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Abstract

The aim of this paper is to introduce several continuous functions via gpc-closed sets in topological spaces namely gpc-continuous function, (gpc)*-continuous functions, minimal gpc-continuous function, maximal gpc-continuous function and study their properties. Furthermore, a decomposition of continuity is obtained.

Key words: Paracontinuous functions, *-paracontinuous functions, gpc-continuous functions, (gpc)*-continuous functions, minimal gpc-continuous function and maximal gpc-continuous function.

MSC: 54C08, 54C05

1 Introduction

Continuous functions in topology found a valuable place in the applications of Mathematics as it has applications to engineering especially to digital signal processing and neural networks. Many researchers have introduced and studied various types of generalization of continuity. In 2001 and 2003, F. Nakaoka and N. Oda^{6,7} introduced and studied minimal open sets and maximal open sets in topology. S.S. Benchalli and Basavaraj M. Ittanagi^{3,2} introduced and studied minimal and maximal open sets and paraopen sets in topological space. Recently, Santhini *et al.* introduced gpc-closed set⁸ via paraopen sets in topology.

The purpose of this paper is to introduce a new class of generalized continuous functions called gpc-continuous function, (gpc)*-continuous function, minimal gpc-continuous function and maximal gpc-continuous function via gpc-closed sets and to elaborate on their basic properties and discuss the interrelations with other variants of continuity. Finally, a decomposition of continuity is obtained.

2 Preliminaries :

Let us recall the following definitions which are used in our paper.

Definition: 2.1. A subset A of a topological space (X, τ) is said to be,

1. semi open [4] if $A \subseteq \text{cl}(\text{int}(A))$.
2. pre open [5] if $A \subseteq \text{int}(\text{cl}(A))$.

3. a g -closed set [1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
4. ω -closed set [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .
5. a g^* -closed set [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .

Definition: 2.2. Let X and Y be topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called,

1. semi-continuous [4] if $f^{-1}(V)$ is semi-closed in X for every closed set V in Y .
2. pre-continuous [5] if $f^{-1}(V)$ is pre-closed in X for every closed set V in Y .
3. g -continuous [1] if $f^{-1}(V)$ is g -closed in X for every closed set V in Y .
4. ω -continuous [9] if $f^{-1}(V)$ is ω -closed in X for every closed set V in Y .
5. g^* -continuous [10] if $f^{-1}(V)$ is closed in X for every g -closed set V in Y .
6. paracotinuuous [2] if $f^{-1}(V)$ is closed in X for every paraclosed set V in Y .
7. $*$ -paracontinuous [2] if $f^{-1}(V)$ is paraclosed in X for every closed set V in Y .
8. parairresolute [2] if $f^{-1}(V)$ is paraclosed in X for every paraclosed set V in Y .
9. minimal paracontinuous [2] if $f^{-1}(V)$ is paraclosed in X for every minimal closed set V in Y .
10. maximal paracontinuous [2] if $f^{-1}(V)$ is paraclosed in X for every maximal closed set V in Y .
11. minimal continuous [3] if $f^{-1}(V)$ is minimal closed in X for every closed set V in Y .
12. maximal continuous [3] if $f^{-1}(V)$ is maximal closed in X for every closed set V in Y .

*Definition: 2.3.*³ A topological space (X, τ) is said to be T_{\min} -space if every nonempty proper open subset of X is minimal open set.

*Definition: 2.4.*³ A topological space (X, τ) is said to be T_{\max} -space if every nonempty proper open subset of X is maximal open set.

*Definition: 2.5.*⁸ A subset A of a topological space (X, τ) is said to be gpc -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is paraopen in X .

The complement of gpc -closed set is gpc -open set. The family of all gpc -open subsets of a space (X, τ) is denoted by $gpc-O(X)$.

*Definition: 2.6.*⁸ A subset A of a space (X, τ) is called gpc^* -set if $A = M \cap N$ where M is paraopen and N is closed in X .

3 gpc -continuous and $(gpc)^*$ -continuous function :

This section is devoted to the introduction and study of new notions such as gpc -continuous and $(gpc)^*$ -continuous functions.

