ISSN 2231-3478 (Print)
JUSPS-B Vol. 29(6), 132-141 (2017). Periodicity-Monthly
Section B
(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

Transient hydromagnetic radiative convection flow of a micropolar fluid past a moving semi- infinite vertical porous plate

ATUL KUMAR SINGH[1], PRATIBHA AGNIHOTRI[2] and N. P. SINGH[3]

[1]Department of mathematics, V. S. S. D. College, Kanpur-208002 (INDIA)
e-mail: atulkumar67singh@gmail.com
[2]Department of mathematics, H. B. T. University, Kanpur-208002 (INDIA)
[3]Department of mathematics, C. L. Jain (P.G.) College, Firozabad-283203 (INDIA)
Corresponding Author Email: pratibha.mishra003@gmail.com
http://dx.doi.org/10.22147/jusps-B/290602

Acceptance Date 14th May, 2017, Online Publication Date 2nd June, 2017

Abstract

Unsteady two-dimensional laminar radiative convection flow of an incompressible, electrically conducting micropolar fluid past a moving semi-infinite vertical porous plate in the presence of magnetic field is studied. The effects of material parameters on the velocity and temperature fields are investigated. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The micropolar fluid is considered to be gray, absorbing-emitting but non-scattering optically thick medium. Numerical results of velocity field and temperature distribution are discussed with the help of figures, while skin-friction and rate of heat transfer are discussed by the use of tables. It is observed that when radiation parameter increases, the velocity increases and temperature decreases, whereas an increase in Grashof number or magnetic parameter decreases the velocity.

Key words: convection flow, incompressible, magnetic field
MSC(2010) : 76E06, 76Dxx, 76E25

Nomenclature :

u', v' dimensional components of velocities along x' and y'-directions
u, v non-dimensional components of velocities along x' and y'-directions
x', y' dimensional spatial coordi-nates along and normal to the plate
x, y non-dimensional spatial co-ordinates along and normal to the plate
g acceleration due to gravity
T' dimensional temperature of the fluid
T non-dimensional temperature of the fluid

This is an open access article under the CC BY-NC-SA license (https://creativecommons.org/licenses/by-nc-sa/4.0)

T'_∞	temperature of the free stream
T'_w	temperature of the wall
B_0	uniform magnetic field
m'	dimensional exponential index
m	non-dimensional exponential index
t'	dimensional time
t	non-dimensional time
j	non - dimensional micro-inertia
j'	dimensional micro-inertia
k	effective thermal conduct-ivity
k_c	absorption coefficient
q'''	non-uniform heat generation /absorption
q'_r	radiative heat flux in y'- direction
U'_∞	free stream velocity
U'_p	dimensional velocity of the plate
U_p	non- dimensional velocity of the plate
A	real positive constant
V_0	scale of suction velocity
U_0	scale of free stream velocity
Pr	Prandtl number
Gr	Grashof number
R	radiation parameter
$A*$	space dependent heat source/ sink parameter
$B*$	temperature dependent heat source/sink parameter

Greek symbols:

ω'	dimensional microrotation variable
ω	non-dimensional microrotation variable
γ'	dimensional spin gradient viscosity
γ	non-dimensional spin gradient viscosity
ρ	density of the fluid
β_r	coefficient of volumetric expansion of the fluid
ϑ	kinematic viscosity of the fluid
ϑ_r	kinematic rotational viscosity of the fluid
σ	electrical conductivity of the fluid
α	effective thermal diffusivity of the fluid
ε	perturbation parameter (<<1)
σ_S	Stefan Boltzmann constant
μ	dynamic viscosity
κ	microratation viscosity

