ISSN 2231-3478 (Print)
JUSPS-B Vol. 29(5), 99-109 (2017). Periodicity-Monthly
(Online) ISSN 2319-8052
Section B
JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

MHD Flow and Heat transfer characteristics of a micropolar nanofluid over an unsteady stretching sheet with prescribed surface heat flux

D. HYMAVATHI[1] and N. KISHAN[2]
[1]Dept of Mathematics , Mahatma Gandhi University, Nalgonda India
[2]Mathematics, Osmania University, Hyderabad India
Corresponding author Emaildyapahyma@yahoo.com
http://dx.doi.org/10.22147/jusps-B/290501

Acceptance Date 31st March, 2017, Online Publication Date 2nd May, 2017

Abstract

In this paper we study the MHD flow and heat transfer of micro polar Nano fluid with prescribed heat flux over an unsteady stretching sheet. The governed partial differential equations are highly nonlinear and they are converted into ordinary differential equations by using the similarity transformations. These equations are solved by implicit finite difference scheme known as Keller Box method. The effects of unsteadiness parameter (S), material parameter (m), viscosity (K), Prandtl number (Pr), Brownian motion parameter (Nb) and thermo phoresis parameter (Nt) on the flow and heat transfer characteristics are studied.

Key words : unsteady flow, Nanofluid, stretching sheet, micro polar fluid and Keller Box method.

Introduction

It is well known that in many industrial processes, heat transfer is an integral part of the flow mechanism. Now there is an abundant literature available on the flow induced by a stretching sheet with heat transfer[1–2]. Heat transfer characteristics are dependent on the thermal boundary conditions. In general, there are four common heating processes representing the wall-to ambient temperature distribution, prescribed surface heat flux distribution and conjugate conditions, where heat transfer through a bounding surface of finite thickness and finite heat capacity is specified. Crane[3] noted that usually the sheet is assumed to be inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching plastic sheet. For examples, materials manufactured by aerodynamic extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on a conveyor belt possess the characteristics of a moving continuous stretching surface. Gorla[4] depicted the application of linearly stretched surface in electro-chemistry. Banks[5] discussed the flow field of a stretching wall with a power-law velocity variation. McLeod and Rajagopal[6] investigated the uniqueness of the flow of a Navier Stokes fluid due to a linear stretching boundary. Many investigators analysed the results for the effect of heat transfer, rotation, MHD, suction/injection, non-Newtonian fluids, chemical reaction etc. The quality of the final product depends on the rate of heat transfer at the stretching surface. The flow problem due to a linearly stretching sheet belongs to a class of exact solutions of the Navier-Stokes equations. Ahmed A. Afify[7] studied heat and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching surface.

Micro polar fluids are fluids with microstructure. They belong to a class of fluids with non-symmetrical stress tensor that we shall call polar fluids and include, as a special case, the well-established Navier-Stokes model of classical fluids that we shall call ordinary fluids. Physically, micro polar fluids may represent fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium, where the deformation of fluid particles is ignored. The model of micro polar fluids introduced by Eringen[8] also developed other forms of micropolar fluids, namely the micro polar fluids with stretch in the study of the thermo-micro polar fluids. Eringen[9] included the thermal effects in an isotropic medium, and then he extended his theory to the anisotropic medium in the study of anisotropic micro polar fluids. More recently, Eringen[10-11] presented the memory-dependent orientable nonlocal micro polar fluids The fluid motion of the micro polar fluid is characterized by the concentration laws of mass, momentum and constitutive relationships describing the effect of couple stress, spin-inertia and micro motion. Hence the flow equation of micro polar fluid involves a micro rotation vector in addition to classical velocity vector. More interesting aspects of the theory and application of micro polar fluids can be found in the books of Peddieson and McNitt[12], Willson[13], Siddheshwar and Pranesh[14,15], Siddheshwar and Manjunath[16] Desseaux and Kelson[17] investigated the flow of a micropolar fluid over a stretching sheet. In another attempt, Kelson and Desseaux[18] have investigated the effects of surface conditions on the flow of a micropolar fluid over a stretching sheet. Bhargava *et al*.[19] studied mixed convection flow of a Micropolar fluid over a porous stretching sheet by implementing finite element method.

Nanofluids are a new class of fluids engineered by dispersing nanometer-sized materials (nanoparticles, nanofibers, nanotubes, nanowires, nanorods, nanosheet, or droplets) in base fluids. In other words, nanofluids are nanoscale colloidal suspensions containing condensed nanomaterials. They are two-phase systems with one phase (solid phase) in another (liquid phase). Nanofluids have been found to possess enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water. It has demonstrated great potential applications in many fields. Madhu, *et.al.*[20] studied the unsteady flow of a Maxwell nanofluid over a stretching surface in the presence of magneto hydrodynamic and thermal radiation effects. Noghrehabadi *et al*.[21] studied and analyzed fluid flow and heat transfer of nanofluids over a stretching sheet near the extrusion slit. Hsiao[22] investigated a nanofluid flow with multimedia physical features for conjugate mixed convection and radiation for the thermal energy conversion problems. Ashorynejad H.P.[23] investigated nanofluid flow over stretching cylinder in the presence of magnetic field. Mustafa *et al.*[24] examined the unsteady boundary layer flow of nanofluid past an impulsively stretching sheet by HAM. Exact analytic solutions of unsteady convective heat transfer problem for various nanofluids have been derived by Turkyilmazoglu[25]. Madhu and Kishan analyzed[26-27] MHD boundary-layer flow of a non-Newtonian nanofluid past a stretching sheet with a heat source/sink, and Boundary layer flow and heat transfer of a non-Newtonian nanofluid over a non-linearly stretching sheet. Numerical solution for non-linear radiation heat transfer problem in nanofluids with an application to solar energy was computed by Mushtaq *et al.*[28]. Flow of nanofluid due to a rotating disk was discussed by Turkyilmazoglu[29]. Magnetic field effects on the flow of Cu-water nanofluid were discussed by Sheikholeslami *et al*.[30]. Malvandi and Ganji[31] examined the flow of water or aluminum based nanofluids through circular channel with magnetic field. Mixed convection flow past a vertical microchannel was addressed by Malvandi and Ganji[32]. In another paper, Malvandi and Ganji[33] forced convection flow of Nano fluid in a cooled plate micro-channel was considered. N. Bachok *et. al.*[34] studied the Flow and heat transfer over an unsteady stretching sheet in a micro polar fluid with prescribed surface heat flux. Kalyani *et.al.*[35] analysed the MHD boundary layer flow of a Nano fluid over an exponentially permeable stretching sheet with radiation and heat source/ sink.