Definition: 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be gpc -continuous if $f^{-1}(V)$ is gpc -closed in X for every closed set V in Y .

Definition: 3.2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $(gpc)^*$ -continuous if $f^{-1}(V)$ is gpc -closed in X for every paraclosed set V in Y .

Theorem: 3.3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is gpc -continuous function if and only if $f^{-1}(V)$ is gpc -open in X for every open set V in Y .

Proof. Let V be a open set in Y . Then V^c is a closed set in Y . Since f is gpc -continuous, $f^{-1}(V)$ is gpc -closed in X . Now $f^{-1}(V^c) = (f^{-1}(V))^c$ implies $f^{-1}(V)$ is gpc -open in X .

Conversely, let F be closed in Y . Then F^c is open in Y . By our assumption $f^{-1}(F^c)$ is gpc -open in X . Now $f^{-1}(F) = (f^{-1}(F^c))^c$ implies $f^{-1}(F)$ is gpc -closed in X . Hence f is gpc -continuous.

Theorem: 3.4. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following are equivalent. Assume that $gpc-O(X)$ is closed under any union.

1. f is gpc -continuous
2. For $x \in X$ and each open set V in Y with $f(x) \in V$ there exists a gpc -open set U in X such that $x \in U$ and $f(U) \subseteq V$.

Proof. (i) \Rightarrow (ii) Let $x \in X$ and V be any open set in Y with $f(x) \in V$. By (i) $f^{-1}(V)$ is gpc -open set containing x . Take $U = f^{-1}(V)$ then $x \in f^{-1}(V)$ and $f(U) \subseteq V$.

(ii) \Rightarrow (i) Let V be an open set in Y and let $x \in f^{-1}(V)$. By (ii) there exists a gpc-openset U_x in X containing x such that $f(U_x) \subset V$ which implies $U_x \subset f^{-1}(V)$. Therefore

$f^{-1}(V) = \cup \{U_x : x \in f^{-1}(V)\}$. Since U_x is gpc-open and gpc- $O(X)$ is closed under any union, f is gpc-continuous.

Theorem: 3.5. Every gpc-continuous function is (gpc)*-continuous but not conversely.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be gpc-continuous and let V be any paraclosed set in Y . Since every paraclosed set is closed², V is closed in Y . Since f is gpc-continuous, $f^{-1}(V)$ is gpc-closed in X . Therefore f is (gpc)*-continuous.

Example: 3.6. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is (gpc)*-continuous but not gpc-continuous.

Theorem: 3.7.

1. Every continuous function is gpc-continuous.
2. Every g-continuous function is gpc-continuous.
3. Every ω -continuous function is gpc-continuous.
4. Every g^* -continuous function is gpc-continuous.
5. Every *-paracontinuous function is gpc-continuous.

Proof.

1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and let V be any closed set in Y . Since f is continuous, $f^{-1}(V)$ is closed in X . By theorem 3.3⁸, $f^{-1}(V)$ is gpc-closed in X . Therefore f is gpc-continuous.

(2)-(5) Similar to the proof of (1).

Remark : 3.8. The converses of the above theorem are not true as seen from the following examples.

Example: 3.9.

1. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=c, f(c)=d$ and $f(d)=a$. Then f is gpc-continuous but not continuous.
2. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=b, f(c)=d$ and $f(d)=a$. Then f is gpc-continuous but not g-continuous.
3. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=d, f(c)=a$ and $f(d)=c$. Then f is gpc-continuous but not ω -continuous.
4. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a, f(b)=c, f(c)=b$ and $f(d)=d$. Then f is gpc-continuous but not g^* -continuous.
5. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=c, f(c)=d$ and $f(d)=a$. Then f is gpc-continuous but not *-paracontinuous.

Theorem: 3.10.

1. Every continuous function is (gpc)*-continuous.
2. Every g-continuous function is (gpc)*-continuous.
3. Every ω -continuous function is (gpc)*-continuous.
4. Every g^* -continuous function is (gpc)*-continuous.
5. Every *-paracontinuous function is (gpc)*-continuous.
6. Every paracontinuous function is (gpc)*-continuous.

