equation

$$HH'' - \frac{1+p}{p-1}H'^2 - \frac{c(1-p)}{2} + \frac{1+p}{2}H' = 0 \tag{69}$$

Transformations of GBE with time –dependent viscosity:

The Generalized Burgers Equations with time dependent viscosity is given by

$$u_t + u^{n-1} u_x = \frac{1}{n} t^{\frac{2}{n}-1} u_{xx}, \quad n \in Z^{+}, \tag{70}$$

Equation (70) is invariant under a one parameter (ε) group of infinitesimal transformations of the form

$$x^{*} = x + \varepsilon\xi(x,t,u) + O(\varepsilon^2),$$

$$t^{*} = t + \varepsilon\eta(x,t,u) + O(\varepsilon^2),$$
$$u^{*} = u + \varepsilon\phi(x,t,u) + O(\varepsilon^2),$$

then

$$-nu^{-2+n}u_x\phi_1 + n^2u^{-2+n}u_x\phi_1 - nu_x(\xi_1)_t - nu_tu_x(\xi_1)_u - nu^{-1+n}u_x^2(\xi_1)_u$$
$$-nu^{-1+n}u_x(\xi_1)_x - nu_t(\xi_2)_t - nu_t^2(\xi_2)_u - nu^{-1+n}u_tu_x(\xi_2)_u$$
$$-nu^{-1+n}u_t(\xi_2)_x + n(\phi_1)_t + nu_t(\phi_1)_u + nu^{-1+n}u_x(\phi_1)_u + nu^{-1+n}(\phi_1)_x$$
$$+2t^{-1+\frac{2}{n}}u_x(\xi_2)_u u_{x,t} + 2t^{-1+\frac{2}{n}}(\xi_2)_x u_{x,t} + t^{-2+\frac{2}{n}}\xi_2 u_{x,x}$$
$$-\frac{2t^{-2+\frac{2}{n}}\xi_2 u_{x,x}}{n} + 3t^{-1+\frac{2}{n}}u_x(\xi_1)_u u_{x,x} + 2t^{-1+\frac{2}{n}}(\xi_1)_x u_{x,x}$$
$$+t^{-1+\frac{2}{n}}u_t(\xi_2)_u u_{x,x} - t^{-1+\frac{2}{n}}(\phi_1)_u u_{x,x} + t^{-1+\frac{2}{n}}u_x^3(\xi_1)_{u,u}$$
$$+2t^{-1+\frac{2}{n}}u_x^2(\xi_1)_{x,u} + t^{-1+\frac{2}{n}}u_x(\xi_1)_{x,x} + t^{-1+\frac{2}{n}}u_tu_x^2(\xi_2)_{u,u}$$
$$+2t^{-1+\frac{2}{n}}u_tu_x(\xi_2)_{x,u} + t^{-1+\frac{2}{n}}u_t(\xi_2)_{x,x}$$
$$-t^{-1+\frac{2}{n}}u_x^2(\phi_1)_{u,u}$$
$$-2t^{-1+\frac{2}{n}}u_x(\phi_1)_{x,u} - t^{-1+\frac{2}{n}}(\phi_1)_{x,x} = 0. \tag{71}$$

The infinitesimals ξ, η and ϕ must be determined from the following over-determined system of linear partial differential equations which are obtained from (71):

$$\xi_u = 0, \tag{72}$$
$$\eta_u = 0, \tag{73}$$
$$-ntu\eta_t - ntu^n\eta_x + ntu\phi_u + t^{2/n}u\eta_{x,x} = 0, \tag{74}$$
$$-ntu^n\phi + n^2tu^n\phi - ntu^2\xi_t - ntu^{1+n}\xi_x$$
$$+ntu^{1+n}\phi_u + t^{2/n}u^2\xi_{x,x} - 2t^{2/n}u^2\phi_{x,u} = 0, \tag{75}$$
$$ntu\phi_t + ntu^n\phi_x - t^{2/n}u\phi_{x,x} = 0, \tag{76}$$
$$\eta_x = 0, \tag{77}$$
$$2\eta - n\eta - 2nt\xi_x + nt\phi_u = 0, \tag{78}$$

$$\phi_{u,u} = 0. \tag{79}$$

The solution of the system (72)-(79) is

$$\xi = k_1 + k_2 x, \tag{80}$$
$$\eta = k_2 nt, \tag{81}$$
$$\phi = -k_2 u. \tag{82}$$

The invariant surface condition is

$$\xi u_x + \eta u_t = \phi, \tag{83}$$

whose auxiliary equations are

$$\frac{dx}{\xi} = \frac{dt}{\eta} = \frac{du}{\phi}. \tag{84}$$

Substituting (80)-(82), with $k_1 = 0, k_2 = 1$, equations (84) become

$$\frac{dx}{x} = \frac{dt}{nt} = \frac{du}{u}. \tag{85}$$

Integration of the first two ratios in (85) gives

$$z(x,t) = t^{-1/n}\ x. \tag{86}$$

where z is the function of integration, the so called, similarity variable.

The similarity function, denoted here by $f(z)$, is determined from the second and third ratios of (85) again by integration, viz.,

$$u(x,t) = t^{-1/n}\ f(z). \tag{87}$$

Substitution of the similarity transformation (86) and (87) into the GBE (70) results in the nonlinear second order ODE

$$f'' - nf^{n-1} f' + zf' + f = 0, \quad {}' \equiv \frac{d}{dz}. \tag{88}$$

Now we replace $f(z)$ by $F(z,\tau)$ in (87), and consider the mapping

$$u(x,t) = t^{-1/n}\ F(z,\tau), \quad \tau = \tau(t). \tag{89}$$

The GBE (70) with variable viscosity, via the mapping (89) and(87), changes to another nonlinear partial differential equation (NLPDE)

$$F_{zz} - nF^{n-1} F_z + zF_z + F = nc\tau\ F_\tau, \tag{90}$$

where we have assumed that the function $\tau(t)$ is a solution of the first order ODE

$$t\frac{d\tau}{dt} = c\,\tau. \tag{91}$$

Solving (91), we write τ as (after setting the constant of integration 0)

$$\tau(t) = t^c. \tag{92}$$

Thus the mapping that transforms the GBE (70) to yet another GBE (90) is given by (86), (89) and (92), viz.,

$$u(x,t) = t^{-1/n}\ F(z,\tau), \quad z(x,t) = t^{-1/n}\ x, \quad \tau(t) = t^c. \tag{93}$$

We remark that the NLPDE may be rewritten as a compatibility condition

$$\frac{d}{dz}G_\tau - \frac{d}{d\tau}G_z \equiv 0, \tag{94}$$

Where $G_z = F,$ (95)

$$nc\tau G_\tau = F_z - F^n + zF. \tag{96}$$

We may interpret (95)-(96) as a Bäcklund transformations relating the solutions of the F -equation (90) and the PDE satisfied by $G(z,\tau)$, viz.,

$$G_{zz} - G_z^n + zG_z = nc\tau G_\tau. \tag{97}$$

If (95) is used to replace F in (93), then thus the mapping that transforms the GBE (70) to yet another GBE (90) is given by (86), (89) and (92), viz.,

$$u(x,t) = t^{-1/n}\ G_z(z,\tau), \quad z(x,t) = t^{-1/n}\ x, \quad \tau(t) = t^c. \tag{98}$$

The governing equation of G is again given by

$$G_{zz} - G_z^m + zG_z = nc\tau\ F_\tau \tag{99}$$

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