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Influence of Heat transfer and Hall effects on a peristaltic pumping of a Newtonian fluid in a planar channel

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Corresponding Author Email:- ravindranathreddy.1982@gmail.com<http://dx.doi.org/10.22147/jusps-B/290502>**Acceptance Date 27th March, 2017, Online Publication Date 2nd May, 2017****Abstract**

In this paper, we studied the effect of hall and heat transfer on the peristaltic pumping of a conducting Newtonian fluid in a channel under the long wavelength approximation. The expressions for the velocity and pressure gradient are obtained analytically. The effects of various pertinent parameters on the pumping characteristics and temperature field are studied in detail with the aid of graphs.

Key words: Hall, Heat transfer, Hartmann number, viscous dissipation, peristaltic pumping.

1. Introduction

Magnetohydrodynamics (MHD) is the science which deals with the movement of a highly conducting fluid in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid². The MHD flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the flow of conductive physiological

fluids, e.g., the blood and blood pump machines, and with the need for theoretical research on the operation of a peristaltic MHD compressor. The effect of moving magnetic field on blood flow was studied by Stud *et al.*¹¹, and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Srivastava and Agrawal¹⁰ considered the blood as an electrically conducting fluid and constitute a suspension of red cell in plasma. Agrawal and Anwaruddin¹ studied the effect of magnetic field on blood flow by taking a simple mathematical model for blood through an equally branched channel with flexible walls executing peristaltic waves using long

wavelength approximation method and observed, for the flow blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations. Hayat *et al.*⁴ discussed the peristaltic transport of a fourth grade fluid in a channel under the effect of magnetic field. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium has discussed by Hayat *et al.*⁵. Srinivas and Kothandapani⁹ have discussed the effect of heat and mass transfer on the MHD peristaltic transport through a porous medium with compliant walls.

The effect of heat transfer on the peristaltic flow of a Newtonian fluid in a vertical annulus under the effect of magnetic field was analyzed by Mekheimer and Elmaboud⁶. Srinivas and Kothandapani⁸ have investigated the influence of MHD and heat transfer on the peristaltic flow of a Newtonian in an asymmetric channel. Vasudev *et al.*¹² have studied the influence of magnetic field and heat transfer on peristaltic flow of Jeffrey fluid through a porous medium in an asymmetric channel. Recently, Ravindranath Reddy *et al.*⁷ have investigated the effect of heat transfer on the peristaltic transport of a Newtonian fluid through a porous medium in a vertical channel with radiation.

In view of these, we studied the effect of hall and heat transfer on the peristaltic pumping of a conducting Newtonian fluid in a channel under the long wavelength approximation. The expressions for the velocity and pressure gradient are obtained analytically. The effects of various pertinent parameters on the pumping characteristics and temperature field are studied in detail with the aid of graphs.

2. Mathematical Formulation :

We consider the peristaltic pumping of a conducting Newtonian fluid flow in a channel of half-width a . A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. A uniform magnetic field B_0 is applied in the transverse direction to the flow. Fig.1 shows the physical model of the problem.

The wall deformation is given by

$$H(X, t) = a + b \sin \left[\frac{2\pi}{\lambda} (X - ct) \right] \quad (2.1)$$

where b is the amplitude, λ the wavelength and c is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y) . The transformation between these two frames is given by

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V \quad \text{and} \quad p(x) = P(X, t), \quad (2.2)$$

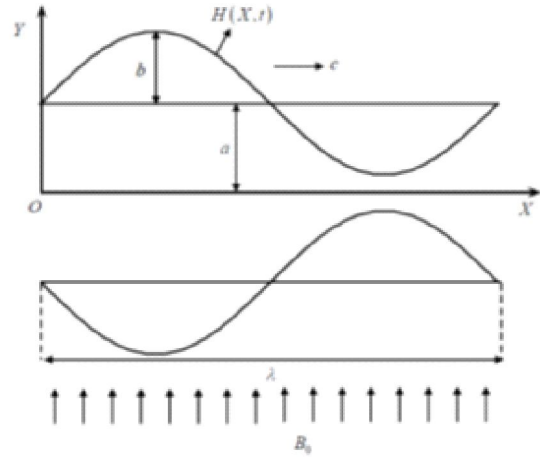


Fig. 1 Physical Model

where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.3)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma B_0^2}{1+m^2} (mv - (u+c)) \quad (2.4)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\sigma B_0^2}{1+m^2} (m(u+c) + v) + \frac{M^2}{1+m^2} (m\delta v - (u+1)) \quad (2.5)$$