1. Introduction

1. A number of fluids exhibit a non-Newtonian fluid behaviour. Physically, non-Newtonian fluids represent fluids consisting of randomly oriented particles suspended in a viscous medium. Eringen[1] has discussed theory of micropolar fluids which takes into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation. A thorough exposition of the continuum mechanics foundation of micromorphic. and micropolar fluids has been provided by Ariman *et. al.*[2] The study of unsteady flow and heat transfer of an electrically conducting, micropolar fluid past a porous plate has attracted the interest of many investigators in view of its applications in many engineering problems such as colloidal solutions, fluids with additives, animal blood, suspension solutions, oil exploration, thermal energy extractions, nuclear reactors, MHD generators and the boundary layer control in the field of aerodynamics Lucaszewicz[3]. Raptis[4] studied numerically, the steady two-dimensional flow of a micropolar fluid past a continuously moving plate embedded in a porous medium in presence of thermal radiation. Kim[5,6] obtained solution for two-dimensional transient convective heat transfer of an electrically conducting viscous fluid past a semi-infinite vertical porous moving plate with variable suction in the presence of magnetic field and unsteady convection flow of micropolar fluids past a vertical porous plate embedded in a porous medium respectively. Kim and Fedorov[7] investigated the case of mixed convection flow of a micropolar fluid past a semi-infinite steadily moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation considering a gray, absorbing-emitting but non-scattering optically thick medium. In the present paper, it is proposed to study the heat and momentum transfer in radiative convective flow of an electrically conducting, incompressible, micropolar fluid along an infinite vertical porous plate moving upward with uniform velocity in the presence of uniform magnetic field and non-uniform heat source for the better application of the model proposed by Kim and Fedorov[7]. Effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation is studied by Makined and Sibanda[8]. Effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame is studied by K.Das[9]. Bakr and Ali[10] has discussed Oscillatory free convection of a micropolar rotating fluid on a vertical plate with variable heat flux and thermal radiation.

2. *Formulation of the problem* :

Let us consider a two-dimensional, laminar, unsteady flow of an incompressible, homogeneous, electrically conducting micropolar fluid past a semi-infinite, vertical porous plate moving steadily with velocity U_p and subjected to a uniform magnetic field and thermal radiation field. The x'-axis is taken along the vertical plate in the upward direction and y'-axis normal to the plate. The acceleration due to gravity g is in the direction opposite to x'- coordinate. It is assumed that the size of holes in the porous plate is much larger than the characteristic microscopic length scale of the micropolar fluid to simplify formulation of the boundary conditions. Also, due to semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and the time t' only. The uniform magnetic field of small magnetic Reynolds number acts transversely to the direction of the flow. Since the magnetic Reynolds number is small, the induced magnetic field is neglected. In the physics of flow, the effect of non-uniform heat generation and radiation on heat transfer phenomenon is taken into account, while viscous dissipation and Ohmic heati is ignored. Further, Boussinesq approximation is followed, so that density differences are only manifested in the thermal buoyancy terms in the liner momentum equation. Under the present configuration, the equations governing the flow are:

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Equation of linear momentum –

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta_r(T' - T'_\infty) + (\vartheta + \vartheta_r)\frac{\partial^2 u'}{\partial y'^2} + 2\vartheta_r\frac{\partial \omega'}{\partial y'} - \frac{\sigma}{\rho}B_0^2 u' \tag{2}$$

Equation of angular momentum-

$$\rho j'\left(\frac{\partial \omega'}{\partial t'} + v'\frac{\partial \omega'}{\partial y'}\right) = \gamma'\frac{\partial^2 \omega'}{\partial y'^2} \tag{3}$$

Equation of energy-

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\left(\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{k}\frac{\partial q'_r}{\partial y'}\right) - q''' \tag{4}$$

The boundary conditions for the velocity, microrotation and temperature fields are:

$$u' = U'_p,\ v' = -V_0\left(1 + \varepsilon A e^{-m't'}\right),$$

$$T' = T'_w + \varepsilon\left(T'_w - T'_\infty\right)e^{-m't'},\ \omega' = -n\frac{\partial u'}{\partial y'} \text{ at } y' = 0$$

$$\frac{\partial u'}{\partial y'} \to 0,\ \frac{\partial T'}{\partial y'} \to 0,\ \omega' \to 0 \text{ as } y' \to \infty \tag{5}$$

The symbols are defined in nomenclature.

