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New Multiplicative Arithmetic-Geometric Indices

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Abstract

In this Paper, we introduce the second, third, fourth and fifth multiplicative arithmetic-geometric indices of a molecular graph. We compute the fifth multiplicative arithmetic-geometric index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

Key words : molecular graph, fifth multiplicative arithmetic-geometric index, nanostructures.
Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. Introduction

Let G be a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of G is the graph obtained from G by replacing each of its edges by a path of length two. We refer to [1, 2] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numeric quantity from structural graph of a molecule. These indices are useful for establishing correlation between the structures of a molecular compound and its physico-chemical properties.

Very recently Kulli[3] introduced the first multiplicative arithmetic-geometric index of a graph G and it is defined as

$$AG_1II(G) = \prod_{uv \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u)d_G(v)}}.$$

Many other multiplicative indices were studied, for example, in[4,5,6,7,8,9,10,11].

Motivated by the definition of the first multiplicative arithmetic-geometric index and by previous research on topological indices, we propose the second, third, fourth and fifth multiplicative arithmetic-geometric indices of a graph as follows:

The second multiplicative arithmetic-geometric index of a graph *G* is defined as

$$AG_2II(G) = \prod_{uv\in E(G)} \frac{n_u + n_v}{2\sqrt{n_u n_v}}$$

where the number n_u of vertices of *G* lying closer to the vertex *u* than to the vertex *v* for the edge *uv* of a graph *G*.

The third multiplicative arithmetic-geometric index of a graph *G* is defined as

$$AG_3II(G) = \prod_{uv\in E(G)} \frac{m_u + m_v}{2\sqrt{m_u m_v}}$$

where the number m_u of edges of *G* lying closer to the vertex *u* than to the vertex *v* for the edge uv of a graph *G*.

The fourth multiplicative arithmetic-geometric index of a graph *G* is defined as

$$AG_4II(G) = \prod_{uv\in E(G)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u)\varepsilon(v)}}$$

where the number ε(u) is the eccentricity of vertex u.

The fifth multiplicative arithmetic-geometric index of a graph *G* is defined as

$$AG_5II(G) = \prod_{uv\in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u) S_G(v)}}, \quad \text{where} \quad S_G(u) = \sum_{uv\in E(G)} d_G(v).$$

We need the following results.

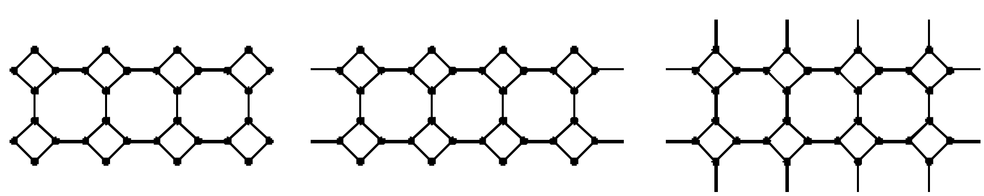
Lemma 1[1]. Let *G* be a *(p, q)* graph. Then *L(G)* has *q* vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

Lemma 2[1]**.** Let *G* be a *(p, q)* graph. Then *S(G)* has *p*+*q* vertices and 2*q* edges.

In this paper, we compute the fifth multiplicative arithmetic-geometric index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

2. 2 D-latice, nanotube and nanotorus of $TUC_4C_8[p, q]$

We consider the graph of 2*D*-lattice, manotube and nanotorus of $TUC_4C_8[p, q]$ where *p* and *q* denote the number of squares in a row and the number of rows of squares respectively. These graphs are shown in Figure 1



G_1 H_1 K_1

(a) (b) (c)

