

## Study of Teacher Student Relationship Using New Average Fuzzy Relational Maps (NAFRMs) Models

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### Abstract

In this paper the influence of a teacher over the students studying in college in Tamil Nadu is analyzed. The class must be made learner centric and the teacher must be devoted to the profession to make the students a better educated generation. Here Fuzzy Relational Maps and the New Average Fuzzy Relational Maps are used for this study. The study is made by a pilot survey. Conclusions based on our study is given using comparative tables.

*Key words:* Fuzzy Relational Maps (FRMs) model, New Average Fuzzy Relational Maps (NAFRMs) model, Hidden pattern, Fixed point.

### 1 Introduction

In this paper a new model constructed in<sup>9</sup> is implemented in the problem. The study of teacher student congenial relation is very vital for that alone can motivate the students to learn and develop interest in the subject. But in these days college teachers do not motivate the students well and also students do not have the patience and mind to learn. Several attributes which can not be measured by numbers are associated with this problem. There are intense feelings; emotional or otherwise so at the outset we are justified in using this new model. Fuzzy relational Maps (FRMs) model was first introduced in<sup>5</sup>. This model is best suited when

the attributes related with the problem can be divided into two disjoint sets. The implementation of this model is it save time and it also gives in addition the effect of attributes of one space over the other space. We have used the new model called New Average Fuzzy Relational Maps (NAFRMs) model<sup>9</sup> to analyze the problem.

This paper has four sections. Section one is introductory in nature. In section two description of the problem is given. Section three studies the problem using NAFRMs. The final section uses the innovative technique of comparison table to analyze the problem.

## 2 Description of the Problem :

Here we briefly describe the problem. For this study, a pilot survey is taken from 40 college students and 15 college teachers. The data is analysed, the attributes are defined by these experts which forms this section.

Since this is a problem involving both college teachers and students, the domain space and the range space are disjoint. So, we are justified in using the Fuzzy Relational Maps (FRMs) model in analyzing the problem. Further the use of FRMs model is justified, as the data is only an unsupervised one<sup>5-7</sup>.

The attributes associated with the problem are described in the following. Consider the attributes of the teachers and students which is described in a line or two. Attributes of the teachers are described by the six nodes  $T_1, T_2, T_3, T_4, T_5$  and  $T_6$  and the attributes associated with the students are described by the nodes  $S_1, S_2, S_3, S_4$  and  $S_5$ .

Description of the attributes associated with teachers is as follows:

- $T_1$  - Motivates the students: Motivating ability is the first and the foremost attribute of the teacher. If only the teacher motivates the students in the proper manner, they develop interest in studies and also perform well.
- $T_2$  - Kind and approachable: Kindness is the master key that can open the heart of all in general and students in particular. This kindness makes the students to contact the teacher easily and makes the teacher approachable.
- $T_3$  - Punctual to the class: If the teacher is in

time to class, she / he becomes their role model and the students will be forced to follow punctuality.

- $T_4$  - Teacher should be serious and make the class interesting: The teacher should make the subject more interesting, which in turn makes the students interested in studies in general.
- $T_5$  - Take interest in students: Teacher should take special interest in the students which can develop their over all personality.
- $T_6$  - Rudeness of teacher: If the teacher is rude, certainly it can hinder and ruin the personality of students and they may loose interest in studies.

Description of the attributes associated with students are as follows:

- $S_1$  - Good and hard working students: A hard working student will certainly achieve success in life.
- $S_2$  - Regular to class: Unless the students are regular to class, they will not be in a position to be good in studies or proper in general in any walk of life.
- $S_3$  - Irregular to class: Absence of regularity and punctuality on the part of a student will surely make him/her poor in his or her studies and useless in life.
- $S_4$  - Does not perform well in studies: This may be due to teachers or problems in the family, or due to bad company and so on.
- $S_5$  - Interested in studies: Students will become interested in studies, if the teacher is kind, approachable, serious and punctual, together with the proper atmosphere to study is created in the class room.