$$\rho \zeta \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \nabla^2 T + \mu \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \quad (2.6)$$

where ρ is the density, T is the temperature, α is the coefficient of linear thermal expansion of the fluid, k , is thermal conductivity, ζ is the specific heat at constant volume, m is the Hall parameter, μ is the viscosity of the fluid and σ is the electrical conductivity.

The dimensional boundary conditions are

$$u = -c \quad \text{at} \quad y = H \quad (2.7)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.8)$$

$$T = T_1 \quad \text{at} \quad y = H \quad (2.9)$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.10)$$

Introducing the non-dimensional quantities

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c\delta}, \delta = \frac{a}{\lambda},$$

$$\bar{p} = \frac{pa^2}{\mu c \lambda}, \bar{t} = \frac{ct}{\lambda}, h = \frac{H}{a}, \phi = \frac{b}{a}, \bar{q} = \frac{q}{ac}$$

$$\Theta = \frac{T - T_0}{T_1 - T_0}, \text{Pr} = \frac{\rho v \zeta}{k}, \text{Ec} = \frac{c^2}{\zeta (T_1 - T_0)}$$

into equations (2.3) to (2.6), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.11)$$

$$\text{Re} \delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$+ \frac{M^2}{1+m^2} (m\delta v - (u+1)) \quad (2.12)$$

$$\text{Re} \delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left(\delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\delta M^2}{1+m^2} (m(u+1) + \delta v) \quad (2.13)$$

$$\text{Re} \delta \left[u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} \right] = \frac{1}{\text{Pr}} \left(\delta^2 \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right) + \text{Ec} \left\{ 4\delta^2 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \delta^4 \left(\frac{\partial v}{\partial x} \right)^2 + 2\delta^2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right\} \quad (2.14)$$

Using long wavelength (*i.e.*, $\delta \ll 1$) approximation, the equations (2.12) and (2.14) become

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2} u = \frac{\partial p}{\partial x} + \frac{M^2}{1+m^2} \quad (2.15)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2.16)$$

$$\frac{1}{\text{Pr}} \left[\frac{\partial^2 \Theta}{\partial y^2} \right] + \text{Ec} \left[\frac{\partial u}{\partial y} \right]^2 = 0 \quad (2.17)$$

From Eq. (2.16), it is clear that p is independent of y . Therefore Eq. (2.15) can be rewritten as

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2} u = \frac{dp}{dx} + \frac{M^2}{1+m^2} \quad (2.18)$$

The corresponding non-dimensional boundary conditions are given as

$$u = -1 \quad \text{at} \quad y = h \quad (2.19)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.20)$$

$$\frac{\partial \Theta}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.21)$$

$$\Theta = 1 \quad \text{at} \quad y = h = 1 + \phi \sin 2\pi x \quad (2.22)$$

Knowing the velocity, the volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u dy. \quad (2.23)$$

The instantaneous flow $Q(X, t)$ in the laboratory frame is

$$Q(X, t) = \int_0^h U dY = \int_0^h (u + 1) dy = q + h \quad (2.24)$$

The time averaged volume flow rate \bar{Q} over

one period $T \left(= \frac{\lambda}{c} \right)$ of the peristaltic wave is given by

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 \quad (2.25)$$

3. Solution

Solving Eq. (2.18) together with the boundary conditions (2.19) and (2.20), we get

$$u = \frac{1}{\alpha^2} \frac{dp}{dx} \left[\frac{\cosh \alpha y}{\cosh \alpha h} - 1 \right] - 1 \quad (3.1)$$