The present paper deals with the study of the MHD flow and heat transfer of micro polar Nano fluid with prescribed heat flux over an unsteady stretching sheet. The effects of different flow parameters are involved in this problem. The effects of different parameters on fluid velocity, micro polar, temperature and Nano particle volume fraction profiles are analysed and plotted.

Problem formulation:

Consider an unsteady, two- dimensional laminar flow of an incompressible micropolar nanofluid over a stretching sheet. At time t=0 the sheet is impulsively stretched with velocity $U_m(x,t)$ along the x- axis, keeping the origin fixed in the fluid of ambient temperature T_∞. The stationary Cartesian coordinate system has its origin located at the leading edge of the sheet with the positive x-axis extending along the sheet while the y- axis is measured normal to the surface of the sheet. The boundary layer equations may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial w}{\partial x}-\frac{\sigma B_0^2(x)}{\rho}u \qquad (2)$$

$$\rho j\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=(\gamma)\frac{\partial^2 w}{\partial y^2}-\kappa\left(2w+\frac{\partial u}{\partial y}\right), \qquad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \qquad (4)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}++\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (5)$$

Subject to the boundary conditions $u=U_w$, $v=0$, $w=-m\frac{\partial u}{\partial y}$, $\frac{\partial T}{\partial y}=-\frac{q_w}{k}$ at y=0,

$$u\to 0,\quad w\to 0,\qquad T\to T_\infty, C\to C_\infty, as\, y\to\infty \qquad (6)$$

where m is the boundary parameter with $0 \le 1 \le m$. u and v are the velocity components in the x - and y - directions, respectively, T is the fluid temperature in the boundary layer, w is the micro rotation or angular velocity, and j, γ, μ, κ, ρ, and α are the micro inertia per unit mass, spin gradient viscosity, dynamic viscosity, vortex viscosity, fluid density and thermal diffusivity, respectively. It is assumed that the stretching velocity $U_w(x,t)$ and the surface heat flux $q_w(x,t)$ are of the forms

$$U_w(x,t)=\frac{ax}{1-ct} \qquad q_w(x,t)=\frac{bx}{1-ct} \qquad (7)$$

where a, b and c are constants with $a > 0$, $b \ge 0$ and $c \ge 0$ (with $ct < 1$) and both a and c have dimension time^{-1}. It should be noted that at t = 0 (initial motion), Eqs. (1) – (5) describe the steady flow over a stretching surface. These particular forms of $U_w(x,t)$ and $q_w(x,t)$ have been chosen in order to be able to devise a new similarity transformation, which transforms the governing partial differential equations (1) – (5) into a set of ordinary differential equations, thereby facilitating the exploration of the effects of the controlling parameters. The spin-gradient viscosity γ can be defined as $\gamma=(\mu+\kappa/2)j=\mu(1+K/2)j$ where $K=\kappa/\mu$ is the dimensionless viscosity ratio and is called the material parameter. The relation (7) is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin w reduces to the angular velocity. The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \qquad (8)$$

The momentum, angular momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation

$$\eta=\left(\frac{U_w}{\nu x}\right)^{1/2},\quad \psi=(\nu x U_w)^{1/2} f(\eta).\quad w=U_w\left(\frac{U_w}{\nu x}\right)^{1/2} g(\eta)$$

$$\theta(\eta)=\frac{k(T-T_\infty)}{q_w}\left(\frac{U_w}{\nu x}\right)^{1/2},\qquad \varphi(\eta)=\frac{k(C-C_\infty)}{q_w}\left(\frac{U_w}{\nu x}\right)^{1/2} \qquad (9)$$

Where ç is the similarity variable. The transformed nonlinear ordinary differential equations are:

$$(1+K)f''' + ff'' - f'^2 + Kg' - S\left(f' + \frac{1}{2}\eta f''\right) - Mf' = 0 \quad (10)$$

$$\left(1+\frac{K}{2}\right)g'' + fg' - f'g - K(2g + f'') - S\left(\frac{3}{2}g + \frac{1}{2}\eta g'\right) = 0 \quad (11)$$

$$\frac{1}{pr}\theta'' + f\theta' - f'\theta - S\left(\theta + \frac{1}{2}\eta\theta'\right) + Nb\theta'\varphi' + Nt\theta'^2 = 0 \quad (12)$$

$$\frac{1}{Sc}\varphi'' + f\varphi' - f'\varphi - S\left(\varphi + \frac{1}{2}\eta\varphi'\right) + \frac{Nt}{Nb}\theta'' = 0 \quad (13)$$

where primes denote differentiation with respect to η, Pr = ϑ/α is the Prandtl number, S = c /a is the unsteadiness parameter, $Nb = \frac{D_B\tau}{\alpha}$ is the Brownian motion parameter and $Nt = \frac{D_B\tau}{T_{\infty,}}$ is the thermophoresis parameter and Sc = ϑ/D is the Schmidt number.