Proof. By theorem 3.5 and by theorem 3.10, proofs follows.

Remark: 3.11. The converses of the above theorem are not true as seen from the following examples.

Example: 3.12.

1. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=c, f(c)=d$ and $f(d)=a$. Then f is (gpc)*-continuous but not continuous.
2. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow$

(Y, σ) by $f(a)=a, f(b)=a, f(c)=b$ and $f(d)=c$. Then f is

$(gpc)^*$ -continuous but not g -continuous.

3. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=d, f(c)=a$ and $f(d)=c$. Then f is

$(gpc)^*$ -continuous but not ω -continuous.

4. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a, f(b)=c, f(c)=b$ and $f(d)=d$. Then f is

$(gpc)^*$ -continuous but not g^* -continuous.

5. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=c, f(c)=d$ and $f(d)=a$. Then f is $(gpc)^*$ -continuous but not $*$ -paracontinuous.

6. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $(gpc)^*$ -continuous but not paracontinuous.

Remark: 3.13. The following table shows the relationships between gpc -continuous and $(gpc)^*$ -continuous functions and other known existing continuous functions. The symbol

"1" in a cell means that a map implies the other maps and symbol "0" means that a map does not imply the other maps.

functions	g	ω	g^*	Para	$*$ -para	gpc	$(gpc)^*$
g	1	0	0	0	0	1	1
ω	1	1	0	0	0	1	1
g^*	1	1	1	1	0	1	1
Para	0	0	0	1	0	0	1
$*$ -para	1	1	0	1	1	1	1
gpc	0	0	0	0	0	1	1
$(gpc)^*$	0	0	0	0	0	0	1

Remark: 3.14. gpc -continuous and semi-continuous, pre-continuous functions are independent of each other.

Example: 3.15. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=c, f(b)=a, f(c)=a$ and $f(d)=b$. Then

f is semi-continuous but not gpc -continuous and the function $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b, f(b)=c, f(c)=a$ and $f(d)=d$. Then f is gpc -continuous but not semi-continuous

Example: 3.16. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b\}, \{b, c, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is gpc -continuous but not pre-continuous and the function $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=d, f(b)=a, f(c)=b$ and $f(d)=c$. Then f is pre-continuous but not gpc -continuous.

Remark : 3.17. Composition of gpc -continuous functions need not be gpc -continuous.

Example: 3.18. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$, $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$ and

$\eta = \{\emptyset, Z, \{c\}, \{d\}, \{c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by

$f(a)=d, f(b)=c, f(c)=a$ and $f(d)=b$ and Define $g : (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a)=d, g(b)=c, g(c)=b, g(d)=a$. Then f and g are gpc -continuous but $g \circ f$ not gpc -continuous.

Theorem: 3.19. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

1. $g \circ f$ is gpc -continuous if f is $(gpc)^*$ -continuous and g is $_$ -paracontinuous.
2. $g \circ f$ is gpc -continuous if f is gpc -continuous and g is continuous.
3. $g \circ f$ is $(gpc)^*$ -continuous if f is gpc -continuous and g is paracontinuous.

Proof.

1. Let V be any closed set in Z . Since g is $*$ -paracontinuous, $g^{-1}(V)$ is paraclosed set in Y . Since f is $(gpc)^*$ -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpc -closed set in X . Consequently, $g \circ f$ is gpc -continuous.

2. Let V be any closed set in Z . Since g is continuous, $g^{-1}(V)$ is closed set in Y . Since f is gpc -continuous, $f^{-1}(g^{-1}(V)) =$

$(g \circ f)^{-1}(V)$ is gpc-closed set in X . Therefore $g \circ f$ is gpc-continuous.

3. Let V be any paraclosed set in Z . Since g is paracontinuous, $g^{-1}(V)$ is closed set in Y . Since f is gpc-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpc-closed set in X . Therefore $g \circ f$ is (gpc)*-continuous.

Theorem: 3.20. Let X and Y be topological spaces and A be a nonempty subset of X .