However FRM model given by six experts who include two educationalists, two professors and two PG students is presented in this section. Finally New Average Fuzzy Relational Maps model is used to give equal importance to every expert and save time.

A large number of experts have to be involved to make such studies to be accurate. Moreover, it is essential to make each expert feel that his/her opinion about the problem has been given equal importance in the study. As this cannot be achieved by considering individual opinions, so the average technique is used. In addition to this, the average model helps in saving time and economy also<sup>1-4</sup>.

**3 The Implementation of the new Average Fuzzy Relational Maps (NAFRMs) Model to this Problem :**

In this section we use the six experts to form the NAFRMs model to analyse the problem.

Here the first experts opinion is given. He has taken the teachers attributes as the domain attributes D and that of students as the range attributes R.

The relational directed graph given by the expert who is an educationalist, has over 20 years of experience in the field of education is given in Figure 3.1 which is as follows:

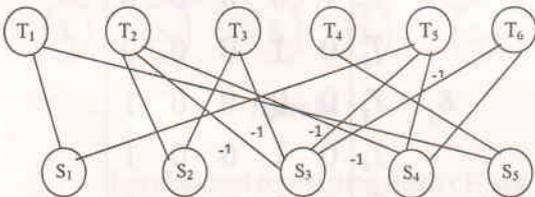


Figure 3.1

The relational connection matrix  $E_1$  got from the above directed graph is as follows:

$$E_1 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Suppose the expert wishes to study the state vector  $x_1 = (1\ 0\ 0\ 0\ 0)$ , that is the on state of the node 'good and hardworking students'  $S_1$  alone in the on state and all other nodes in the off state.

The effect of  $x_1$  on the dynamical system is

$$x_1 E_1^T = (1\ 0\ 0\ 0\ 1\ 0) = y_1 \in D$$

$y_1 E_1 = (2\ 0\ -1\ -1\ 1) \rightarrow (1\ 0\ 0\ 0\ 1) = x_2 \in R$  ( $\rightarrow$  denotes the resultant vector has been updated and thresholded.)

$$x_2 E_1^T = (2\ 0\ 0\ 1\ 1\ 0) \rightarrow (1\ 0\ 0\ 1\ 1\ 0) = y_2 \in D.$$

$$y_2 E_1 = (2\ 0\ -1\ -1\ 2) \rightarrow (1\ 0\ 0\ 0\ 1) = x_3 (= x_2) \in R.$$

Thus the hidden pattern of the state vector  $x_1$  is a fixed pair given by  $\{(1\ 0\ 0\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 1)\}$ .

By keeping the node, 'good and hardworking students' in the on state; the expert finds that it is due to the good attributes of the teacher evident from the on state of the nodes  $T_1$ ,  $T_4$  and  $T_5$  in the domain space.

The relational directed graph as given by the second expert who is a final year PG student is as follows:

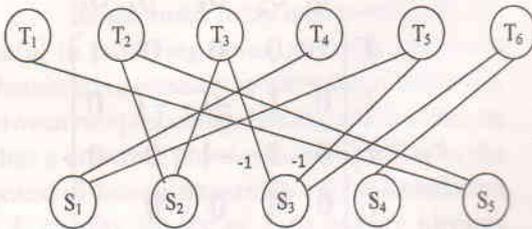


Figure 3.2

The relational connection matrix  $E_2$  got from the above directed graph is as follows;

$$E_2 = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{matrix} \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Suppose the expert want to study the on state of the node 'Motivates the students' alone in the on state and all other nodes in the off state, from the domain space; that is  $a_1 = (1\ 0\ 0\ 0\ 0\ 0)$ .

The effect of  $a_1$  on the dynamical system is,  $a_1 E_2 = (0\ 0\ 0\ 0\ 1) = b_1 \in R$ .

$b_1 E_2^T = (1\ 1\ 0\ 0\ 0\ 0) = a_2 \in D$ .

$a_2 E_2 = (0\ 1\ 0\ 0\ 2) \rightarrow (0\ 1\ 0\ 0\ 1) = b_2 \in R$ .

