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Common Fixed Point for Gregus Maps on Intuitionistice Fuzzy Metric Space

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Abstract

In this paper, we have two sections. In the first section we establish a common fixed point theorem for Gregus type mappings in intuitionistic fuzzy metric spaces using the (CLRg) property. Our result generalizes and improves upon, among others the corresponding results of Gregus[10], Fisher and Sessa[8], Huang and Cho[11], Jungck[12], Ciric[1], Diwan and Rajlaxmi Gupta[2] and Mukherjee and Verma[14].

Key words: Gregus type mappings; weakly compatible mappings; (CLRg) property; property (E.A); intuitionistics fuzzy metric spaces.

1 Introduction

Zadeh[16] investigated the concept of a fuzzy set in his seminal paper. Since then, with a view to utilize this concept in topology and analysis, many authors have extensively developed the theory of fuzzy sets along with their applications. In the last two decades there has been a tremendous development and growth in fuzzy mathematics.

The concept of fuzzy metric space was introduced by Kramosil and Michalek in[13] which opened an avenue for further development of analysis in such spaces. Further, George and Veeramani[9] modified the concept of fuzzy metric space introduced by Kramosil and Michalek[13] with a view to obtain a Hausdoroff topology which has very important applications in quantum particle physics, particularly in connection with both string and ε^{∞} theory. Fuzzy set theory also has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc.

Intuitionistic fuzzy metric notion is also useful in modeling some physical problems wherein it is necessary to study the relationship between two probability functions. For instance, it has a concrete physical visualization in the context of two-slit experiment as the foundation of E-infinity theory of high energy physics whose details are available in El Naschie in[3], El Naschie in[4], El Naschie in[5], El Naschie in[6], El Naschie[7]. Since the topology induced by intuitionistic fuzzy metric coincides with the topology induced by fuzzy metric, Saadati *et al.*[15] reframed the idea of intuitionistic fuzzy metric spaces and proposed a new notion under the name of modified intuitionistic fuzzy metric spaces by introducing the idea of continuous *t*-representable.

2 Definitions and Preliminaries:

2.1 Definition: A negator on L^* is any decreasing mapping $\mathcal{N}: L^* \to L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L^*$, then $\mathcal{N}$ is called an involutive negator. A negator on [0, 1] is a decreasing mapping: $[0, 1] \to [0, 1]$ satisfying $\mathcal{N}(0)=1$ and $\mathcal{N}(1)=0$. $\mathcal{N}_s$ denotes the standard negator on [0, 1] defined as (for all $x \in [0, 1]$) $\mathcal{N}_s(x) = 1 - x$.

2.2 Definition: Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be an intuitionistic fuzzy metric space. For $t > 0$, define the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$, as

$$B(x, r, t) = \{y \in X : \mathcal{M}_{M,N}(x, y, t) >_{L^*} (\mathcal{N}_s(r), r)\}$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subseteq A$. Let $\mathcal{T}_{\mathcal{M}_{M,N}}$ denote the family of all open subsets of X. $\mathcal{T}_{\mathcal{M}_{M,N}}$ is called the topology induced by intuitionistic fuzzy metric.

2.3 Definition: Two self-mappings f and g of a metric space (X, d) are said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that for some $t \in X$.

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t,$$

for some $t \in X$.

2.4 Definition: Two self-mappings f and g of a metric space (X, d) are said to satisfy the common limit in the range of g property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$.

In what follows, the common limit in the range of g property will be denoted by the (CLRg) property.

Now, we give examples of mappings f and g which are satisfying the (CLRg) property.

3 Main Results

3.1 Theorem: Let f and g be two weakly compatible self-mappings of a metric space (X, d) such that

(i) f and g satisfy the (CLRg) property.

(ii) $M^p(fx, fy, kt) \geq a \min M^p(gx, gy, t), M^p(gx, fy, t)\}$
$+ b \min\{M^p(fx, gx, t), M^p(fy, gy, t)\}$
$+ c \min\{M^p(gx, gy, t), M^p(fx, gx, t), M^p(fy, gy, t)\}$

(iii) $N^p(fx, fy, kt) \leq a \max\{N^p(gx, gy, t), N^p(gx, fy, t)\}$
$+ b \max\{N^p(fx, gx, t), N^p(fy, gy, t)\}$
$+ c \max\{N^p(gx, gy, t), N^p(fx, gx, t), N^p(fy, gy, t)\}$

for all $x, y \in X$, where $a, b, c > 0$, $p \geq 1$, $0 < k < 1$ and $a + b + c = 1$, then f and g have a unique common fixed point.

Proof: Since f and g satisfy the (CLRg) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu, \text{ for some u in X.}$$

Now we show that fu = gu. Suppose that fu ≠ gu. Then using above conditions (ii) and (iii) with $x = x_n$ and $y = u$, we get

$$M^p(fx_n, fu, kt) \geq a \min\{M^p(gx_n, gu, t), M^p(gx_n, fu, t)\} + b \min\{M^p(fx_n, gx_n, t), M^p(fu, gu, t)\} + c \min\{M^p(gx_n, gu, t), M^p(fx_n, gx_n, t), M^p(fu, gu, t)\}$$

$$N^p(fx_n, fu, kt) \leq a \max\{N^p(gx_n, gu, t), N^p(gx_n, fu, t)\} + b \max\{N^p(fx_n, gx_n, t), N^p(fu, gu, t)\} + c \max\{N^p(gx_n, gu, t), N^p(fx_n, gx_n, t), N^p(fu, gu, t)\}.$$

if $n \to \infty$ then this yields

$$M^p(gu, fu, kt) \geq a \min\{M^p(gu, gu, t), M^p(gu, fu, t)\} + b \min\{M^p(gu, gu, t), M^p(fu, gu, t)\} + c \min\{M^p(gu, gu, t), M^p(gu, gu, t), M^p(fu, gu, t)\}$$

$$M^p(gu, fu, kt) \geq a\, M^p(gu, fu, t) + b\, M^p(fu, gu, t) + c\, M^p(fu, gu, t)$$

$$M^p(gu, fu, kt) \geq a + b + c)\, M^p(gu, fu, t)$$

$$M^p(gu, fu, kt) \geq M^p(gu, fu, t).$$

which is a contradiction. Hence gu = fu = u.