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is gpc-continuous then the restriction map $f_A : (A, \tau) \rightarrow (Y, \sigma)$ is gpc-continuous.

Proof. Let U be any closed set in Y . Since f is gpc-continuous, $f^{-1}(U)$ is gpc-closed in X . Now $(f|_A)^{-1}(U) = A \cap f^{-1}(U)$, and so $A \cap f^{-1}(U)$ is gpc-closed in A . Therefore f_A is a gpc-continuous.

4 gpc-irresolute function

Definition: 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be gpc-irresolute if $f^{-1}(V)$ is gpc-closed in X for every gpc-closed set V in Y .

Theorem: 4.2.

1. Every gpc-irresolute function is gpc-continuous.
2. Every gpc-irresolute function is (gpc)*-continuous.
3. Every parairresolute function is (gpc)*-continuous.

Proof.

1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be gpc-irresolute and let V be any closed set in Y . By theorem 3.3⁸, V is gpc-closed in Y . Since f is gpc-irresolute, $f^{-1}(V)$ is gpc-closed in X . Therefore f is gpc-continuous.
2. By theorem 3.5, proof follows.
3. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be parairresolute and let V be any paraclosed set in Y . Since f is parairresolute, $f^{-1}(V)$ is paraclosed in X . By theorem 3.3⁸, $f^{-1}(V)$ is gpc-closed in X . Therefore f is (gpc)*-continuous.

Remark: 4.3. The converses of the above theorem are not true as seen from the following examples.

Example: 4.4.

Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{d\}, \{b,d\}, \{c, d\}, \{b, c, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=b, f(b)=d, f(c)=c$ and $f(d)=a$. Then f is gpc-continuous and (gpc)*-continuous but not gpc-irresolute and parairresolute.

Theorem: 4.5. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

1. $g \circ f$ is (gpc)*-continuous if f is gpc-irresolute and g is paracontinuous.
2. $g \circ f$ is gpc-continuous if f is gpc-irresolute and g is ω -continuous.
3. $g \circ f$ is gpc-irresolute if f is gpc-irresolute and g is gpc-irresolute.

Proof.

1. Let V be any paraclosed set in Z . Since g is paracontinuous, $g^{-1}(V)$ is closed set in Y . By theorem 3.3⁸, $g^{-1}(V)$ is gpc-closed set in Y . Since f is gpc-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpc-closed set in X . Therefore $g \circ f$ is (gpc)*-continuous.
2. Let V be any closed set in Z . Since g is ω -continuous, $g^{-1}(V)$ is ω -closed set in Y . By theorem 3.3⁸, $g^{-1}(V)$ is gpc-closed in Y . Since f is gpc-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpc-closed set in X . Therefore $g \circ f$ is gpc-continuous.
3. Let V be any gpc-closed set in Z . Since g is gpc-irresolute, $g^{-1}(V)$ is gpc-closed set in Y . Since f is gpc-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpc-closed set in X . Therefore $g \circ f$ is gpc-irresolute.

5 Minimal (Maximal) gpc-continuous function :

In this section, the notion of minimal gpc-continuous function and maximal gpc-continuous function are defined and some of their basic properties are studied.

Definition: 5.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be minimal (maximal) gpc-continuous if $f^{-1}(M)$ is gpc-closed set in X for every minimal (maximal) closed set M in Y .

Theorem: 5.2.

1. Every continuous function is minimal (maximal) gpc-continuous.
2. Every minimal (maximal) continuous function is minimal (maximal) gpc-continuous.
3. Every minimal (maximal) paracontinuous function is minimal (maximal) gpc-continuous.
4. Every *-paracontinuous function is minimal (maximal) gpc-continuous.

5. Every gpc-continuous function is minimal (maximal) gpc-continuous.

Proof.

1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and let V be any minimal (maximal) closed set in Y . Since every minimal (maximal) closed set is closed², V is a closed set in Y .

Since f is continuous, $f^{-1}(V)$ is closed in X . By theorem 3.3⁸, $f^{-1}(V)$ is gpc-closed in X . Therefore f is minimal (maximal) gpc-continuous.

