ISSN 2231-3478 (Print)
JUSPS-B Vol. 29(6), 142-148 (2017). Periodicity-Monthly
Section B
(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

Hall effects on peristaltic pumping of a Newtonian fluid in a channel with long wavelength approximation

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http://dx.doi.org/10.22147/jusps-B/290603

Acceptance Date 14th May, 2017, Online Publication Date 2nd June, 2017

Abstract

In this paper, the effect of hall on the peristaltic pumping of a Newtonian fluid in a two dimensional channel under the assumption of long wavelength is investigated. A closed form solutions are obtained for axial velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient, time-averaged volume flow rate and frictional force are discussed with the aid of graphs.

Key words : Hall, Newtonian fluid, Hartmann number, long wavelength, Peristaltic pumping.

1. Introduction

The study of the mechanism of peristalsis in both mechanical and physiological situations has recently become the object of scientific research, since the first investigation of Latham[5]. Several theoretical and experimental attempts have been made to understand peristaltic action in different situations. A review of much of the early literature is presented in an article by Jaffrin and Shapiro[4]. A summary of most of the experimental and theoretical investigations reported with details of the geometry, fluid Reynolds number, wavelength parameter wave amplitude parameter and wave shape has been given by Srivastava and Srivastava[11].

The magnetohydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids, (e.g., the blood flow in arteries). The effect of magnetic field on blood flow was studied by Sud *et al*.[12] and it is observed that the effect of suitable magnetic field accelerates the speed of blood. Srivastava and Agrawal[10] and Prasad and

Ramacharyulu[8] by considering the blood as an electrically conducting fluid and constitutes a suspension of red cell in plasma. Also, Agrawal and Anwaruddin[1] studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Li *et al.*[6] have used an impulsive magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract. It was found that the impulsive Magnetic field (IMF) activates the impulsive activity of the ureteral smooth muscles in 100% of cases. Mekheimer[7] studied the peristaltic transport of blood under effect of a magnetic field in non uniform channels. Hayat *et al.*[3] have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid trough a porous medium in channel. Recently, Eldabe[2] have studied the Hall Effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer.

In view of these, we studied the effect of hall on the peristaltic flow of a Newtonian fluid in a two dimensional channel under the assumption of long wavelength. A closed form solution is obtained for axial velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient, time-averaged volume flow rate and frictional force are discussed with the aid of graphs.

2. *Mathematical Formulation*

We consider the peristaltic pumping of a conducting Newtonian fluid flow in a channel of half-width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half-width of the channel as shown in the Fig.1. The wall deformation is given by

$$H\left(X,t\right)=a+b\sin\left[\frac{2\pi}{\lambda}\left(X-ct\right)\right] \qquad (2.1)$$

where b is the amplitude, λ the wavelength and c is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wavelength and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame moving with velocity c away from the fixed (laboratory) frame . The transformation between these two frames is given by

$$x=X-c\,t,\ y=Y,\ \ u=U-c,\ \ v=V \text{ and}$$

$$p(x)\ =\ P(X,\ t), \qquad (2.2)$$

where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (2.3)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$+\frac{\sigma B_0^2}{1+m^2}\left(mv-\left(u+c\right)\right) \qquad (2.4)$$

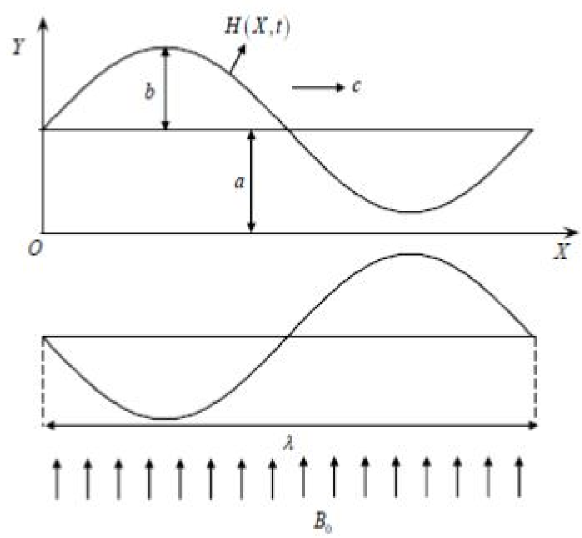

Fig. 1 The Physical Model

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$-\frac{\sigma B_0^2}{1+m^2}\left(m\left(u+c\right)+v\right) \tag{2.5}$$