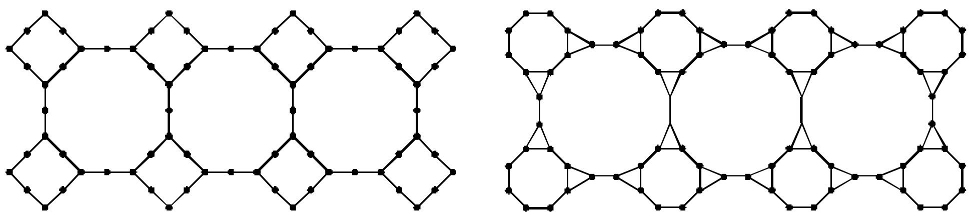
Figure 1

(a) 2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4, 2]$ nanotorus

By algebraic method, we get $|V(G_1)| = 4pq$, $|E(G_1)| = 6pq - p - q$; $|V(H_1)| = 4pq$, $|E(H_1)| = 6pq - p$; $|V(K_1)| = 4pq$, $|E(K_1)| = 6pq$.

3. Results for 2*D*-lattice of $TUC_4C_8[p, q]$

The line graph of the subdivision graph of 2*D*-lattice of $TUC_4C_8[p, q]$ is shown in Figure 2(b).



(a) (b)

Figure 2

(a) subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$ (b) line graph of the subdivision graph of $TUC_4C_8[p, q]$

Theorem 1. Let G be the line graph of the subdivision graph of 2*D*-lattice of $TUC_4C_8[p, q]$. Then

$$AG_5II(G) = \left(\frac{9}{4\sqrt{5}}\right)^{8} \times \left(\frac{13}{4\sqrt{10}}\right)^{4(p+q-2)} \times \left(\frac{17}{12\sqrt{2}}\right)^{8(p+q-2)}, \qquad \text{if } p>1, q>1,$$

$$= \left(\frac{9}{4\sqrt{5}}\right)^{4} \times \left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)} \times \left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)}, \qquad \text{if } p>1, q=1.$$

Proof: The 2*D* - lattice of $TUC_4C_8[p, q]$ is a graph with $4pq$ vertices and $6pq - p - q$ edges. By Lemma 2, the subdivision graph of 2*D*-lattice of $TUC_4C_8[p, q]$ is a graph with $10pq - p - q$ vertices and $2(6pq - p - q)$ edges. Thus by Lemma 1, G has $2(6pq - p - q)$ vertices and $18pq - 5p - 5q$ edges. It is easy to see that the vertices of G are either of degree 2 or 3, see Figure 2(b). Therefore we have partition of the edge set of G as follows.

Table 1. Edge partition of G with $p>1$ and $q>1$

$S_G(u), S_G(v)\backslash uv\in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	4	8	$2(p+q-4)$	$4(p+q-2)$	$8(p+q-2)$	$2(9pq+10) - 19(p+q)$

Table 2. Edge partition of G with $p>1$ and $q = 1$

$S_G(u), S_G(v)\backslash uv\in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	6	4	$2(p-2)$	$4(p-1)$	$2(p-1)$	$4(p-1)$	$p-1$

Case 1. Suppose $p > 1$ and $q > 1$.

By algebraic method, we obtain $|V_4|=8$, $|V_5|=4(p+q-2)$, $|V_8|=4(p+q-2)$ and $|V_9|=2(6pq-5p-5q+4)$. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 1.

To compute $AG_5II(G)$, we see that

$$AG_5II(G)=\prod_{uv\in E(G)}\frac{S_G(u)+S_G(v)}{2\sqrt{S_G(u)S_G(v)}}$$

$$=\left(\frac{4+4}{2\sqrt{4\times4}}\right)^4\times\left(\frac{4+5}{2\sqrt{4\times5}}\right)^8\times\left(\frac{5+5}{2\sqrt{5\times5}}\right)^{2(p+q-4)}\times\left(\frac{5+8}{2\sqrt{5\times8}}\right)^{4(p+q-2)}$$

$$\times\left(\frac{8+9}{2\sqrt{8\times9}}\right)^{8(p+q-2)}\times\left(\frac{9+9}{2\sqrt{9\times9}}\right)^{2(9pq+10)-19(p+q)}$$

$$=(1)^4\times\left(\frac{9}{4\sqrt{5}}\right)^8\times(1)^{2(p+q-4)}\times\left(\frac{13}{4\sqrt{10}}\right)^{4(p+q-2)}\times\left(\frac{17}{12\sqrt{2}}\right)^{8(p+q-2)}\times(1)^{2(9pq+10)-19(p+q)}$$

$$=\left(\frac{9}{4\sqrt{5}}\right)^8\times\left(\frac{13}{4\sqrt{10}}\right)^{4(p+q-2)}\times\left(\frac{17}{12\sqrt{2}}\right)^{8(p+q-2)}.$$