Solving Eq. (2.17) using the Eq. (3.1) and the boundary conditions (2.21) and (2.22) to get the temperature distribution as

$$\theta = 1 + \frac{\text{Pr} Ec}{\alpha^2 \cosh^2 \alpha h} \left(\frac{dp}{dx} \right)^2 \left\{ \left(\frac{\cosh 2\alpha y - \cosh 2\alpha h}{4\alpha^2} \right) - \left(\frac{y^2 - h^2}{2} \right) \right\} \quad (3.2)$$

The volume flow rate q in a wave frame of reference is given by

$$q = \frac{1}{\alpha^3} \frac{dp}{dx} \left[\frac{\sinh \alpha h - \alpha h \cosh \alpha h}{\cosh \alpha h} \right] - h \quad (3.3)$$

From Eq. (3.3), we write

$$\frac{dp}{dx} = \frac{(q + h) \alpha^3 \cosh \alpha h}{\sinh \alpha h - \alpha h \cosh \alpha h} \quad (3.4)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.5)$$

4. Results and Discussion

Fig. 2 illustrates the variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number M for $\phi = 0.6$ and $m = 0.3$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing M .

The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall parameter m for $\phi = 0.6$ and $M = 1$ is shown in Fig. 3. It is observed that, the axial pressure

gradient $\frac{dp}{dx}$ decreases with increasing m .

Fig. 4 depicts the variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio ϕ for $M = 1$ and $m = 0.3$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing ϕ .

The variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of Hartmann number M with $\phi = 0.6$ and $m = 0.3$ is depicted in Fig. 5. It is found that, the time-averaged flow rate \bar{Q}

increases in the pumping region ($\Delta p = 0$) with increasing M , while it decreases in both the free-pumping ($\Delta p = 0$) and co-pumping ($\Delta p < 0$) regions with increasing M .

Fig. 6 shows the variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of Hall parameter m with $\phi = 0.6$ and $M = 1$. It is observed that, the time-averaged flow rate \bar{Q} decreases in the pumping region with an increase in m , while it increases in both the free-pumping and co-pumping regions with increasing m .

The variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.3$ is shown in Fig. 7. It is found that the time-averaged flow rate \bar{Q} increases with increasing amplitude ratio ϕ in both the pumping and free pumping regions, while it decreases with increasing amplitude ratio ϕ in the co-pumping region for chosen $\Delta p (< 0)$.

Fig. 8 shows the temperature profiles for different values of Hartmann number M with $\phi = 0.6$, $Pr Ec = 1$, $\bar{Q} = -1$ and $m = 0.3$. It is observed that, the temperature θ decreases with increasing the Hartmann number M .

Temperature profiles for different values of Hall parameter m with $\phi = 0.6$, $Pr Ec = 1$, $\bar{Q} = -1$ and $M = 1$ is shown in Fig. 9. It is found the temperature θ increases with increase in Hall parameter m .

Fig. 10 illustrates the temperature profiles for different values of $Pr Ec$ with $\phi = 0.6$, $M = 1$, $\bar{Q} = -1$ and $m = 0.3$. It is noted that, the temperature θ increases with increasing $Pr Ec$.

Temperature profiles for different values of

amplitude ratio ϕ with $M = 1$, $Pr Ec = 1$, $\bar{Q} = -1$ and $m = 0.3$ is depicted in Fig. 11. It is observed that, the temperature θ decreases with increasing ϕ in the middle of the channel, while it increases with ϕ near the wall of the channel.

5. Conclusions

In this paper, the effects of heat transfer and hall on the peristaltic flow of a conducting fluid in a symmetric channel under the assumption of long wavelength approximation is investigated. The expressions for the velocity, temperature and pressure gradient are obtained analytically. It is found that, the pressure gradient and the time-averaged flow rate in the pumping region are increases with increasing Hartmann number M and amplitude ratio ϕ , while they decreases with increasing hall parameter m . The temperature θ increases with increasing Hall parameter m and $Pr Ec$, while it decreases with increasing Hartmann number M and amplitude ratio ϕ .