The dimensional boundary conditions are

$$u=-c \quad \text{at} \quad y=H \tag{2.6}$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \tag{2.7}$$

Introducing the non-dimensional quantities

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c\delta},\delta=\frac{a}{\lambda},$$

$$\bar{p}=\frac{pa^2}{\mu c\lambda},\bar{t}=\frac{ct}{\lambda},h=\frac{H}{a},\phi=\frac{b}{a},\bar{q}=\frac{q}{ac}$$

Into equations (2.3) to (2.5), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.8}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$+\frac{M^2}{1+m^2}\left(m\delta v-\left(u+1\right)\right) \tag{2.9}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$-\frac{\delta M^2}{1+m^2}\left(m\left(u+1\right)+\delta v\right) \tag{2.10}$$

where $M=aB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number and

$\mathrm{Re}=\frac{\rho ac}{\mu}$ is the Reynolds number.

Using long wavelength (*i.e.*, $\delta <<1$) approximation, the equations (2.9) and (2.10) become

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{\partial p}{\partial x}+\frac{M^2}{1+m^2} \tag{2.11}$$

$$\frac{\partial p}{\partial y}=0 \tag{2.12}$$

From Eq. (2.12), it is clear that p is independent of y. Therefore Eq. (2.11) can be rewritten as

$$\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}u=\frac{dp}{dx}+\frac{M^2}{1+m^2} \tag{2.13}$$

The corresponding non-dimensional boundary conditions are given as

$$u=-1 \quad \text{at} \quad y=h \tag{2.14}$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \tag{2.15}$$

Knowing the velocity, the volume flow rate q in a wave frame of reference is given by

$$q=\int_0^h u\,dy. \tag{2.16}$$

The instantaneous flow $Q(X, t)$ in the laboratory frame is

$$Q(X,t)=\int_0^h U\,dY=\int_0^h\left(u+1\right)dy=q+h \tag{2.17}$$

The time averaged volume flow rate $\bar{Q}$ over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is given by

$$\bar{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{2.18}$$

3. *Solution*

Solving Eq. (2.13) together with the boundary conditions (2.14) and (2.15), we get

$$u=\frac{1}{\alpha^2}\frac{dp}{dx}\left[\frac{\cosh\alpha y}{\cosh\alpha h}-1\right]-1 \tag{3.1}$$

The volume flow rate in a wave frame of reference is given by

$$q = \frac{1}{\alpha^3}\frac{dp}{dx}\left[\frac{\sinh \alpha h - \alpha h \cosh \alpha h}{\cosh \alpha h}\right] - h \quad (3.2)$$

From Eq. (3.2), we write

$$\frac{dp}{dx} = \frac{(q+h)\alpha^3 \cosh \alpha h}{\sinh \alpha h - \alpha h \cosh \alpha h} \quad (3.3)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.4)$$

The dimensionless frictional force per one wavelength in the wave frame is defined as

$$F = \int_0^1 (-h)\frac{dp}{dx} dx \quad (3.5)$$

Note that, $M \to 0$ as our results coincide with the results of Shapiro *et al*.[9].

4. Results and Discussion

Fig. 2 shows the variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number M for $\phi = 0.5$ and $m = 0.2$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing M.

The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall parameter m for $\phi = 0.5$ and $M = 1$ is shown in Fig. 3. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing m.

Fig. 4 depicts the variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio ϕ for $M = 1$ and $m = 0.2$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing ϕ.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5$ and $m = 0.2$ is depicted in Fig. 5. It is found that, the time-averaged flow rate $\overline{Q}$ increases in the pumping region $(\Delta p > 0)$ with increasing M, while it decreases in both the free-pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions with increasing M.

Fig. 6 illustrates the variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5$ and $M = 1$. It is observed that, the time-averaged flow rate $\overline{Q}$ decreases in the pumping region with an increase in m, while it increases in both the free-pumping and co-pumping regions with increasing m.