Case 2. Suppose p>1 and $q = 1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 2.

$$AG_5II(G)=\prod_{uv\in E(G)}\frac{S_G(u)+S_G(v)}{2\sqrt{S_G(u)S_G(v)}}$$

$$=\left(\frac{4+4}{2\sqrt{4\times4}}\right)^6\times\left(\frac{4+5}{2\sqrt{4\times5}}\right)^4\times\left(\frac{5+5}{2\sqrt{5\times5}}\right)^{2(p-2)}\times\left(\frac{5+8}{2\sqrt{5\times8}}\right)^{4(p-1)}$$

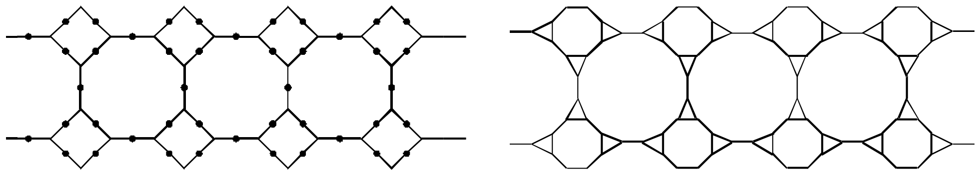
$$\times\left(\frac{8+8}{2\sqrt{8\times8}}\right)^{2(p-1)}\times\left(\frac{8+9}{2\sqrt{8\times9}}\right)^{4(p-1)}\times\left(\frac{9+9}{2\sqrt{9\times9}}\right)^{(p-1)}$$

$$=(1)^6\times\left(\frac{9}{4\sqrt{5}}\right)^4\times(1)^{2(p-2)}\times\left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)}\times(1)^{2(p-1)}\times\left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)}\times(1)^{(p-1)}$$

$$=\left(\frac{9}{4\sqrt{5}}\right)^4\times\left(\frac{13}{4\sqrt{10}}\right)^{4(p-1)}\times\left(\frac{17}{12\sqrt{2}}\right)^{4(p-1)}.$$

4. Results for $TUC_4C_8[p, q]$ nanotube

The line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube is shown in Figure 3(b).



(a) (b)

Figure 3

(a) Subdivision graph of $TUC_4C_8[4, 2]$ nanotube (b) line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube

Theorem 2. Let H be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then

$$AG_5II(H) = \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{8p}, \qquad \text{if } p>1 \text{ and } q>1,$$

$$= \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p}, \qquad \text{if } p>1 \text{ and } q=1.$$

Proof: The $TUC_4C_8[p, q]$ nanotube is a graph with $4pq$ vertices and $6pq - p$ edges. By Lemma 2, the subdivision graph of $TUC_4C_8[p, q]$ nanotube is a graph with $10pq - p$ vertices and $12pq - 2p$ edges. Thus by Lemma 1, H has $12pq - 2p$ vertices and $18pq - 5p$ edges. We see that in H, there are $4p$ vertices, are of degree 2 and remaining all vertices are of degree 3. Therefore we have partition of the edge set of H as follows:

Table 3. Edge partition of H with $p>1$ and $q>1$.

$S_H(u), S_H(v) \backslash uv \in E(H)$	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$8p$	$18pq - 19p$

Table 4. Edge partition of H with $p>1$ and $q = 1$

$S_H(u), S_H(v) \backslash uv \in E(H)$	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$2p$	$4p$	p

Case 1. Suppose $p>1$ and $q>1$.