By using the Rosseland approximation, the radiative heat flux in the y'-direction is expressed as-

$$q'_r = -\frac{4}{3}\frac{\sigma_s}{k_c}\frac{\partial T'^4}{\partial y'}, \tag{6}$$

where σ_S is the Stefan-Boltzmann constant and k_c is the mean absorption coefficient. For sufficient small temperature differences within the flow, the radiative heat flux described in (6) can be linearized by expanding T'^4, neglecting higher order terms, into Taylor series about T'_∞ to give

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{7}$$

The spin-gradient viscosity γ', which defines the relationship between the coefficient of viscosity and micro-inertia, is given by:

$$\gamma' = \left(\mu + \frac{\kappa}{2}\right) j' = \mu j' \left(1 + \frac{1}{2}\beta\right)$$

$\beta = \dfrac{\kappa}{\mu}$ (ratio of microrotation viscosity and dynamic viscosity)

Also q''' is the space and temperature dependent internal heat generation/absorption (non-uniform heat source/sink Abel *et. al.* [2007]), which can be expressed in simplest form as

$$q''' = \frac{k}{\rho C_p}\frac{V_0^2}{\vartheta^2}\left[A^*\left(T'_w - T'_\infty\right)\left(1 + \varepsilon e^{-mt}\right) + B^*\left(T' - T'_\infty\right)\right] \quad (8)$$

where A^* and B^* are parameters of space and temperature dependent internal heat generation / absorption. It is to be noted $A^* < 0$ and $B^* < 0$ correspond to internal heat generation, while $A^* > 0$ and $B^* > 0$ correspond to internal heat absorption.

It should be noted that by using the Rosseland approximation to describe the radiative heat flux in energy equation, limits our analysis to absorbing-emitting but non-scattering optically thick medium. The microrotation variable ω', which describes its relationship with the surface stress in the relation $\omega' = -n\dfrac{\partial u'}{\partial y}$, is shown in boundary condition (5).

The parameter n is the number between 0 and 1 that relates the micro-gyration vector to the shear stress. The value $n = 0$ corresponds to the case, where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value $n = 0.5$ is indicative of week concentrations, while $n = 1$ represents turbulent boundary layers see Kim and Fedorov[7].

The equation (1) implies that the velocity component v' is either constant or a function of t only. Therefore the suction velocity v' can be taken as-

$$v' = -V_0\left(1 + \varepsilon A e^{-m't'}\right) \quad (9)$$

where V_0 is real positive constant, $\varepsilon << 1$, εA is small and m is a positive parameter. The negative sign indicates that the suction acts towards the wall.

$$u = \frac{u'}{U_0},\ v = \frac{v'}{V_0},\ y = \frac{y'V_0}{\vartheta},\ U_p = \frac{u_p'}{U_0},$$

$$\omega = \frac{\omega'\vartheta}{U_0 V_0},\ t = \frac{t'V_0^2}{\vartheta},\ T = \frac{T' - T'_\infty}{T_w' - T_\infty'},$$

$$m = \frac{m'\vartheta}{V_0^2},\ j = \frac{j'V_0^2}{\vartheta^2},\ \gamma = \frac{\gamma' V_0^2}{\vartheta^2},$$

$$P_r = \frac{\vartheta\rho C_p}{k} = \frac{\vartheta}{\alpha} \text{ (Prandtl number)},$$

$$Gr = \frac{\vartheta\beta_r g(T_W' - T_\infty')}{U_0 V_0^2} \text{ (Grashof number)}$$

$$R = \frac{kk_c}{4\sigma_s T_\infty'^3} \text{ (Radiation parameter)},$$

$$R_1 = \left(1 - \frac{4}{3R + 4}\right) \text{ (Modified radiation parameter)},$$

$$M = \frac{\sigma\vartheta}{\rho}\frac{B_0^2}{V_0^2} \text{ (Magnetic parameter)},$$

$$\beta_1 = \frac{\vartheta_r}{\vartheta} = \frac{\vartheta_r}{\vartheta} \text{ (Ratio of microrotation viscosity and dynamic viscosity)}, \quad (10)$$

In view of (6) - (10), the governing Eqs. (1) – (4), in non-dimensional form are :

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{-mt}\right)\frac{\partial u}{\partial y} = (1 + \beta)\frac{\partial^2 u}{\partial y^2} + GrT + 2\beta\frac{\partial \omega}{\partial y} - Mu \quad (11)$$

$$\frac{\partial \omega}{\partial t} - \left(1 + \varepsilon A e^{-mt}\right)\frac{\partial \omega}{\partial y} = \frac{1}{\beta_1}\frac{\partial^2 \omega}{\partial y^2} \quad (12)$$

$$\frac{\partial T}{\partial t}-\left(1+\varepsilon Ae^{-mt}\right)\frac{\partial T}{\partial y}=\frac{1}{R_1 Pr}\frac{\partial^2 T}{\partial y^2}-\frac{1}{Pr}\left[A^*\left(1+\varepsilon e^{-mt}\right)+B^* T\right] \tag{13}$$