$b_2 E_2^T = (1\ 2\ 1\ 0\ 0\ 0) \rightarrow (1\ 1\ 1\ 0\ 0\ 0) = a_3 \in D$ .

$a_3 E_2 = (0\ 2\ -1\ 0\ 2) \rightarrow (0\ 1\ 0\ 0\ 1) = b_3 = b_2 \in R$ .

Thus the hidden pattern of the initial state vector  $a_1$  is a fixed pair given by  $\{(1\ 1\ 1\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 1)\}$ .

By keeping the node, 'motivating the students' in the on state, we find that the students become regular to class and get more interested in studies, evident from the on state of the nodes  $S_2$  and  $S_5$  in the range space.

The relational directed graph given by the third expert who is a professor, who has nearly 15 years experience is as follows:

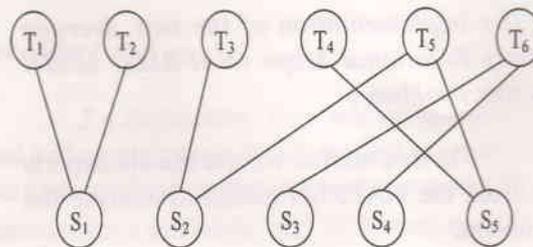


Figure 3.3

The relational connection matrix  $E_3$  got from the above directed graph is as follows:

$$E_3 = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{matrix} \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Now the expert studies the on state of the vector given in  $c_1 = (0\ 0\ 1\ 0\ 0)$ , that is the on state of the node 'irregular to class' and all other nodes are in the off state.

The effect of  $c_1$  on the dynamical system is,

$$c_1 E_3^T = (0\ 0\ 0\ 0\ 0\ 1) = d_1 \in D$$

$$d_1 E_3 = (0\ 0\ 1\ 1\ 0) = c_2 \in R$$

$$c_2 E_3^T = (0\ 0\ 0\ 0\ 0\ 2) \rightarrow (0\ 0\ 0\ 0\ 0\ 1) = d_2 (= d_1) \in D$$

Thus the hidden pattern of the state vector  $c_1$  is a fixed pair given by  $\{(0\ 0\ 0\ 0\ 0\ 1), (0\ 0\ 1\ 1\ 0)\}$ .

By keeping the node 'irregular to class' in the on state, it is found that, "it is due to the rudeness of the teachers," evident from the on state of the node  $T_6$  in domain space.

The relational directed graph given by fourth expert who is an educationalist, who has 12 years experience in the field of education is as follows:

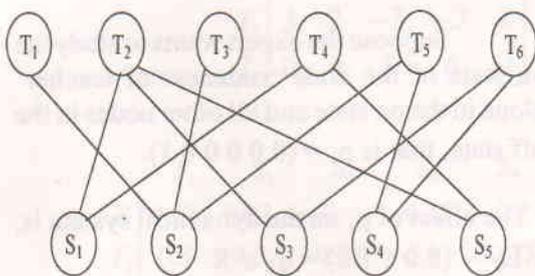


Figure 3.4

The relational connection matrix  $E_4$  got from the above directed graph is as follows:

$$E_4 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Now the expert wishes to study with the initial state vector  $j_1 = (0\ 0\ 0\ 1\ 0\ 0)$ , that is the node 'Teacher should be serious and make the class interesting' alone in the on state and all other nodes in the off state<sup>3-7</sup>.

The effect of  $j_1$  on the dynamical system is,

$$j_1 E_4 = (1\ 0\ 0\ 0\ 1) = k_1 \in R.$$

$$k_1 E_4^T = (0\ 2\ 0\ 2\ 0\ 0) \rightarrow (0\ 1\ 0\ 1\ 0\ 0) = j_2 \in D$$

$$j_2 E_4 = (2\ 0\ -1\ 0\ 2) \rightarrow (1\ 0\ 0\ 0\ 1) = k_2 (= k_1) \in R.$$

Thus the hidden pattern of the state vector  $j_1$  is a fixed pair given by  $\{(0\ 1\ 0\ 1\ 0\ 0), (1\ 0\ 0\ 0\ 1)\}$

By keeping the node 'the teacher should be serious and make the class interesting' in the on state, we find that the students become good, hardworking and get more interested in studies, evident from the on state of the nodes  $S_1$  and  $S_5$ .