The boundary conditions (6) now become

$$f(0) = 0, \quad f'(0), \quad g(0) = -mf''(0), \quad \theta'(0) = -1, \quad \varphi'(0) = -1,$$
$$f'(\eta) \to 0, \quad g(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \varphi(\eta) \to 0 \quad as \quad \eta \to \infty \quad (14)$$

Method of solution:

The nonlinear ordinary differential equations (10) – (14) have been solved numerically by a finite-difference scheme known as the Keller-box method. It has been widely applied for the solutions of boundary layer equations in fluid mechanics. This method has several attractive features such as simplicity and ease of programming, unconditional stability, second order accuracy and ability to use extrapolation as step size approaches to zero. Moreover it applies in a simple fashion to both linear and non-linear differential equations. Unlike shooting method, it can also be applied for solving non-linear partial differential equations. We solve the Equations (10)-(13) subject to the boundary conditions (14) by using Keller-Box method. A detailed description of the method can be found in the book by Cebeci and Bradshaw[36]. The equations are transformed to first-order system by using appropriate substitutions and then reduced to difference equations using central difference. The resulting algebraic equations are linearized by using Newton's method and written in matrix-vector form. At the end, the linear system is solved by using block-tridiagonal elimination technique. The step size Δη in η , and the position of the edge of the boundary-layer η_∞ have to be adjusted for difference values of the parameters to maintain the necessary accuracy. In this study, the values of Δη between 0.001 and 0.1 were used, depending on the values of the parameters considered, in order that the numerical values obtained are mesh independent, at least to four decimal places. However, a uniform grid of Δη= 0.01 was found to be satisfactory for a convergence criterion of η which gives accuracy to five decimal places, in nearly all cases. On the other hand, the boundary-layer thickness was chosen where the infinity boundary condition is achieved.

Results and Discussions

The numerical results are given to carry out a parametric study showing the influences of the unsteadiness parameter S, viscosity parameter K, boundary parameter m, Brownian motion Nb, thermophoresis parameter Nt and Prandtl number Pr. For validation of the numerical method used in this study, the case when S = 0 (steady-state flow) and K=0 (viscous fluid) has also been considered.

The effects of viscosity K is shown in figures 1(a)-(d), it depicts that with the increase of viscosity value the velocity profile, micropolar profile and temperature profiles are increasing, while the nano particle volume fraction profile is decreasing. And it is evident that the boundary layer thickness increases with the increase of K. The velocity gradient at the surface decreases as K increases. Thus the micropolar fluids show drag reduction compared to viscous fluids.

Figures 2(a)-(d) are depicts that the effects of unsteady parameter S on velocity, micropolar, temperature and nano particle volume fraction profile. With an increasing of unsteady parameter S the velocity profile, temperature profile and nano particle volume fraction profile are decreasing where as the micropolar profile is increasing, which represents the heat transfer rate at the surface is increasing.

From figures 3(a)-(d) we analyze that the effects of boundary parameter 'm' on velocity, micropolar fluid, temperature and concentration of nanofluid particles. With the increase of boundary parameter the velocity profile and the temperature profiles are increasing, but the micropolar fluid profile and the concentration of nanoparticle profiles are decreasing.

Figures 4(a)-(b) depicts that the effects of Brownian motion parameter Nb on temperature and concentration profiles. With the increase of Brownian motion parameter the temperature profile is increasing where as the nano particle volume fraction profile is decreasing.

Figures 5(a)-(b) depicts that with an increase of thermophresis parameter Nt the temperature profile and nano particle volume fraction profile is increasing.

Figure 6(a)-(b) shows that the effects of Prandtl number Pr on temperature and nano particle volume fraction profile. With an increase of Prandtl number the temperature is decreasing and the nano particle volume fraction profiles is increasing. When Pr >>1 viscous forces are in balance in the thermal boundary layer and viscous and inertia are in balance in the larger viscous boundary layer. From the governed momentum and micro polar angular velocity equations show that the Prandtl number not influences the velocity and micropolar fluid motion.

Figures 7(a)-(b) we can observe that the effects of Schmidt number on the temperature profile and nano particle volume fraction profile, Schmidt number is the ratio of fluid boundary layer to mass transfer boundary layer thickness. Therefore the figures depicts that with the increasing of Schmidt number the temperature profile and the nano particle volume fraction profile are decreasing.

Figures 8(a)-(d) we seen that the effects of magnetic parameter on various flow parameters that the velocity profile starts from maximum value at the surface and then decreasing until it reaches to the minimum value at the end of the boundary layer for all the values M. It is interesting to note that the effect of magnetic field is more prominent at the point of peak value, because the presence of M in an electrically conducting fluid introduces a force like Lorentz force