The boundary conditions (5) in non-dimensional form are:

$$u=U_p,\ \ T=1+\varepsilon e^{-mt}\ \ \omega=-n\frac{\partial u}{\partial y},\text{ at } y=0$$

$$\frac{\partial u}{\partial y}\to 0,\ \ \frac{\partial T}{\partial y}\to 0,\ \ \omega\to 0,\text{ as } y\to\infty \tag{14}$$

3. *Solution of the problem*:

In order to reduce the above stated system of partial differential equations to a system of ordinary differential equations in non- dimensional form, we perform an asymptotic analysis by representing the linear velocity, microrotation and temperature as:

$$u(y,t)=u_0(y)+\varepsilon u_1(y)e^{-mt}+O\left(\varepsilon^2\right) \tag{15}$$

$$\omega(y,t)=\omega_0(y)+\varepsilon\omega_1(y)e^{-mt}+O\left(\varepsilon^2\right) \tag{16}$$

$$T(y,t)=T_0(y)+\varepsilon T_1(y)e^{-mt}+O\left(\varepsilon^2\right) \tag{17}$$

Substituting (15)–(17) into Eqs.(11)–(13), neglecting the terms of $O\left(\varepsilon^2\right)$, we obtain following pairs of ordinary differential equations:

$$\frac{d^2\omega_0}{dy^2}+\beta_1\frac{d\omega_0}{dy}=0 \tag{18}$$

$$\frac{d^2T_0}{dy^2}+R_1P_r\frac{dT_0}{dy}-R_1B^*T_0=R_1A^* \tag{19}$$

$$(1+\beta)\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-Mu_0=-GrT_0-2\beta\frac{d\omega_0}{dy} \tag{20}$$

$$\frac{d^2\omega_1}{dy^2}+\beta_1\frac{d\omega_1}{dy}+m\beta_1\omega_1=-A\beta_1\frac{d\omega_0}{dy} \tag{21}$$

$$\frac{d^2T_1}{dy^2}+R_1P_r\frac{dT_1}{dy}-R_1B^*M_1T_1=-R_1PrA\frac{dT_0}{dy}+R_1A^* \tag{22}$$

$$(1+\beta)\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-MM_2u_1=-GrT_1-2\beta\frac{d\omega_1}{dy}-A\frac{du_0}{dy} \tag{23}$$

where $M_1=1-\frac{mPr}{B^*}$ and $M_2=1-\frac{m}{M}$

Substituting (15) – (17) into the boundary conditions (14), the corresponding boundary conditions are given by:

$$u_0=U_p,\ u_1=0,\ T_0=1,\ T_1=1,$$

$$\omega_0=-n\frac{du_0}{dy},\ \ \omega_1=-n\frac{du_1}{dy},\ \ \text{at } y=0$$

$$\frac{du_0}{dy}\to 0,\ \frac{du_1}{dy}\to 0,\ \frac{dT_0}{dy}\to 0,$$

$$\frac{dT_1}{dy}\to 0,\ \omega_0\to 0,\ \omega_1\to 0\ \text{ as } y\to\infty \tag{24}$$

The solution of Eqs. (18) – (23) satisfying the boundary conditions (24) is given by:

$$\omega_0(y)=C_1e^{-\beta_1 y} \tag{25}$$

$$T_0(y)=A_1e^{-H_1y}-\frac{A^*}{B^*} \tag{26}$$

$$u_0(y)=B_1e^{-H_2y}+C_1B_2e^{-\beta_1 y}+B_3e^{-H_1y}+B_4 \tag{27}$$

$$\omega_1(y)=C_2e^{-H_3y}+\frac{A\beta_1}{m}C_1e^{-\beta_1 y} \tag{28}$$

$$T_1(y)=A_2e^{-H_4y}+A_3e^{-H_1y}+A_4 \tag{29}$$

$$u_1(y)=B_5e^{-H_5y}+B_6e^{-H_1y}+B_7e^{-H_2y}+B_8C_2e^{-H_3y}+B_9e^{-H_4y}+C_1B_{10}e^{-\beta_1 y}+B_{11} \tag{30}$$