The relational directed graph given by the fifth expert who is a first year post graduate student is as follows:

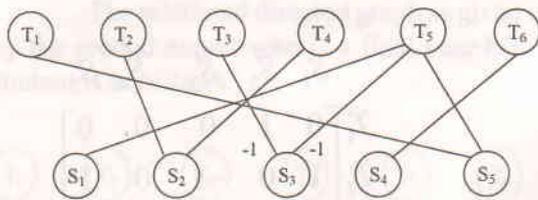


Figure 3.5

The relational connection matrix  $E_5$  got from the above directed graph is as follows:

$$E_5 = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{matrix} \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

Suppose the expert studies the on state of the node 'interested in studies' alone in the on state and all other nodes in the off state, that is  $m_1 = (0\ 0\ 0\ 0\ 1)$ .

The effect of  $m_1$  on the dynamical system is,

$$m_1 E_5^T = (1\ 0\ 0\ 0\ 1\ 0) = n_1 \in D$$

$$n_1 E_5 = (1\ 0\ -1\ 0\ 2) \rightarrow (1\ 0\ 0\ 0\ 1) = m_2 \in R$$

$$m_2 E_5^T = (1\ 0\ 0\ 0\ 2\ 0) \rightarrow (1\ 0\ 0\ 0\ 1\ 0) = n_2 (=n_1) \in D.$$

Thus the hidden pattern of the state vector  $m_1$  is a fixed pair given by  $\{(1\ 0\ 0\ 0\ 1\ 0), (1\ 0\ 0\ 0\ 1)\}$ . The node 'interested in studies' in the on state, gives the resultant that it is due to the good attributes of the teacher, evident from the on state of the nodes  $T_1$  and  $T_5$ .

The relational directed graph given by the sixth expert is a professor, who has 7 years experience is as follows:

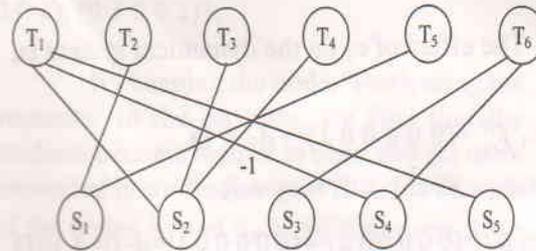


Figure 3.6

The relational connection matrix  $E_6$  got from the above directed graph is as follows:

$$E_6 = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{matrix} \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Suppose the expert wants to study the on state of the node 'rudeness of teacher' alone in the on state and all other nodes in the off state, that is  $p_1 = (0\ 0\ 0\ 0\ 0\ 1)$ .

The effect of  $p_1$  on the dynamical system is,

$$p_1 E_6 = (0\ 0\ 1\ 1\ 0) = q_1 \in R$$

$$q_1 E_6^T = (-1\ 0\ 0\ 0\ 0\ 2) \rightarrow (0\ 0\ 0\ 0\ 1) = p_2 (=p_1) \in D.$$

Thus the hidden pattern of the state vector  $p_1$  is a fixed pair given by  $\{(0\ 0\ 0\ 0\ 1), (0\ 0\ 0\ 0\ 1)\}$ .

(0 0 1 1 0)}.

By keeping the node, 'rudeness of teachers' in the on state, we find that the students become irregular to class and does not perform well in studies, evident from the on state of the nodes  $S_3$  and  $S_4$  in the range space.

Next we use all the six experts opinion to get the New Average Fuzzy Relational Maps model.