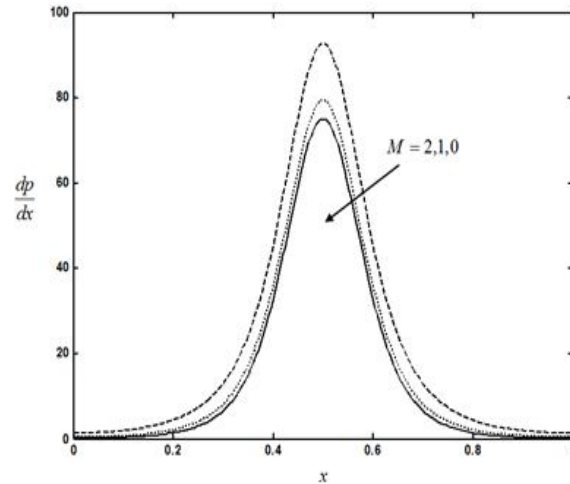


Fig. 2. The variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number M for $\phi = 0.6$, $\bar{Q} = -1$ and $m = 0.3$.

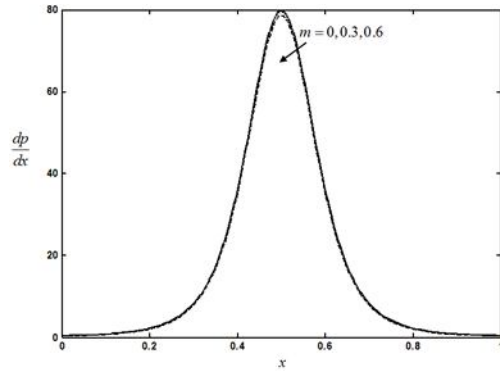


Fig. 3. The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall parameter m for $\phi = 0.6$, $\bar{Q} = -1$ and $M = 1$.

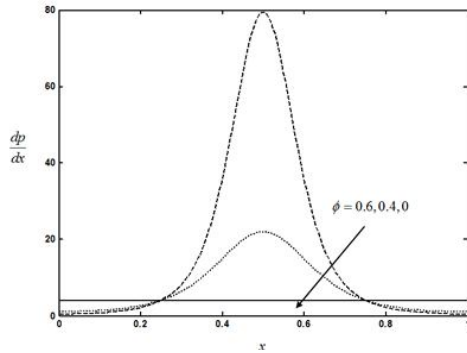


Fig. 4. The variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio ϕ for $M = 1$, $\bar{Q} = -1$ and $m = 0.3$.

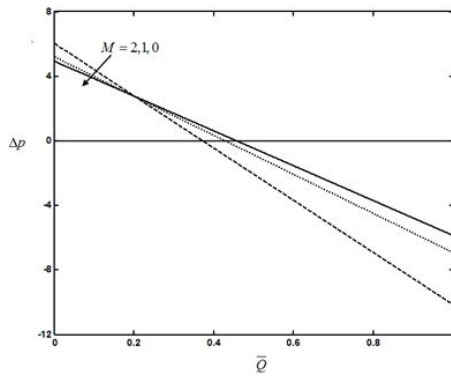


Fig. 5. The variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of Hartmann number M with $\phi = 0.6$ and $m = 0.3$.

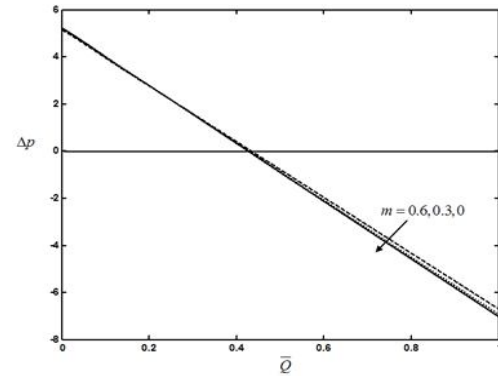


Fig. 6. The variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of Hall parameter m with $\phi = 0.6$, and $M = 1$.