4. *Skin -friction and rate of heat transfer :*

After obtaining the stream wise velocity, micro-rotation and temperature in the boundary layer, the quantity of physical interest is the skin-friction coefficient at the porous wall, which is given by

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} = K_1 + \varepsilon K_2 e^{-mt} \tag{31}$$

Where $K_1 = -\left(H_2 B_1 + \beta_1 B_2 C_1 + H_1 B_3\right)$

$$K_2 = -(H_5 B_5 + H_1 B_6 + H_2 B_7 + H_3 B_8 C_2$$
$$H_4 B_9 + \beta_1 C_1 B_{10})$$

We can also calculate the heat transfer coefficient at the wall in terms of Nusselt number as follow:

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$= H_1 A_1 + \varepsilon\left(H_4 A_2 + H_1 A_3\right) e^{-mt} \tag{32}$$

5. Verification of problem through simple cases:

(i) If the influence of uniform magnetic field and the supply of heat through variable heat source is not considered, the results of the study are exactly same as obtained by Kim and Fedorov[7].

(ii) If the supply of variable heat source is remove and the presence of uniform magnetic field is adjusted as flow in the porous medium, the results of the study are similar to those of Kim[6].

(iii) If the supply of the heat by means of variable heat source is removed, the results are similar to those obtained by Kim[5].

6. Results and Discussion

Fig. 1 shows variations in the fluid velocity (u) against span wise coordinate (y) for different numerical values of the ratio of microrotation viscosity and dynamic viscosity β at fixed values of Prandtl number (Pr), radiation parameter (R), maganetic parameter (M), Grashof number (Gr), space and temperature dependent internal heat generation/ absorption parameters A^* and B^*. For numerical computation these values are chosen to be $Pr=0.71$, $R=1.0$, $M=1.0$, $Gr=2.0$, $A^*=0.3$ and $B^*=0.3$. It is observed that an increase in the ratio of microrotation and dynamic viscosity (β) enhances the velocity of the micropolar fluid. In the vicinity of the plate, the velocity increases more rapidly and after attaining a maximum magnitude at $y=0.6$ approximately, it decreases and becomes asymptotic to horizontal axis and ultimately dies away.

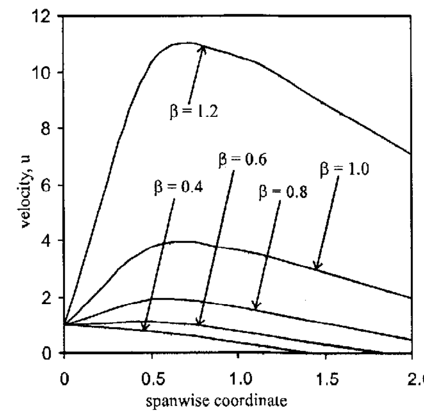

Fig. 1. Velocity profiles against spanwise coordinate y for different values of ratio of microrotation viscosity and dynamic viscosity parameter β at $Pr = 0.71$, $R = 1.0$, $Gr = 2.0$, $A^* = 0.3$, $B^* = 0.3$, $M = 1.0$.

Fig. 2 illustrates variations in the fluid velocity (u) against span wise coordinate (y) for different numerical values of magnetic parameter (M) at $Pr=0.71$, $\beta=0.4$, $Gr=2.0$, $A^*=0.3$ and $B^*=0.3$ choosing $R=1.0$ and $R=1.5$. It is demonstrated that increase in magnetic parameter decreases the velocity, i.e., retards the flow for cooling case ($Gr>0$) *i.e.*, heat removal from the surface. This is an important controlling mechanism in nuclear energy system heat transfer, where momentum development can be reduced in porous plate flow regimes by enhancing the magnetic field. Obviously an increase in the radiation parameter increases the velocity. In fact, due to increase in radiation, the medium resistance is lowered and this increases the momentum development of the flow regime, there by enhancing the velocity.