The average of these relational connection matrices  $E_1, E_2, \dots, E_6$  of the FRM is given as  $E'$  where

$$E' = \left( \frac{E_1 + E_2 + E_3 + E_4 + E_5 + E_6}{6} \right).$$

$$E' = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 2 & 2 & 0 & 1 & 3 \\ 3 & 3 & -2 & -1 & 2 \\ 0 & 5 & -3 & 0 & 0 \\ 2 & 2 & 0 & 0 & 3 \\ 3 & 2 & -3 & -2 & 2 \\ 0 & 0 & 5 & 6 & 0 \end{bmatrix} \end{matrix} /6.$$

$$E' = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0.33 & 0.33 & 0 & -0.17 & 0.5 \\ 0.5 & 0.5 & -0.33 & -0.17 & 0.33 \\ 0 & 0.83 & -0.5 & 0 & 0 \\ 0.33 & 0.33 & 0 & 0 & 0.5 \\ 0.5 & 0.33 & -0.5 & -0.33 & -0.33 \\ 0 & 0 & 0.83 & 1 & 0 \end{bmatrix} \end{matrix}.$$

Now using the parameter  $\alpha = 0.5 \in [0, 1]$  we write  $E'$  as follows. Let  $E'$  be the thresholded using  $\alpha = 0.5$ . All elements in  $E'$  which are greater than or equal to  $\pm 0.5$  is replaced by  $\pm 1$  respectively and other terms by 0.

Let

$$E = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}.$$

Now the expert wishes to study with the state vector  $s_1 = (0 1 0 0 0)$ , that is the node 'kind and approachable' alone in the on state and all other nodes in the off state.

The effect of  $s_1$  on the dynamical system is,

$$s_1 E = (1 1 0 0 0) = t_1 \in R$$

$$t_1 E^T = (0 2 1 0 1 0) \rightarrow (0 1 1 0 1 0) = s_2 \in D$$

$$s_2 E = (2 2 -2 0 0) \rightarrow (1 1 0 0 0) = t_2 (= t_1) \in R.$$

Thus the hidden pattern of the state vector  $s_1$  is a fixed pair given by  $\{(0 1 1 0 1 0), (1 1 0 0 0)\}$ .

By keeping the node, 'kind and approachable' in the on state, it was found that the students become good, hardworking and are regular to class, evident from the on state of the nodes  $S_1$  and  $S_2$  in the range space.

Suppose the expert wants to study the on state of the node 'take interest in students' alone in the on state and all other nodes in the off state, that is  $u_1 = (0\ 0\ 0\ 0\ 1\ 0)$ .

The effect of  $u_1$  on the dynamical system is,  
 $u_1 E = (1\ 0\ -1\ 0\ 0) \rightarrow (1\ 0\ 0\ 0\ 0) = v_1 \in R$   
 $v_1 E^T = (0\ 1\ 0\ 0\ 1\ 0) = u_2 \in D$   
 $u_2 E = (2\ 1\ -1\ 0\ 0) \rightarrow (1\ 1\ 0\ 0\ 0) = v_2 \in R$   
 $v_2 E^T = (0\ 2\ 1\ 0\ 1\ 0) \rightarrow (0\ 1\ 1\ 0\ 1\ 0) = u_3 \in D$   
 $u_3 E = (2\ 2\ -2\ 0\ 0) \rightarrow (1\ 1\ 0\ 0\ 0) = v_3 (= v_2) \in R$ .

Thus the hidden pattern of the state vector  $u_1$  is a fixed pair given by  $\{(0\ 1\ 1\ 0\ 1\ 0), (1\ 1\ 0\ 0\ 0)\}$ .

By keeping the node, 'take interest in students' in the on state, we find that the students become good, hardworking and regular to class, evident from the on state of the nodes  $S_1$  and  $S_2$  in the range space.

Now the expert studies the on state of the vector given in  $x_1 = (0\ 1\ 0\ 0\ 0)$ , that is the on state of the node 'regular to class' and all other nodes in the off state.