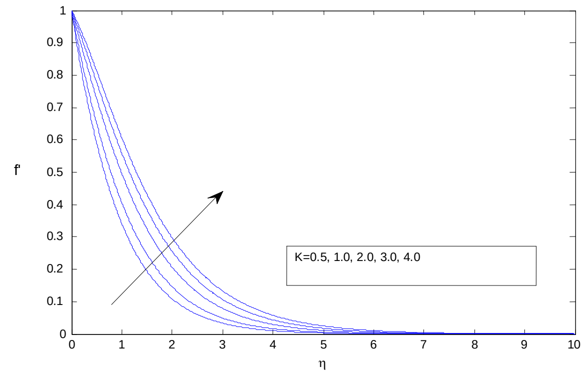

Figure 1(a) Effects of viscosity on velocity profile

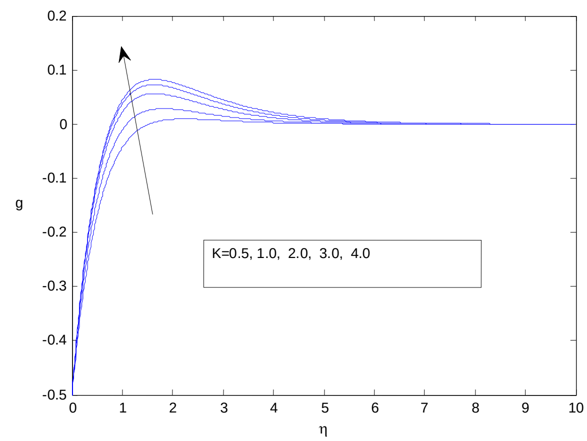

Figure 1(b) Effects of viscosity on Micropolar profile

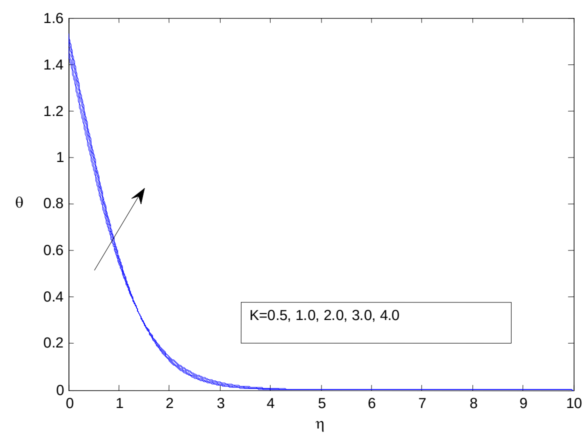

Figure 1(c) Effects of viscosity on Temperature profile

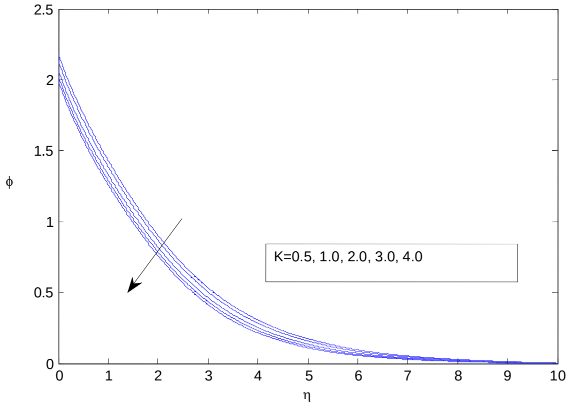

Figure 1(d) Effects of viscosity on nano particle volume fraction profile

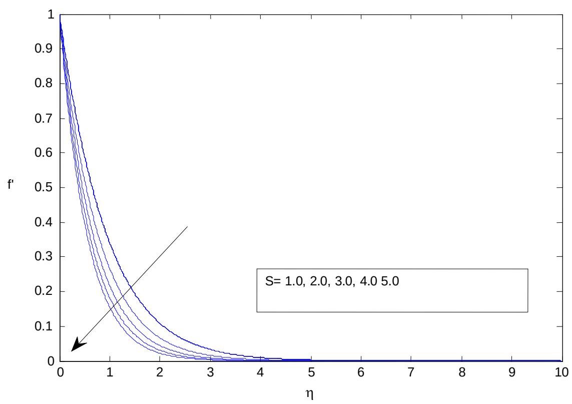

Figure 2(a) Effects of unsteady parameter on velocity profile

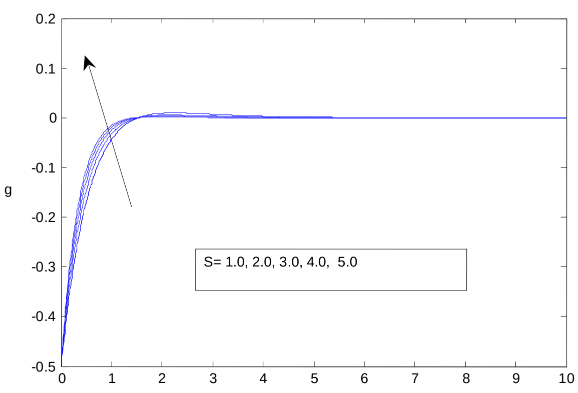

Figure 2(b) Effects of unsteady parameter on micropolar profile

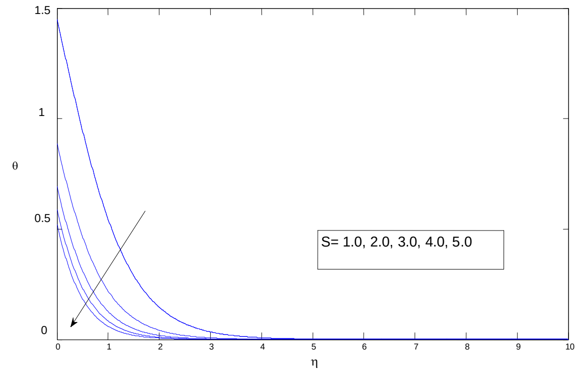

Figure 2(c) Effects of unsteady parameter on temperature profile

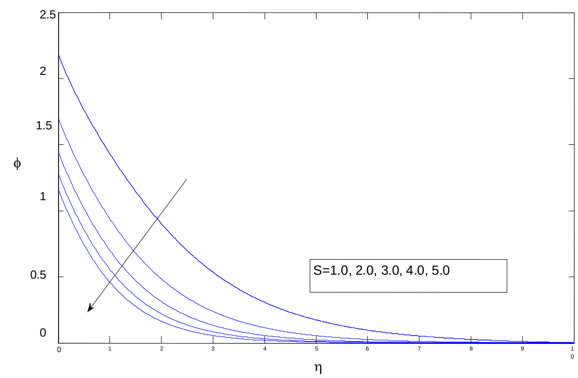

Figure 2(d) Effects of unsteady parameter on nano particle volume fraction profile