Fig. 3 shows the variations in the velocity (u) against span wise coordinate (y) for different numerical values of Grashof number (Gr). Two different cases, namely, cooling of the plate ($Gr>0$)

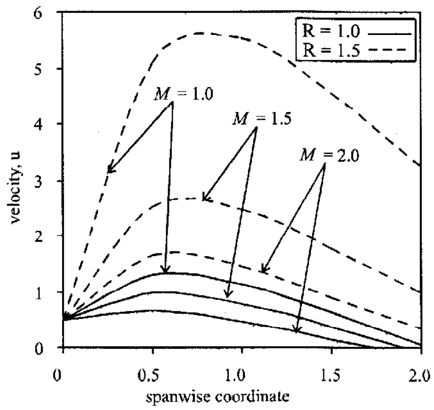

Fig. 2. Velocity profiles against spanwise coordinate y for different values of Magnetic parameter M at $Pr = 0.71$, $Gr = 2.0$, $A^* = 0.3$, $B^* = 0.3$, $\beta = 0.4$.

and heating of the plate ($Gr<0$) are chosen. To observe these effects, we choose Pr=0.71, R=1.0, M=1.0, β=0.4, A^*=0.3 and B^*=0.3. It is observed that an increase in Grashof number (Gr) increases the velocity in cooling case of the plate while reverse effect is noted in heating case. Hence, again we note that buoyancy parameter (Gr), has dominant effect in escalating transient velocity of the fluid.

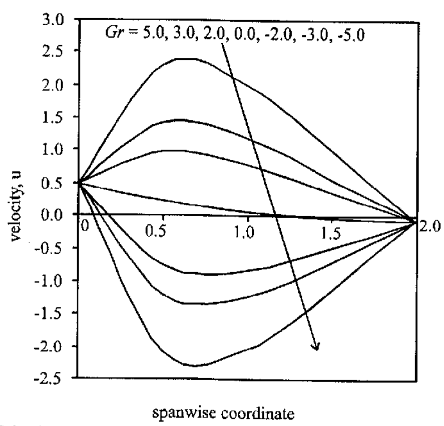

Fig. 3. Velocity profiles against spanwise coordinate y for different values of Grashof number Gr at $Pr = 0.71$, $M = 1.0$, $R = 1.0$, $A^* = 0.3$, $B^* = 0.3$, $\beta = 0.4$.

Fig. 4 shows variations in the velocity (u) against span wise coordinate (y) for different numerical values of space dependent heat source/sink parameter A^* at Pr=0.71, R=1.0, β=0.4, Gr=2.0, M=1.0 and B^*=0.3. It is observed that the velocity of the fluid decreases for space dependent heat source parameter but increases for space dependent heat sink parameter.

Fig. 5 illustrates variations in the velocity (u) against span wise coordinate (y) for different numerical values of temperature dependent heat source/sink parameter B^* at Pr=0.71, R=1.0, β=0.4, Gr=2.0, M=1.0 and A^*=0.3. It is observed that the velocity of the fluid increases for temperature dependent heat source parameter. As expected, the temperature dependent heat source favors the velocity of the fluid while space dependent heat source apposes the velocity.

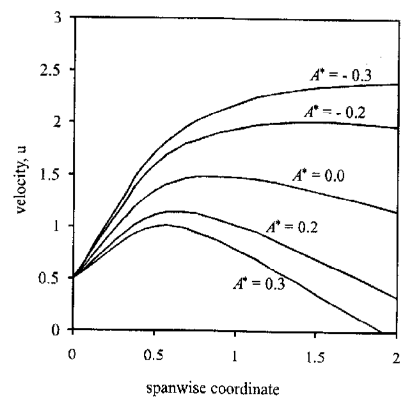

Fig. 4. Velocity profiles against spanwise coordinate y for different values of space dependent heat source parameter A^* at $Pr = 0.71$, $M = 1.0$, $Gr = 2.0$, $B^* = 0.3$, $R = 1.0$, $\beta = 0.4$.

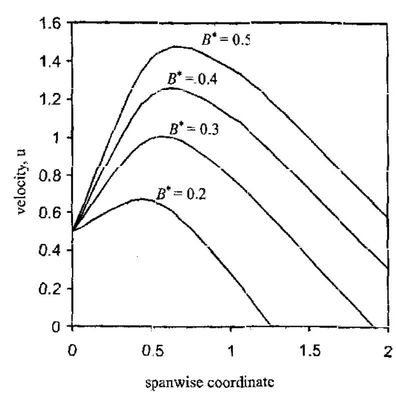

Fig. 5. Velocity profiles against spanwise coordinate y for different values of temperature dependent heat source parameter B^* at $Pr = 0.71$, $M = 1.0$, $Gr = 2.0$, $A^* = 0.3$, $R = 1.0$, $\beta = 0.4$.