The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.2$ is shown in Fig. 7. It is found that that the time-averaged flow rate $\overline{Q}$ increases with increasing amplitude ratio ϕ in both the pumping and free pumping regions, while it decreases with increasing amplitude ratio ϕ in the co-pumping region for chosen $\Delta p(< 0)$.

Fig. 8 shows the variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5$ and $m = 0.2$. It is noticed that, the frictional force F initially decreases and then increases with increasing M.

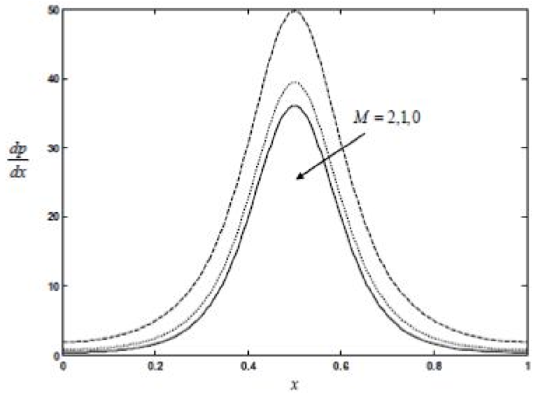

Fig. 2. The variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number M for $\phi = 0.5$ and $m = 0.2$.

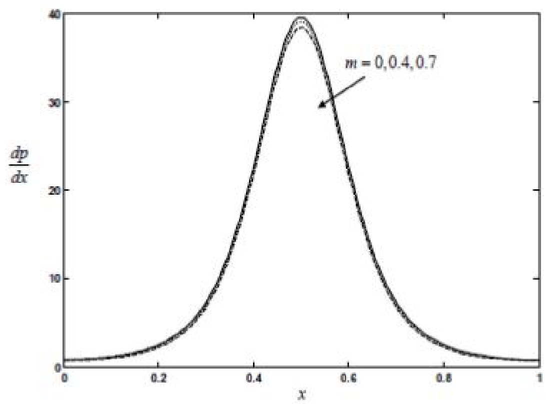

Fig. 3. The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall parameter m for $\phi = 0.5$ and $M = 1$.

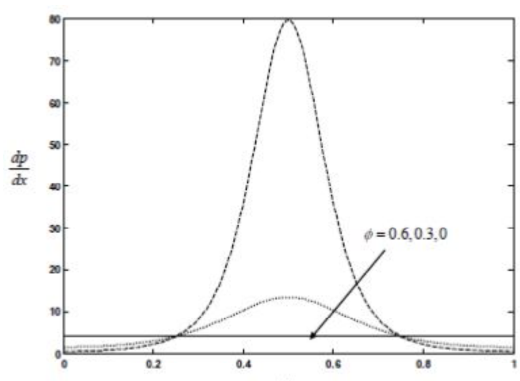

Fig. 4. The variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio ϕ for $M = 1$ and $m = 0.2$.

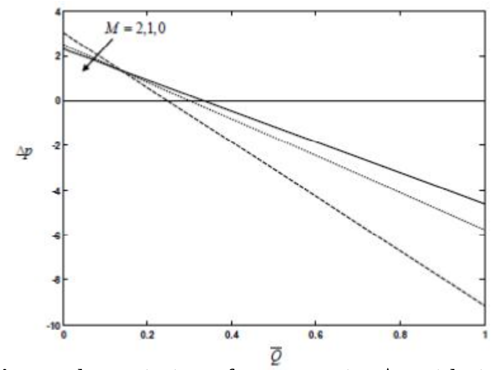

Fig. 5. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5$ and $m = 0.2$.

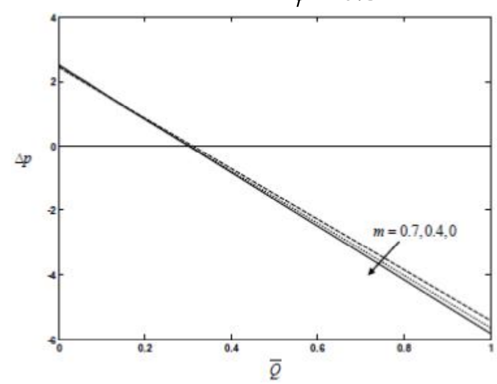

Fig. 6. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5$ and $M = 1$.