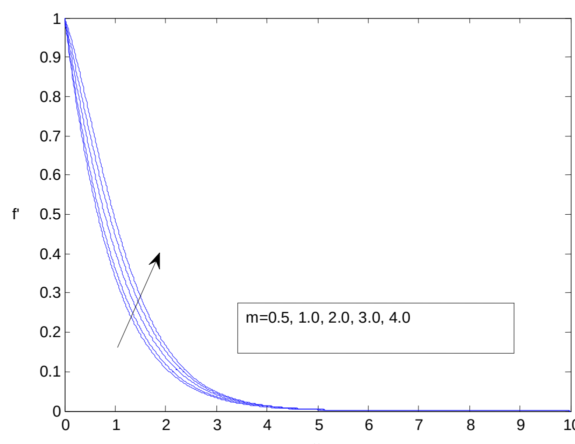

Figure 3(a) Effects of boundary parameter on velocity profile

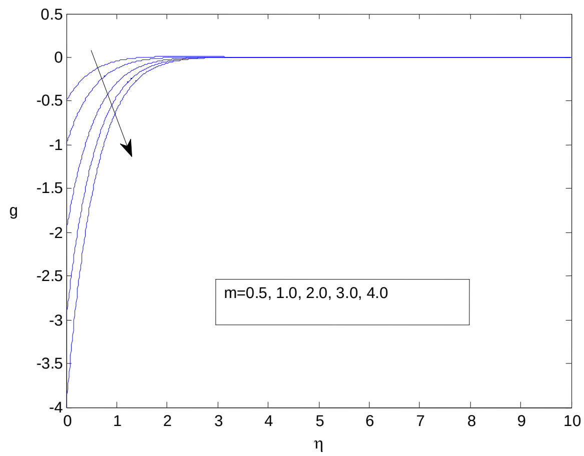

Figure 3(b) Effects of boundary parameter on micropolar profile

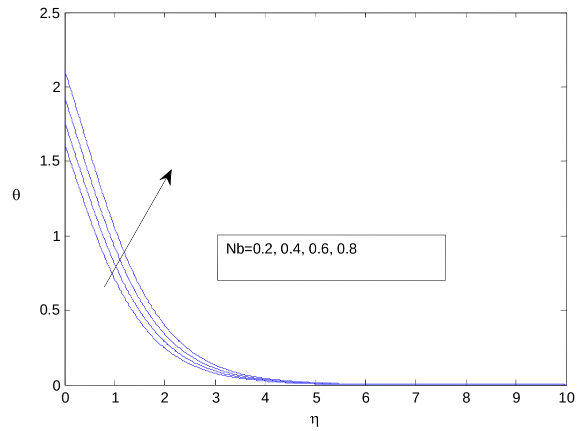

Figure 1(a) Effects of Brownian motion parameter on temperature.

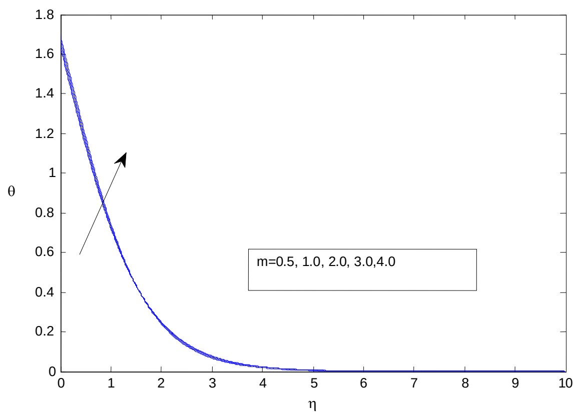

Figure 3(c) Effects of boundary parameter on temperature profile

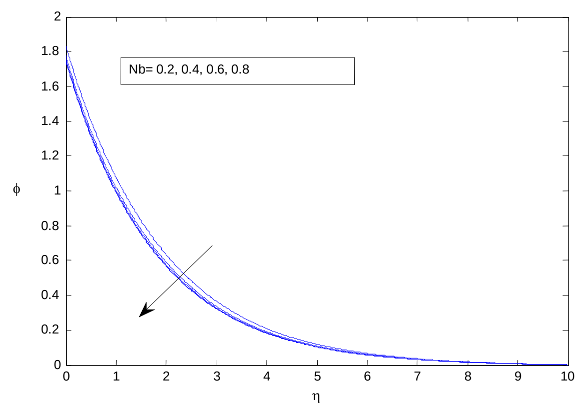

Figure 4(b) Effects of Brownian motion parameter on nano particle volume fraction profile.

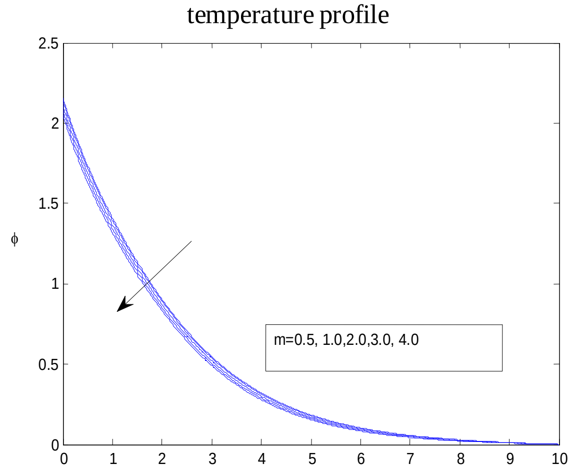

Figure 3(d) Effects of boundary parameter on nanoparticle volume fraction profile

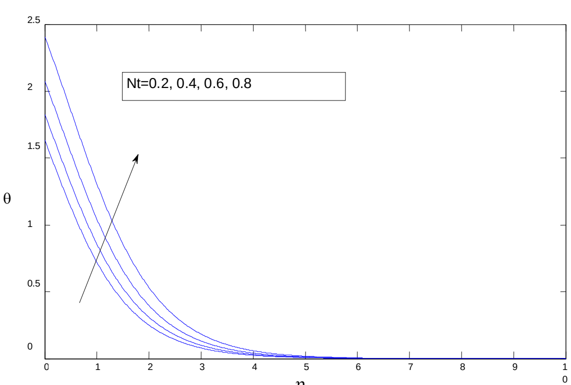

Figure 5(a) Effects of Brownian motion parameter on Temperature profile.