Fig. 6 shows variations in the angular velocity (ω) against span wise coordinate (y) for different numerical values of the ratio of microrotation viscosity and dynamic viscosity β at Pr=0.71, R=1.0, Gr=2.0, M=1.0, A^*=0.3 and B^*=0.3. It is observed that the angular velocity of the fluid decreases with increase in the ratio of microrotation and dynamic viscosity. In the boundary condition at the plate, the angular velocity is negatively related to the rate of change of velocity, *i.e.*, shear stress at the plate. Hence, effects of various parameters on the fluid velocity and the angular velocity are observed in reverse order.

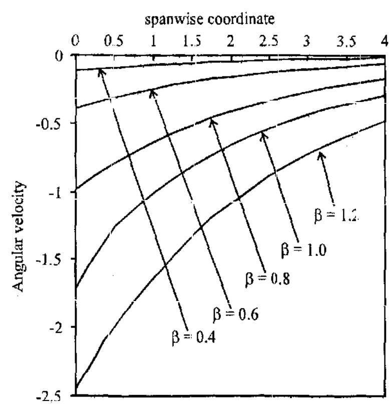

Fig. 6. Angular velocity profiles against spanwise coordinate y for different values of ratio of microrotation viscosity and dynamic viscosity parameter β at Pr = 0.71, R = 1.0, Gr = 2.0, A^*= 0.3, B^* = 0.3, M = 1.0.

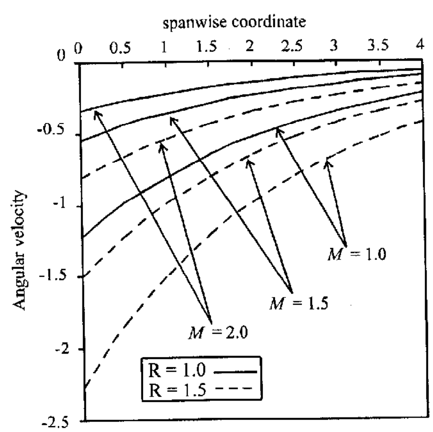

Fig. 7. Angular velocity profiles against spanwise coordinate y for different values of Magnetic parameter M at Pr = 0.71, Gr = 2.0, A^*= 0.3, B^* = 0.3, β = 0.4.

Fig. 7 illustrates variations in the angular velocity (ω) against span wise coordinate (y) for different numerical values of magnetic parameter (M) at Pr=0.71, β=0.4, Gr=2.0, A^*=3.0 and B^*=3.0 choosing R=1.0 and R=1.5. Obviously, an increase in the radiation parameter decreases the angular velocity. The physics behind this phenomenon is that due to an increase in radiation, the microrotation of the micropolar fluid particles becomes less intrusive so that the flow regime becomes more dense, which in turn retards the angular velocity.

7. Conclusions

The equations governing the flow of an unsteady incompressible micro-polar fluid past a semi-infinite vertical porous plate with variable suction under the influence of uniform magnetic field and variable heat source are solved using regular perturbation technique. The solutions are presented in (25)-(30) and the expressions for skin-friction and rate of heat transfer are presented in (31)-(32) respectively. To illustrate the details of the flow and heat transfer characteristics and their dependence on material parameters, the numerical results are presented in the form of figures for specific values of flow parameters. The study concludes the following results:

- An increase in the ratio of microrotation and dynamic viscosity (β) enhances the velocity of the micropolar fluid, while decreases the angular velocity.
- An increase in the radiation parameter increases the velocity but decreases angular velocity, while an increase in magnetic parameter decreases the velocity but increases angular velocity.
- The velocity of the fluid increases with an increase in temperature dependent heat source parameter, while angular velocity decreases.