The effect of  $x_1$  on the dynamical system is,  
 $x_1 E^T = (0\ 1\ 1\ 0\ 0\ 0) = y_1 \in D$   
 $y_1 E = (1\ 2\ -1\ 0\ 0) \rightarrow (1\ 1\ 0\ 0\ 0) = x_2 \in R$   
 $x_2 E^T = (0\ 2\ 1\ 0\ 1\ 0) \rightarrow (0\ 1\ 1\ 0\ 1\ 0) = y_2 \in D$   
 $y_2 E = (2\ 2\ -2\ 0\ 0) \rightarrow (1\ 1\ 0\ 0\ 0) = x_3 (= x_2) \in R$ .

Thus the hidden pattern of the state vector  $x_1$  is a fixed pair given by  $\{(0\ 1\ 1\ 0\ 1\ 0), (1\ 1\ 0\ 0\ 0)\}$ .

By keeping the node 'regular to class'

in the on state, it is found that this is due to the good attributes of the teachers, evident from the on state of the node  $T_2, T_3$  and  $T_5$  in domain space.

Suppose the expert studies the on state of the node 'does not perform well in studies' alone in the on state and all other nodes in the off state, that is  $a_1 = (0\ 0\ 0\ 1\ 0)$ .

The effect of  $a_1$  on the dynamical system is,  
 $a_1 E^T = (0\ 0\ 0\ 0\ 0\ 1) = b_1 \in D$   
 $b_1 E = (0\ 0\ 1\ 1\ 0) = a_2 \in R$   
 $a_2 E^T = (0\ 0\ -1\ 0\ -1\ 2) \rightarrow (0\ 0\ 0\ 0\ 1) = b_2 (= b_1) \in D$ .

Thus the hidden pattern of the state vector  $a_1$  is a fixed pair given by  $\{(0\ 0\ 0\ 0\ 1), (0\ 0\ 1\ 1\ 0)\}$ .

By keeping the node 'does not perform well in studies' in the on state, it is found that it is due to the rudeness of the teachers, evident from the on state of the node  $T_6$  in domain space.

#### *4 Comparison of the Experts Opinion – an Analysis of the FRM Models for Conclusions:*

It is an innovative method to use tables to find the closeness or deviation of experts for a given initial state vectors from the domain or range space. In this section we give the table of comparison which acts as a ready reconer for comparison of one expert with the other and also of each and every expert with the new NAFRMs constructed using all the experts opinion.

The Table 4.1 gives the comparison table given by the six experts and the resultant

vectors given by the New Average FRMs in the columns  $E_1, E_2, E_3, E_4, E_5, E_6$  and  $E$  respectively.

The effect of state vector on the relational connection matrix and the resultant hidden pattern are given below.

Table 4.1: Comparison table of FRMs of the 6 experts and the NAFRMs

State Vector	$E_1$	$E_2$	$E_3$
(100000)	{{(100110), (10001)}}	{{(111000), (01001)}}	{{(110000), (10000)}}
(010000)	{{(111110), (11001)}}	{{(111000), (01001)}}	{{(110000), (10000)}}
(001000)	{{(111110), (11001)}}	{{(111000), (01001)}}	{{(001110), (01001)}}
(000100)	{{(100110), (10001)}}	{{(000110), (10000)}}	{{(001110), (01001)}}
(000010)	{{(100110), (10001)}}	{{(000110), (10000)}}	{{(001110), (01001)}}
(000001)	{{(000001), (00110)}}	{{(000001), (00110)}}	{{(000001), (00110)}}
(10000)	{{(100110), (10001)}}	{{(000110), (10000)}}	{{(110000), (10000)}}
(01000)	{{(111110), (11001)}}	{{(111000), (01001)}}	{{(001110), (01001)}}
(00100)	{{(000001), (00110)}}	{{(000001), (00110)}}	{{(000001), (00110)}}
(00010)	{{(000001), (00110)}}	{{(000001), (00110)}}	{{(000001), (00110)}}
(00001)	{{(100110), (10001)}}	{{(111000), (01001)}}	{{(001110), (01001)}}

