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# Bianchi Type II Bulk Viscous Stiff Fluid Model in General Relativity

B.L. MEENA

Department of Mathematics, Govt. P.G. College, Tonk, Rajasthan (India)
E-mail : drblmina@rediffmail.com

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**Abstract**

Bianchi Type II stiff fluid model in presence of bulk viscosity in General Relativity is investigated to get the deterministic model of the universe, we have used the stiff fluid condition ($\rho$=p), $\rho$ the matter density, p the isotropic pressure, $\zeta\theta$ = constant, $\zeta$ is the coefficient of bulk viscosity $\theta$ the expansion in the model and $\sigma$ is proportion to $\theta$ which leads to $R = S^n$ where R and S are metric potentials and n is a constant. We find that reality condition $\rho > 0$ is satisfied and the bulk viscosity prevent the matter density to vanish. The model starts with a big bang and the expansion in the model decreases as time increases. The model represents the accelerating and decelerating phases of the universe which matches with latest astronomical observation. The model in general represents anisotropic phase of universe.

*Key words* : Bianchi II, Bulk viscous, Stiff Fluid, General Relativity.

2010 Mathematics Subject Classifications : 83CXX, 83FXX

## 1. Introduction

It has been argued for long time that dissipative process may well account for the high degree of isotropy, we observe today[1]. Dissipative effects including bulk viscosity play a significant role in the study of early evolution of universe. Eckart[2] first developed the relativistic theory of non-equilibrium thermodynamics to study the effect of viscosity. Padmanabhan and Chitre[3] have discussed that the presence of bulk viscosity leads to inflationary like solution in general relativity. The effect of bulk viscosity on cosmological evolution has been investigated by many authors viz. Sahni and Starobinsky[4], Mak and Harko[5,6], Peebles[7], Saha[8], Hu and Meng[9], Ren and Meng[10], Singh *et al.*[11], Bali and Singh[12], Montiel and Braton[13], Gagnon and Lesgoulrgues[14].

*2. Metric and Field Equations :*

We consider Bianchi Type II metric in the form

$$ds^2 = -dt^2 + R^2(dx^2 + dz^2) + S^2(dy - x\,dz)^2 \quad (2.1)$$

where R and S are functions of t alone.

Energy momentum tensor in presence of bulk viscosity is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \zeta\theta(g_i^j + v_i v^j) \quad (2.2)$$

with

$$g_{ij} v^i v^j = -1 \quad (2.3)$$

We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1 \quad (2.4)$$

where $\rho$ the matter density, p the isotropic pressure, $\zeta$ the coefficient of bulk viscosity and $\theta$ the expansion in the model, $v^i$ the flow vector satisfying (2.3).

Now

$$T_1^1 = p - \zeta\theta, T_2^2 = p - \zeta\theta, T_3^3 = p - \zeta\theta, T_4^4 = -\rho \quad (2.5)$$

The Einstein field equations

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi T_i^j \quad (2.6)$$

(in geometrized units G=1, c=1 and taking $\Lambda$=0)

For the line element (2.1) leads to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{S^2}{4R^4} = -8\pi(p - k) \quad (2.7)$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi(p - k) \quad (2.8)$$

$$\frac{2R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{S^2}{4R^4} = -8\pi\rho \quad (2.9)$$

where subscript 4 denotes differentiation with respect to t and we assume

$$\zeta\theta = k \text{ as given by Brevik } et\ al.^{15} \quad (2.10)$$

*3. Solution of Field Equations :*

Equations (2.7) and (2.9) lead to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{3R_4 S_4}{RS} + \frac{R_4^2}{R^2} = -8\pi(\rho - p - k) \quad (3.1)$$

To get determinate solution, we assume

(i) stiff fluid condition $\rho = p$ (3.2)

(ii) $\sigma \alpha \theta$ which leads to $R = S^n$ (3.3)