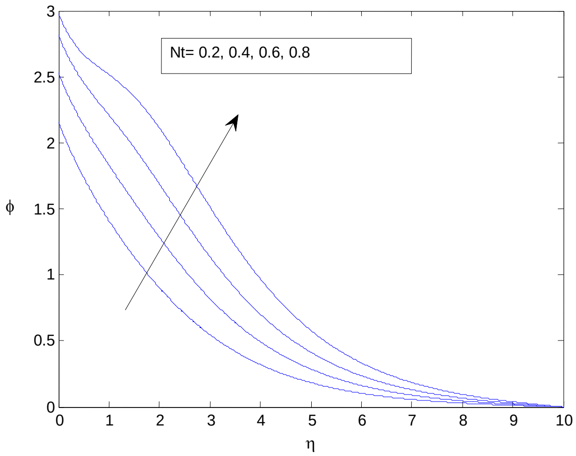

Figure 5(b) Effects of Brownian motion parameter on nanoparticle volume fraction profile.

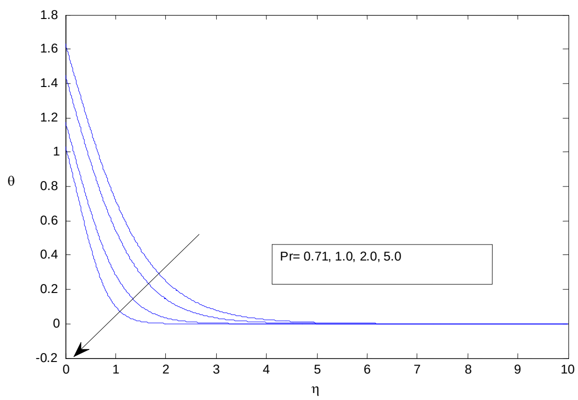

Figure 6(a) Effects of Prandtle number on temperature profile.

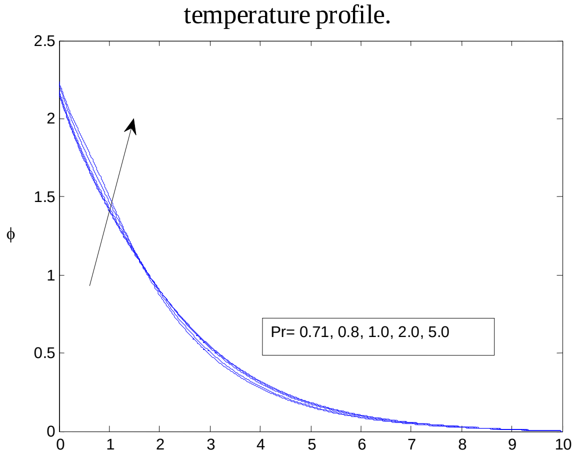

Figure 6(b) Effects of Prandtl number on nanoparticle volume fraction profile.