7. *Appendix*

$$A_1 = 1 + \frac{A^*}{B^*},\ A_2 = 1 - A_3 - A_4,$$

$$M_1 = 1 - \frac{m\Pr}{B^*}, \quad M_2 = 1 - \frac{m}{M},$$

$$A_3 = \frac{AH_1A_1R_1\Pr}{H_1^2 - R_1\Pr H_1 - R_1B^*M_1}$$

$$H_1 = \frac{R_1\Pr + \sqrt{R_1^2\Pr^2 + 4R_1B^*}}{2}$$

$$H_2 = \frac{1 + \sqrt{1 + 4(1+\beta)M}}{2(1+\beta)}$$

$$H_3 = \frac{\beta_1}{2}\left[1 + \sqrt{1 - \frac{4m}{\beta_1}}\right],$$

$$H_4 = \frac{R_1Pr + \sqrt{R_1^2Pr^2 + 4R_1B^*M_1}}{2},$$

$$H_5 = \frac{1 + \sqrt{1 + 4(1+\beta)MM_2}}{2(1+\beta)},$$

$$B_1 = U_p - B_2C_1 - B_3 - B_4,$$

$$B_2 = \frac{2\beta\beta_1}{(1+\beta)\beta_1^2 - \beta_1 - M},$$

$$B_3 = \frac{-GrA_1}{(1+\beta)H_1^2 - H_1 - M},$$

$$C_1 = \frac{n\left[H_1B_3 + H_2\left(U_p - B_3 - B_4\right)\right]}{1 + nB_2\left(H_2 - \beta_1\right)}$$

$$B_4 = \frac{-GrA^*}{B^*M},$$

$$B_5 = -\left(B_6 + B_7 + B_8C_2 + B_9 + B_{10}C_1 + B_{11}\right)$$

$$B_6 = \frac{AH_1B_3 - GrA_3}{(1+\beta)H_1^2 - H_1 - MM_2},$$

$$B_7 = \frac{AH_2B_1}{(1+\beta)H_2^2 - H_2 - MM_2},$$

$$B_8 = \frac{2\beta H_3}{(1+\beta)H_3^2 - H_3 - MM_2},$$

$$B_9 = \frac{-GrA_2}{(1+\beta)H_4^2 - H_4 - MM_2},$$

$$B_{10} = \frac{A\beta_1\left(B_2 + 2\beta\beta_1m^{-1}\right)}{(1+\beta)\beta_1^2 - \beta_1 - MM_2},$$

$$B_{11} = \frac{GrA_4}{MM_2},$$

$$B_{12} = H_1B_6 + H_2B_7 + H_4B_9 - H_5\left(B_6 + B_7 + B_9 + B_{11}\right),$$

$$C_2 = \frac{nB_{12} - C_1\left[nB_{10}\left(H_5 - \beta_1\right) + A\beta_1m^{-1}\right]}{1 + nB_8\left(H_5 - H_3\right)}$$

References

1. Eringen, A.C., Theory of Micropolar Fluids, *J. Math Mech.*, 16,1-18(1966).
2. Ariman, T., Turk, M. A., Sylvester, N. D., Applications of Micro-continuum Fluid Mechanics, *Int. J. Engng. Sci.*, *11*, 905 – 930 (1974).
3. Lucaszewick, G., Micropolar Fluids: Theory and Applications, Birkhuser, Boston (1990).
4. Raptis, A., Radiation and free convection flow through a porous medium, *Int Comm. Heat Mass Transfer*, *25*, 289-295 (1998).
5. Kim, Y.J., Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, *Int. J. Engng. Sci.*, 38, 833-845 (2000).
6. Kim, Y.J., Unsteady convection flow of micropolor fluids past a vertical porous plate embedded in a porous medium, *Acta. Mech.*, *148*, 105-116 (2001).

7. Kim, Y.J. Fedorov, A.G., Transient mixed radiative convection flow of a micropolor fluid past a moving, semi-inifinite vertical porous plate, *Int. J. Heat Mass Transfer*, *46,* 1751-1758 (2003).
8. El- Arabawy, H.A.M., Effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation, *Int. J. Heat Mass Transfer, 46,* 1471-1477 (2003).
9. Das, K., Effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame of reference, *Int. J. Heat Mass Transfer , 54,* 3505-3513 (2011).
10. Bakr, A.A., Chamkha, Ali. J., Oscillatory free convection of a micropolar rotating fluid on a vertical plate with variable heat flux and thermal radiation, *Heat Transfer research 48(2),*139-159 (2017).