Now after using (3.2) and (3.3) in (3.1) leads to

$$(n+1)\frac{S_{44}}{S} + (2n^2 + 3n)\frac{S_4^2}{S^2} = 8\pi k \quad (3.4)$$

which leads to

$$2S_{44} + 4n\frac{S_4^2}{S} = \frac{16\pi k}{n+1} S \quad (3.5)$$

Let $S_4 = f(S)$

$\therefore \quad S_{44} = f f', f' = df/dS$

I.F. = $S^{4n}$

Solution is

$$f^2 S^{4n} = \frac{16\pi k}{n+1}\int S^{4n+1}\,dS$$

$$= \frac{16\pi k}{n+1}\frac{S^{4n+2}}{4n+2} + \alpha$$

where $\alpha$ is constant of integration. Thus

$$f^2 = \frac{8\pi k}{(n+1)(2n+1)} S^2 + \alpha S^{-4n} \quad (3.6)$$

which leads to

$$\frac{dS}{\sqrt{\beta S^2 + \alpha S^{-4n}}} = dt \quad (3.7)$$

where

$$\beta = \frac{8\pi k}{(n+1)(2n+1)} \quad (3.8)$$

To get determinate solution, we assume

$$n = 1/2 \quad (3.9)$$

Using (3.8) and (3.9) in (3.7), we have

$$\frac{S\,dS}{\sqrt{\beta (S^2)^2 + \alpha}} = dt$$

which leads to

$$\frac{S\,dS}{\sqrt{(S^2)^2 + \gamma^2}} = \sqrt{\beta}\,dt \quad (3.10)$$

where

$$\frac{\alpha}{\beta} = \gamma^2$$

Let

$$S^2 = \tau$$

$$S\,dS = \frac{d\tau}{2} \quad (3.11)$$

Equation (3.10) leads to

$$\sinh^{-1}\left(\frac{\tau}{\gamma}\right) = \sqrt{\beta}\,t + b$$

which leads to

$$\tau = \gamma \sinh(at+b)$$

Thus we have

$$S^2 = \gamma \sinh(at+b) \quad (3.12)$$

where

$$a = \sqrt{\beta}$$

Therefore

$$R^2 = S = \sqrt{\gamma}\sinh^{1/2}(at+b) \quad (3.13)$$

The metric (2.1) leads to

$$ds^2 = -dt^2 + \sqrt{\gamma}\sinh^{1/2}(at+b)(dx^2 + dz^2)$$

$$+ \gamma \sinh(at+b)(dy - x\,dz)^2 \quad (3.14)$$

*4. Some Physical and Geometrical Features:*

We have

$$S^2 = \gamma \sinh(at+b) \quad (4.1)$$

$$\therefore \frac{S_4}{S} = \frac{a}{2}\coth(at+b) \quad (4.2)$$

The matter density from equation (2.9) is given by

$$8\pi\rho = \frac{2R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{S^2}{4R^4} \quad (4.3)$$

$$=2\frac{1}{2}\frac{S_4^2}{S^2}+\frac{1}{4}\frac{S_4^2}{S^2}-\frac{1}{4}$$

$$=\frac{5}{4}\frac{S_4^2}{S^2}-\frac{1}{4} \tag{4.4}$$

$$=\frac{5}{4}\frac{a^2}{4}\coth^2(at+b)-\frac{1}{4}$$

$$=\frac{5a^2}{16}\coth^2(at+b)-\frac{1}{4}$$

$$=8\pi p \tag{4.5}$$

The expansion ($\theta$), the shear ($\sigma$), the spatial volume ($\gamma^3$), the deceleration parameter (q) are given by

$$\theta=\frac{R_4}{R}+\frac{S_4}{S} \tag{4.6}$$

$$=\frac{3}{2}\frac{S_4}{S}$$

$$=\frac{3a}{2}\operatorname{cotR}(at+b) \tag{4.7}$$

$$\sigma=\frac{1}{\sqrt{3}}\left|\frac{R_4}{R}-\frac{S_4}{S}\right|$$

$$=\frac{a}{4\sqrt{3}}\coth(at+b) \tag{4.8}$$

$$V^3=R^2S=S^2=\gamma\sinh(at+b) \tag{4.9}$$

$$q=-\frac{\ddot{V}/V}{\dot{V}^2/V^2} \tag{4.10}$$

Now

$$V^3=\gamma\sinh(at+b) \tag{4.11}$$

$$\therefore\ \frac{\dot{V}}{V}=\frac{a}{3}\coth(at+b)$$

$$\frac{\ddot{V}}{V}-\frac{\dot{V}^2}{V^2}=-\frac{a^2}{3}\operatorname{cosech}^2(at+b)$$

$$\frac{\ddot{V}}{V}=-\frac{a^2}{3}\operatorname{cosech}^2(at+b)+\frac{a^3}{9}\coth^2(at+b)$$

$$=-\frac{a^2}{3}[\coth^2(at+b)-1]+\frac{a^2}{9}\coth^2(at+b)$$

$$=\frac{a^2}{3}-\frac{2a^2}{9}\coth^2(at+b)$$

$$\therefore\ q=-\frac{9\left[\frac{a^2}{3}-\frac{2a^2}{9}\coth^2(at+b)\right]}{a^2\coth^2(at+b)}$$

$$=-3\tanh^2(at+b)+2 \tag{4.12}$$

Particle horizon

$$=\int_{t_0}^{t}\frac{dt}{\gamma^3(t)}$$

$$=\int_{t_0}^{t}\frac{dt}{\gamma\sinh(at+b)}$$

$$=\text{finite} \tag{4.13}$$

## 5. Discussion and Conclusion

The reality condition $\rho > 0$ leads to

$$\coth^2(at+b) > \frac{4}{5a^2} \qquad (4.14)$$

The equation (4.4.1) indicates that bulk viscosity prevents the matter density to vanish. The model (4.3.13) starts with a big-bang at t = -b/a and the expansion decreases with time. Since $\frac{\sigma}{\theta} \neq 0$, hence anisotropy is maintained throughout. The spatial volume increases exponentially with time. Thus the model represents inflationary scenario. The model also represents accelerating and decelerating phases of universe matching with recent astronomical observations. Also the particle horizon is finite, hence the particles are in communicable region. The model in general represents realistic model of the universe.

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