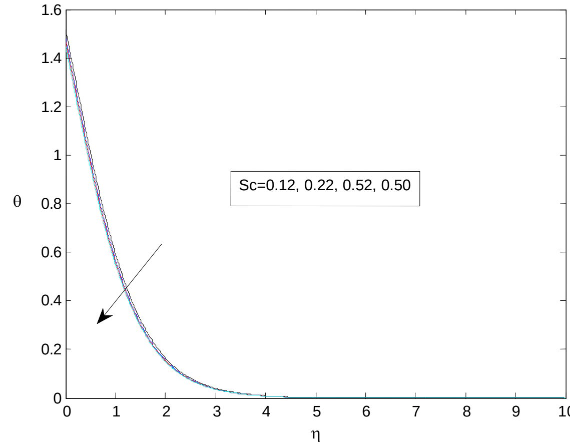

Figure 7(a) effects of Schmidt number on temperature

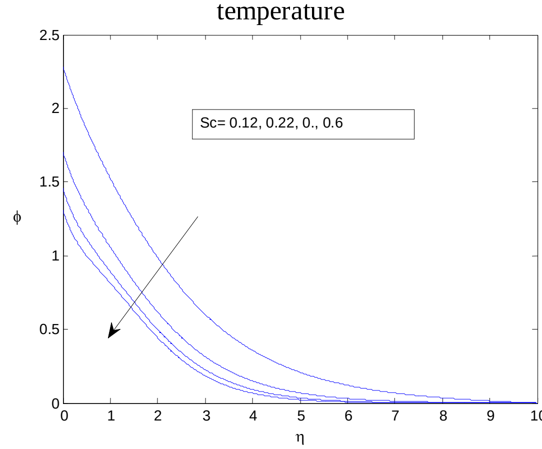

Figure 7(b) effects of Schmidt number on nano particle volume fraction profile

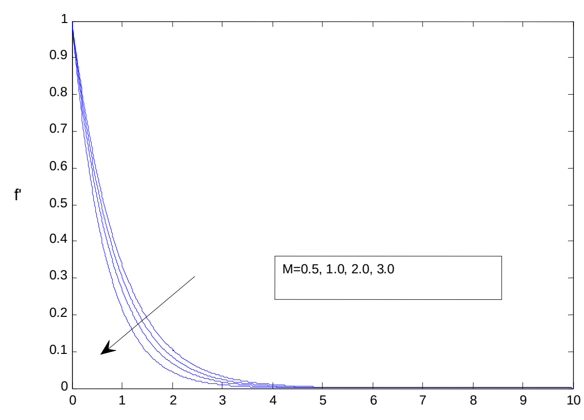

Figure 8(a) effects of Magnetic parameter on velocity profile

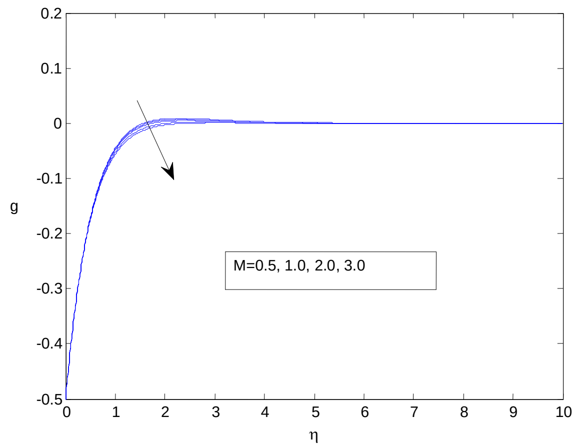

Figure 8(b) effects of Magnetic parameter on micropolar profile

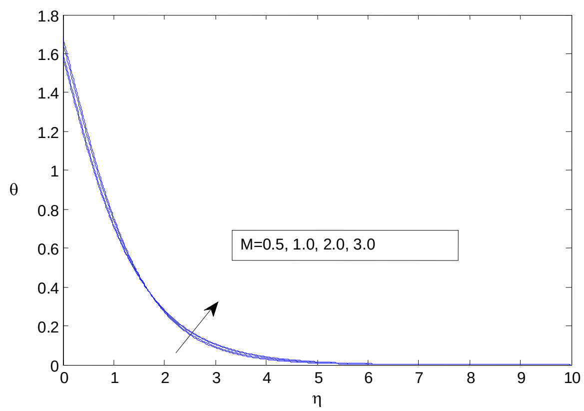

Figure 8(c) effects of Magnetic parameter on temperature profile

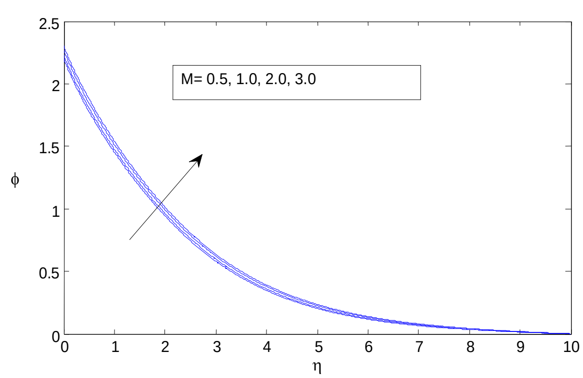

Figure 8(d) effects of Magnetic parameter on nano particle volume fraction profile

which acts against the flow if the magnetic field is applied in the normal direction as in the present problem. As a result velocity profile is decreased. With the increasing magnetic parameter the velocity and micropolar profiles are decreasing. But the temperature profile and nanoparticle volume fraction profile are increasing with the increase of magnetic parameter.

Conclusions

In this paper we investigated the analysis of the flow and heat transfer of micropolar nanofluid with prescribed heat flux over an unsteady stretching sheet and the results are carried out through graphs and we observed that:

- With the increase of viscosity value the velocity profile, micropolar profile and temperature profiles are increasing, while the nano particle volume fraction profile is decreasing.
- With an increasing of unsteady parameter S the velocity profile, temperature profile and nano particle volume fraction profile are decreasing but the micropolar profile is increasing.
- The velocity profile and the temperature profiles are increasing where as the micropolar fluid profile and the concentration of nanoparticle profiles are decreasing with the increase of boundary parameter.
- With the increase of Brownian motion parameter the temperature profile is increasing and the nano particle volume fraction profile is decreasing.
- With an increase of thermophresis parameter Nt the temperature profile and nano particle volume fraction profile is increasing.
- With an increase of Prandtl number the temperature is decreasing and the nano particle volume fraction profiles is increasing.
- With the increasing of Schimdt number the temperature and nano particle volume fraction profile are decreasing.
- The velocity profile and micropolar profile are decreasing but the temperature and nano particle volume fraction profiles are increasing with increase of Magnetic field parameter.