(2)-(5) Similar to the proof of (1)

Remark: 5.3. The converses of the above theorem are not true as seen from the following examples.

Example: 5.4.

1. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is minimal (maximal) gpc-continuous but not continuous.

2. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$

and $\sigma = \{\emptyset, Y, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a, f(b)=c, f(c)=b$ and $f(d)=a$. Then f is minimal (maximal) gpc-continuous but not minimal (maximal) continuous.

3. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, b, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by

$f(a)=b, f(b)=d, f(c)=c$ and $f(d)=a$. Then f is minimal (maximal) gpc-continuous but not minimal (maximal) paracontinuous.

4. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is minimal (maximal) gpc-continuous but not *-paracontinuous.

5. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a)=a, f(b)=a, f(c)=c$ and $f(d)=d$. Then f is minimal (maximal) gpc-continuous but not gpc-continuous.

Theorem: 5.5. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

1. $g \circ f$ is minimal (maximal) gpc-continuous if f is paracontinuous and g is minimal (maximal) paracontinuous

2. $g \circ f$ is minimal (maximal) gpc-continuous if f is pairresolute and g is minimal (maximal) paracontinuous.

Proof.

1. Let V be minimal (maximal) closed set in Z . Since g is minimal (maximal) paracontinuous, $g^{-1}(V)$ is paraclosed in Y . Since f is paracontinuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is closed in X . By theorem 3.3⁸, $(g \circ f)^{-1}(V)$ is gpc-closed in X . Therefore $g \circ f$ is minimal (maximal) gpc-continuous.

2. Let V be a minimal (maximal) closed set in Z . Since g is minimal (maximal) paracontinuous, $g^{-1}(V)$ is paraclosed in Y . Since f is pairresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is paraclosed in X . By theorem 3.3⁸, $(g \circ f)^{-1}(V)$ is gpc-closed in X . Therefore $g \circ f$ is minimal (maximal) gpc-continuous.

Theorem: 5.6. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is minimal (maximal) gpc-continuous and Y is a T_{\min} -space (T_{\max} -space) and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is minimal (maximal) paracontinuous then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is minimal (maximal) gpc-continuous.

Proof. Let V be any minimal (maximal) closed set in Z . Since g is minimal (maximal) paracontinuous, $g^{-1}(V)$ is paraclosed in Y . Since every paraclosed set is closed², $g^{-1}(V)$ is closed in Y . Since Y is a T_{\min} -space (T_{\max} -space), $g^{-1}(V)$ is minimal (maximal) closed in Y . Since f is minimal (maximal) gpc-continuous, $(g \circ f)^{-1}(V)$ is gpc-closed in X and so $g \circ f$ is minimal (maximal) gpc-continuous.

6 Decomposition of Continuity :

In this section we establish the notion of gpc**-continuity and its properties.

Definition: 6.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be gpc**-continuous if $f^{-1}(V)$ is a gpc*-set in X for every closed set V in Y .

Theorem: 6.2. Every continuous function is gpc**-continuous.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and let V be any closed set in Y . Since f is continuous, $f^{-1}(V)$ is closed in X , and by theorem 6.3⁸, $f^{-1}(V)$ is a gpc*-set in X . Therefore f is gpc**-continuous.

Remark: 6.3. The converses of the above theorem are not true as seen from the following examples.

Example: 6.4. Let $X=Y=\{a,b,c,d\}$ with $\tau = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, c, d\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is gpc**-continuous but not continuous.

Theorem: 6.5. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following are equivalent.

- (i) f is continuous.
- (ii) f is gpc-continuous and gpc**-continuous.

Proof: (i) \Rightarrow (ii) By theorem 3.7 and by theorem 6.2 (ii) holds.

(ii) \Rightarrow (i) Suppose that f is both gpc-continuous and gpc**-continuous. Let V be any closedset in Y . Then $f^{-1}(V)$ is both gpc-closed and gpc*-set. By theorem 6.9⁸, $f^{-1}(V)$ is closed in X . Therefore f is continuous.

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