State Vector	$E_4$	$E_5$	$E_6$
(100000)	{{(101010), (01000)}}	{{(100010), (10001)}}	{{(101100), (01001)}}
(010000)	{{(010100), (10001)}}	{{(110010), (11001)}}	{{(010010), (10000)}}
(001000)	{{(101010), (01000)}}	{{(000000), (00000)}}	{{(101100), (01001)}}
(000100)	{{(010100), (10001)}}	{{(110010), (11001)}}	{{(101100), (01001)}}
(000010)	{{(101010), (01000)}}	{{(100010), (10001)}}	{{(010010), (10000)}}
(000001)	{{(000001), (00110)}}	{{(000001), (00010)}}	{{(000001), (00110)}}
(10000)	{{(010100), (10001)}}	{{(100010), (10001)}}	{{(010010), (10000)}}
(01000)	{{(101010), (01000)}}	{{(110010), (11001)}}	{{(101100), (01001)}}
(00100)	{{(000001), (00110)}}	{{(000000), (00000)}}	{{(000001), (00110)}}
(00010)	{{(000001), (00110)}}	{{(000001), (00010)}}	{{(000001), (00110)}}
(00001)	{{(010100), (10001)}}	{{(100010), (10001)}}	{{(101100), (01001)}}

State Vector	E
(100000)	{{(100100), (00001)}
(010000)	{{(011010), (11000)}
(001000)	{{(011010), (11000)}
(000100)	{{(100100), (00001)}
(000010)	{{(011010), (11000)}
(000001)	{{(000001), (00110)}
(10000)	{{(011010), (11000)}
(01000)	{{(011010), (11000)}
(00100)	{{(000001), (00110)}
(00010)	{{(000001), (00110)}
(00001)	{{(100100), (00001)}

$$= \frac{1}{6} \{(6\ 2\ 3\ 2\ 3\ 0), (3\ 3\ 0\ 0\ 4)\}$$

$$= \{(1, 0.33, 0.5, 0.33, 0.5, 0), (0.5, 0.5, 0, 0, 0.66)\}.$$

This average hidden pattern is thresholded using the parameter  $\alpha = 0.5 \in [0, 1]$ ; if entries are greater than or equal to  $\alpha$  then it is replaced by 1 if  $\alpha < 0.5$  it is replaced by 0. So the average is  $\{(1\ 0\ 1\ 0\ 1\ 0), (1\ 1\ 0\ 0\ 1)\}$ . The same procedure is performed for each and every row given in Table 4.1.

These averages are tabulated in the following Table 4.2.

Table 4.2. Average Hidden Pattern from Table 4.1

Initial state vector from the domain and range space	Average of the hidden pattern pair of 6 experts opinion after thresholding by 0.5
(1 0 0 0 0 0)	{{(1 0 1 0 1 0), (1 1 0 0 1)}
(0 1 0 0 0 0)	{{(1 1 0 0 1 0), (1 1 0 0 1)}
(0 0 1 0 0 0)	{{(1 0 1 1 0 0), (0 1 0 0 1)}
(0 0 0 1 0 0)	{{(1 0 0 1 1 0), (1 1 0 0 1)}
(0 0 0 0 1 0)	{{(1 0 0 1 1 0), (1 0 0 0 1)}
(0 0 0 0 0 1)	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(1 0 0 0 0)	{{(1 1 0 1 1 0), (1 0 0 0 1)}
(0 1 0 0 0)	{{(1 1 1 1 1 0), (0 1 0 0 1)}
(0 0 1 0 0)	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(0 0 0 1 0)	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(0 0 0 0 1)	{{(1 0 1 1 1 0), (1 1 0 0 1)}

For the average of the 6 resultant vectors using the same thresholding function  $\alpha$  given in section three of this paper is calculated. The sum of the average of the six experts opinion for each of the state vectors given in the first column of the Table 4.1 is calculated as follows:

Average for the six hidden pattern pairs of the state vector (1 0 0 0 0 0) of the six expert is as follows:

$$\frac{1}{6} (\{(1\ 0\ 0\ 1\ 1\ 0), (1\ 0\ 0\ 0\ 1)\} + \{(1\ 1\ 1\ 0\ 0\ 0), (0\ 1\ 0\ 0\ 1)\} + \{(1\ 1\ 0\ 0\ 0\ 0), (1\ 0\ 0\ 0\ 0)\} + \{(1\ 0\ 1\ 0\ 1\ 0), (0\ 1\ 0\ 0\ 0)\} + \{(1\ 0\ 0\ 0\ 1\ 0), (1\ 0\ 0\ 0\ 1)\} + \{(1\ 0\ 1\ 1\ 0\ 0), (0\ 1\ 0\ 0\ 1)\})$$

$$= \frac{1}{6} \{(1\ 0\ 0\ 1\ 1\ 0) + (1\ 1\ 1\ 0\ 0\ 0) + (1\ 1\ 0\ 0\ 0\ 0) + (1\ 0\ 1\ 0\ 1\ 0) + (1\ 0\ 0\ 0\ 1\ 0) + (1\ 0\ 1\ 1\ 0\ 0), (1\ 0\ 0\ 0\ 1) + (0\ 1\ 0\ 0\ 1) + (1\ 0\ 0\ 0\ 0) + (0\ 1\ 0\ 0\ 0) + (1\ 0\ 0\ 0\ 1) + (0\ 1\ 0\ 0\ 1)\}$$

Table 4.3. Comparison table of Hidden pattern pairs of NAFRMs and the Average from Table 4.1

Initial state vector from the domain and range space	Average of the hidden pattern pair of 6 experts opinion after thresholding by 0.5	Average of the hidden pattern pair of 6 experts opinion after thresholding by 0.5
(1 0 0 0 0)	{{(1 0 1 0 1 0), (1 1 0 0 1)}	{{(1 0 0 1 0 0), (0 0 0 0 1)}
(0 1 0 0 0)	{{(1 1 0 0 1 0), (1 1 0 0 1)}	{{(0 1 1 0 1 0), (1 1 0 0 0)}
(0 0 1 0 0)	{{(1 0 1 1 0 0), (0 1 0 0 1)}	{{(1 0 0 1 0 0), (0 0 0 0 1)}
(0 0 0 1 0)	{{(1 0 0 1 1 0), (1 1 0 0 1)}	{{(0 1 1 0 1 0), (1 1 0 0 0)}
(0 0 0 0 1 0)	{{(1 0 0 1 1 0), (1 0 0 0 1)}	{{(1 0 0 1 1 0), (1 0 0 0 1)}
(0 0 0 0 1 0)	{{(1 0 0 1 1 0), (1 0 0 0 1)}	{{(0 1 1 0 1 0), (1 1 0 0 0)}
(0 0 0 0 0 1)	{{(0 0 0 0 0 1), (0 0 1 1 0)}	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(1 0 0 0 0)	{{(1 1 0 1 1 0), (1 0 0 0 1)}	{{(0 1 1 0 1 0), (1 1 0 0 0)}
(0 1 0 0 0)	{{(1 1 1 1 1 0), (0 1 0 0 1)}	{{(0 1 1 0 1 0), (1 1 0 0 0)}
(0 0 1 0 0)	{{(0 0 0 0 0 1), (0 0 1 1 0)}	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(0 0 0 1 0)	{{(0 0 0 0 0 1), (0 0 1 1 0)}	{{(0 0 0 0 0 1), (0 0 1 1 0)}
(0 0 0 0 1)	{{(1 0 1 1 1 0), (1 1 0 0 1)}	{{(1 0 0 1 0 0), (0 0 0 0 1)}

Next the average hidden pattern pairs from the Table 4.2 and the last column of Table 4.2 is tabulated in Table 4.3 so as to compare the hidden pattern pairs from real average and that of from the NAFRMs in Table 4.1.

It is clear from this table the deviation in all cases is not very large. Some case there are three differences. The large deviations were discussed and from the sayings of Kosko the result reflects the efficiency or ignorance

of the expert. Finally all the hidden pattern pairs of the all the experts and the NAFRMs happened to be fixed point pair there by indicating these attributes are not changeable. Further by using this new NAFRMs model one can save both time and economy<sup>8-10</sup>.

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