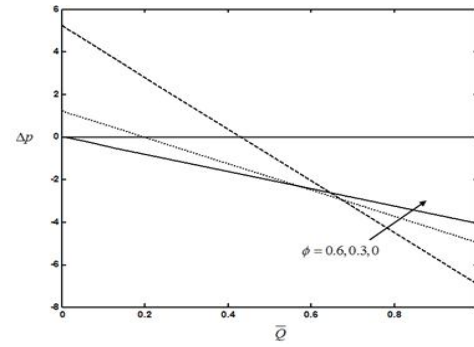


Fig. 7. The variation of pressure rise Δp with time-averaged flow rate \bar{Q} for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.3$.

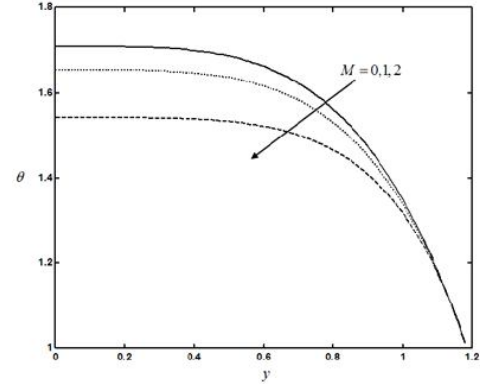


Fig. 8. Temperature profiles for different values of Hartmann number M with $\phi = 0.6$, $Pr Ec = 1$, $\bar{Q} = -1$ and $m = 0.3$.

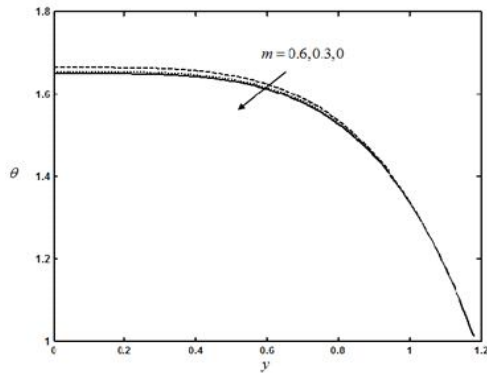


Fig. 9. Temperature profiles for different values of Hall parameter m with $\phi = 0.6$, $Pr Ec = 1$, $\bar{Q} = -1$ and $M = 1$.

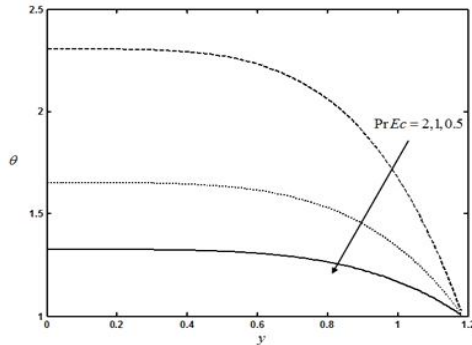


Fig. 10. Temperature profiles for different values of $Pr Ec$ with $\phi = 0.6$, $M = 1$, $\bar{Q} = -1$ and $m = 0.3$.

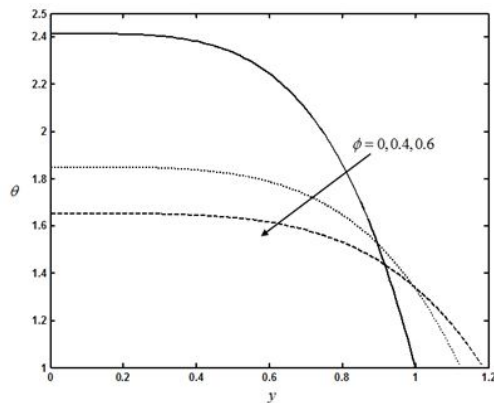


Fig. 11. Temperature profiles for different values of amplitude ratio ϕ with $M = 1$, $Pr Ec = 1$, $\bar{Q} = -1$ and $m = 0.3$.

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