By algebraic method, we obtain $|V_5| = 4p$, $|V_8| = 4p$ and $|V_9| = 2(6pq - 5p)$ in H. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given in Table 3.

$$AG_5II(H) = \prod_{uv \in E(H)} \frac{S_H(u) + S_H(v)}{2\sqrt{S_H(u) S_H(v)}}$$

$$= \left(\frac{5+5}{2\sqrt{5\times 5}}\right)^{2p} \times \left(\frac{5+8}{2\sqrt{5\times 8}}\right)^{4p} \times \left(\frac{8+9}{2\sqrt{8\times 9}}\right)^{8p} \times \left(\frac{9+9}{2\sqrt{9\times 9}}\right)^{18pq-19p}$$

$$= (1)^{2p} \times \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{8p} \times (1)^{18pq-19p} = \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{8p}.$$

Case 2. Suppose $p>1$ and $q=1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained, as given Table 4.

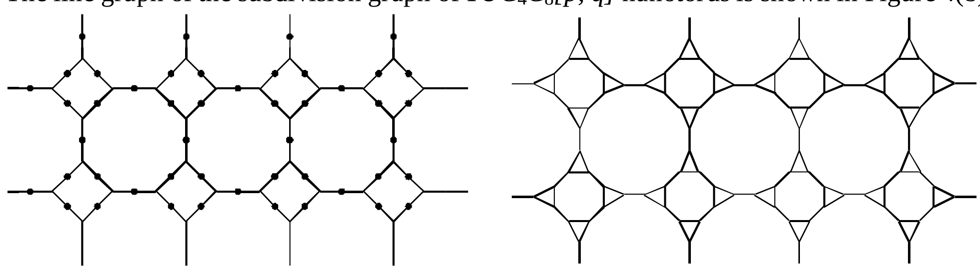
$$AG_5II(H) = \prod_{uv\in E(H)} \frac{S_H(u)+S_H(v)}{2\sqrt{S_H(u)S_H(v)}}$$

$$= \left(\frac{5+5}{2\sqrt{5\times 5}}\right)^{2p} \times \left(\frac{5+8}{2\sqrt{5\times 8}}\right)^{4p} \times \left(\frac{8+8}{2\sqrt{8\times 8}}\right)^{2p} \times \left(\frac{8+9}{2\sqrt{8\times 9}}\right)^{4p} \times \left(\frac{9+9}{2\sqrt{9\times 9}}\right)^{p}$$

$$= (1)^{2p} \times \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times (1)^{2p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p} \times (1)^{p} = \left(\frac{13}{4\sqrt{10}}\right)^{4p} \times \left(\frac{17}{12\sqrt{2}}\right)^{4p}.$$

5 Results for $TUC_4C_8[p, q]$ nanotorus

The line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotorus is shown in Figure 4(b).



(a) (b)

Figure 4

(a) subdivision graph of $TUC_4C_8[4,2]$ nanotorus

(b) line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotorus.

Theorem 3. Let K be the line graph of the subdivision graph of $TUC_4C_8[p, q]$ nanotorus. Then $AG_5II(K) = 1$.

Proof: The graph of $TUC_4C_8[p, q]$ nanotorus has $4pq$ vertices and $6pq$ edges. Then by Lemma 2, the subdivision graph of $TUC_4C_8[p, q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1, K has $12pq$ vertices and $18pq$ edges. We see easily that in K, $|V_9| = 12pq$ and we have edge partition based on the degree sum of neighbor vertices of each vertex, as given in Table 5.

Table 5. Edge partition of K.

$S_K(u), S_K(v)\backslash uv\in E(K)$	(9, 9)
Number of edges	$18pq$

$$AG_5II(K) = \prod_{uv\in E(K)} \frac{S_K(u)+S_K(v)}{2\sqrt{S_K(u)S_K(v)}} = \left(\frac{9+9}{2\sqrt{9\times 9}}\right)^{18pq} = 1.$$

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