References

1. Takhar, H.S., Bhargava, R., Agrawal, R.S. and Balaji, A.V.S., 'Finite element solution of micropolar fluid flow and heat transfer between two porous discs', Int. J. Eng. Sci.vol. *38*, no 17, pp. 1907–1922 (1992).
2. Mahmoud M.A.A., 'Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity', Physica A. Statistical mechanics and its applications, vol. *375* no 2, pp. 401–410 (2007).
3. Crane, L.J., 'Flow past a stretching plate', Journal of applied mathematics and physics, vol. *21*, no. 4, pp. 645–647 (1970).
4. Gorla, R.S.R., Unsteady mass transfer in the boundary layer on a continuous moving sheet electrod. J. Electrochem. Soc. *125(6)*, 865–869 (1978).
5. Banks, W.H.H., 'Similarity solutions of the boundary layer equation for a stretching Wall', J. Mec. Theor. Appl., vol. *2*, no 3, pp. 375–392 (1983).
6. McLeod, J.B. and Rajagopal, K.R., 'On the uniqueness of flow of a Navier-Stokes fluid due to a stretching boundary', Arch. Ration. Mech. Anal, vol. *98*, no 4, pp. 386–395 (1987).
7. Ahmed A. Afify., 'MHD free convective flow and mass transfer over a stretching sheet with chemical reaction', journal of Heat and Mass Transfer, vol. *40*, no 6, pp. 495–500 (2004).
8. Eringen, A.C., ' Micropolar fluids with stretch', Int. J. Engng. Sci, vol *7*, no 1, pp. 115-127 (1969).
9. Eringen, A.C., 'Theory of thermomicro-fluids', J. Math. Analysis and Applications, vol. *38*, no 2, pp. 480-496 (1972).
10. Eringen, A.C., ' Theory of anisotropic micropolar fluids', Int. J. Engng. Sci., vol. *18*, no 1, pp. 5-17 (1980).
11. Eringen, A.C., 'Memory-dependent orientable nonlocal micropolar fluids', Int. J. Engng. Sci., vol. *29* no 12, pp. 1515-1529 (1991).
12. Peddieson abd John, 'Boundary layer theory for micropolar fluid', Recent Adv Eng Sci., vol *5*, pp. 405–426 (1970).
13. Willson, A.J., 'Boundary layers in micropolar liquids', Proc. Camb Phil Soc, vol. *67*, no 2, pp. 469–476 (1970).
14. Siddheshwar, P. G. and Pranesh, S., 'Effect of a non-uniform basic temperature gradient on Rayleigh-Benard convection in a micropolar fluid', Int J. Eng Sci., vol. *36*, no 11, pp. 1183–1196 (1998).
15. Siddheshwar, P.G. and Pranesh, S., 'Magneto convection in fluids with suspended parti-cles under lg and mg', Aerospace Sci. Tech, vol. *6*, no 2, pp. 105–114 (2002).
16. Siddheshwar, P.G. and Manjunath, S., 'Unsteady convective diffusion with heterogeneous chemical reaction in a plane-Poiseuille flow of a micropolar fluid', Int J Eng Sci., vol. *38*, no 7, pp. 765–783 (2000).
17. Desseaux, A. and Kelson, N.A., 'Flow of a micropolar fluid bounded by a stretching sheet', Anziam J vol. *42*, pp. 536–560 (2000).
18. Kelson, N.A. and Desseaux, A., 'Effects of surface conditions on flow of a micropolar fluid driven by a porous stretching sheet', Int. J. Eng Sci., vol. *39*, pp. 1881– 1897 (2001).
19. Bhargava, R., Kumar, L. and Thakar, H.S., 'Finite element solution of mixed convection micropolar flow driven by a porous stretching sheet', Int. J. Eng Sci. vol. *41*, pp. 2161–2178 (2003).
20. Madhu, M., Kishan, N. and Chamkha, A.J., 'Unsteady flow of a Maxwell nanofluid over a stretching surface in the presents of MHD and thermal radiation effects', Journal of Propulsion and power research.http://dx.doi.org/10.1016/j.jppr.2017.01.002 (2017).
21. Noghrehabadi, A., Izadpanahi, E. and Ghalambaz, M. 'Analyze of fluid flow and heat transfer of nanofluids over a stretching sheet near the extrusion slit', Comput. Fluids, vol. *100*, pp. 227–236 (2014).
22. Hsiao, K. L., 'Nanofluid flow with multimedia physical features for conjugate mixed convection

and radiation', Comput. Fluids, vol. *104,* pp. 1–8 (2014) .

23. Ashorynejad, H.R., Sheikholeslami, M., Pop, I. and Ganji, D.D., 'Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field', Heat Mass Tranf, vol. *49,* no 3, pp. 427–436 (2013).
24. Mustafa, M., Hayat, T. and Alsaedi, A., 'Unsteady boundary layer flow of nanofluid past an impulsively stretching sheet', Journal of Mechanics, vol. *29*, no 3, pp. 423–432 (2013).
25. Turkyilmazoglu, M., 'Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer', Journal of Heat Transfer, ASME, vol. *136,* no 3, 031704 (2013).
26. Madhu, M. and Kishan, N., 'MHD boundary layer flow of a non Newtonian nanofluid past a stretching sheet with a heat source/ sink', Journal of Applied Mechanics and Technical physics, Vol. *57,* no 5, pp. 908-915 (2016).
27. Madhu, M., Kishan, N. and Chamkha, A., 'Bounday layer flow and heat transfer of a non-Newtonian nanofluid over anon linearly stretching sheet', International journal of Numerical methods for Heat and fluid flow, vol. *26,* no 7, pp. 2198-2217 (2016).
28. Mushtaq, A., Mustafa, M., Hayat, T. and Alsaedi, A., 'Nonlinear radiative heat transfer in the flow of nanofluid due to solar energy: A numerical study', J Taiwan Inst Chem Eng, vol. *45,* no 4, pp. 1176–1183 (2014).
29. Turkyilmazoglu, M., 'Nanofluid flow and heat transfer due to a rotating disk', Comp Fluids, vol. *94*, pp. 139–146 (2014).
30. Sheikholeslami, M., Bandpy, M.G., Ellahi, R., Hassan, M. and Soleimani, S., 'Effects of MHD on Cu—water nanofluid flow and heat transfer by means of CVFEM', J Magnetism and Magnetic Materials, vol. *349,* pp. 188–200 (2014).
31. Malvandi, A. and Ganji, D.D., 'Brownian motion and thermophoresis effects on slip flow of alumina/water nanofluid inside a circular microchannel in the presence of a magnetic field', Int. J. Therm Sci., vol. *84,* pp. 196–206 (2014).
32. Malvandi A. and Ganji D.D., 'Mixed convective heat transfer of water/alumina nanofluid inside a vertical microchannel', Powder Technology, vol. *263,* pp. 37–44 (2014).
33. Malvandi, A. and Ganji, D.D., 'Effects of nanoparticle migration on force convection of alumina/water nanofluid in a cooled parallel-plate channel', Adv Powd Technol,vol. *25*, pp. 1369–1375 (2014).
34. Bachok, N., Ishak, A., and Nazar, R.,'Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with prescribed surface heat flux', Int. J. of Mathematical models and methods in applied sciences. Vol. *4*, no 3, pp. 167-176 (2010).
35. C. Kalyani, N. Kishan, and M. Chenna Krishna Reddy., 'MHD boundary layer flow of a nanofluid over an exponentially permeable stretching sheet with radiation and heat source/ sink', Journal of Transport phenomina in nano and micro scales vol. *4,* no 1, Pp. 44-51 (2016).
36. Cebeci and Bradshaw., 'Momentum transfer in boundary layers, thermal and fluids engineering thermal and fluids engineering series. vol. *1* New York, McGraw-Hill Book Co, 407 p. (1977).