Since f and g are weakly compatible: fu = gu implies fgu = gtu and hence ffu = fgu = gfu.

Finally, we show that fu is a common fixed point of f and g. Let fu ≠ ffu. Then using above conditions (ii) and (iii) with x = u and y = fu, we get

$$M^p(fu, ffu, kt) \geq a \min\{M^p(gu, gfu, t), M^p(gu, ffu, t)\} + b \min\{M^p(fu, gu, t), M^p(ffu, gfu, t)\} + c \min\{M^p(gu, gfu, t), M^p(fu, gu, t), M^p(ffu, gfu, t)\}$$

$$\geq a \min\{M^p(fu, ffu, t), M^p(fu, ffu, t)\} + b \min\{M^p(fu, fu, t), M^p(ffu, ffu, t)\} + c \min\{M^p(fu, ffu, t), M^p(fu, fu, t), M^p(ffu, ffu, t)\}$$

$$\geq (a + b + c)\, M^p(ffu, ffu, t)\}$$

$$N^p(fu, ffu, kt) \leq a \max\{N^p(gu, gfu, t), N^p(gu, ffu, t)\} + b \max\{N^p(fu, gu, t), N^p(ffu, gfu,$$

$+ c \max\{N^p(gu, gfu, t), N^p(fu, gu, t), N^p(ffu, gfu, t)$

$\leq a \max\{N^p(fu, ffu, t), N^p(fu, ffu, t)\}$
$+ b \max\{N^p(fu, fu, t), N^p(ffu, ffu, t)\}$
$+ c \max\{N^p(fu, ffu, t), N^p(fu, fu, t), N^p(ffu, ffu, t)$

$\leq (a + b + c) N^p(fu, ffu, t).$

which is a contradiction. Hence fu = ffu and gfu = ffu = fu. Thus fu is a common fixed point of mappings f and g.

Let t and v two common fixed points of mappings f and g. Then using above conditions (ii) and (iii), we have

$M^p(t, v, kt)=M^p(ft, fv, kt)\geq a \min\{M^p(gt, gv, t)\}$
$+ b \min\{M^p(ft, gt, t), M^p(fv, gv, t)\}$
$+ c \min\{M^p(gt, gv, t), M^p(ft, gt, t), M^p(fv, gv, t)\}$

$= a M^p(t, v, t)+b \min\{M^p(t, t, t), M^p(v, v, t)\}$
$+c \min\{M^p(t, v, t), M^p(t, t, t), M^p(v, v, t)\}$

$M^p(t, v, kt) = (a + b + c) M^p(t, v, t).$

$N^p(t, v, kt)= N^p(gt, v, kt) \leq a \max\{N^p(gt, gv, t), N^p(gt, fv, t)\}$
$+ b \max\{N^p(ft, gt, t), N^p(fv, gv, t)\}$
$+ c \max\{N^p(gt, gv, t), N^p(ft, gt, t), N^p(fv, gv, t)\}$

$= a N^p(t, v, t)+b \max\{N^p(t, t, t), N^p(v, v, t)\}$
$+ c \max\{N^p(t, v, t), N^p(t, t, t), N^p(v, v, t)\}$

$N^p(t, v, kt) = (a + b + c) N^p(t, v, t)$

which is a contradiction. Hence t = v. Thus f and g have a unique common fixed point in X.

3.2 Corollary: If p = 1 in condition (ii) and (iii) of the theorem 2.1.3.1, we get the following corollary:

Let f and g be two weakly compatible self-mappings of a metric space (X, d) such that

(i) S and T satisfy the (CLRg) property,

(ii) $M(fx, fy, kt) \geq a \min M(gx, gy, t), M(gx, fy, t)\}$
$+ b \min\{M(fx, gx, t), M(fy, gy, t)\}$
$+c \min\{M(gx, gy, t), M(fx, gx, t), M^p(fy, gy, t)\}$

(iii) $N(fx, fy, kt) \leq a \max\{N(gx, gy, t), N(gx, fy, t)\}$
$+ b \max\{N(fx, gx, t), N(fy, gy, t)\}$
$+ c \max\{N(gx, gy, t), N(fx, gx, t), N(fy, gy, t)\}$

for all x, y ∈ X, where a, b, c> 0, a + b + c = 1, then f and g have a unique common fixed point.

3.3 Corollary: If p = 1 and c = 0 in condition (ii) and (iii) of the theorem 2.1.3.1, we get the following corollary:

Let f and g be two weakly compatible self-mappings of a metric space (X, d) such that

(i) S and T satisfy the (CLRg) property,

(ii) $M(fx, fy, kt) \geq a \min M(gx, gy, t), M(gx, fy, t)\}$
$+ b \min\{M(fx, gx, t), M(fy, gy, t)\}$

(iii) $N(fx, fy, kt)$ a $\max\{N(gx, gy, t), N(gx, fy, t)\}$
$+ b \max\{N(fx, gx, t), N(fy, gy, t)\}$

for all x, y ∈ X, where a, b > 0, a + b = 1, then f and g have a unique common fixed point.

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