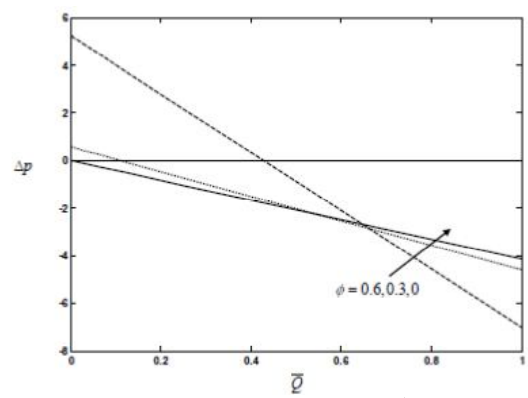

Fig. 7. The variation of pressure rise Δp with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.2$.

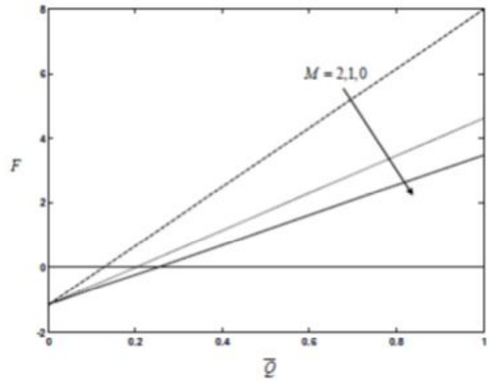

Fig. 8. The variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number M with $\phi = 0.5$ and $m = 0.2$.

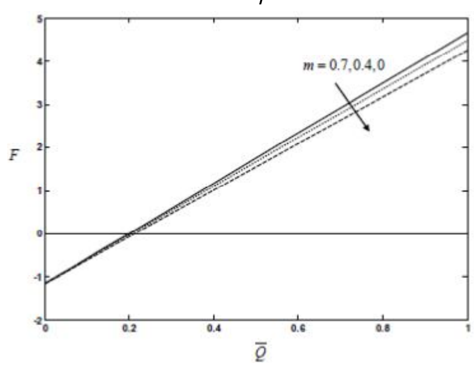

Fig. 9. The variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5$ and $M = 1$.

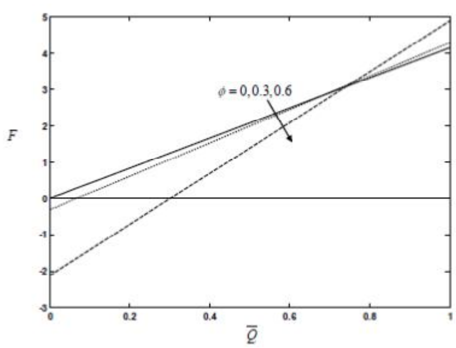

Fig. 10. The variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.2$.

The variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter m with $\phi = 0.5$ and $M = 1$ is shown in Fig. 9. It is observed that, the frictional force F initially increases and then decreases with increasing m.

Fig. 10 depicts the variation of frictional force F with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio ϕ with $M = 1$ and $m = 0.2$. It is found that, the frictional force F first decreases and then increases with increasing ϕ.

5. Conclusions

In this paper, the effect of hall on the peristaltic flow of a conducting fluid in a symmetric channel under the assumption of long wavelength approximation is investigated. The expressions for the velocity and pressure gradient are obtained analytically. It is found that, the pressure gradient and the time-averaged flow rate in the pumping region increases with increasing Hartmann number and amplitude ratio, while they decrease with increasing hall parameter. Further it is observed that, the frictional force initially decreases and then increases with increasing Hartmann number and amplitude ratio, whereas it initially increases and then decreases with increasing hall parameter.

References

1. Agrawal, H.L. and Anwaruddin, B., Peristaltic flow of blood in a branch, *Ranchi University Math. J. 15*, pp. 111-121 (1984).
2. Eldabe, N.T.M., Ahmed Y. Ghaly, A.Y., Sallam, S.N., Elagamy, K. and Younis, Y.M., Hall effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer, *Journal of Applied Mathematics and Physics, 3*, pp. 1138-1150 (2015).
3. Hayat, T., Ali, N. and Asghar, S., Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, *Phys. Letters A*, 363, pp. 397-403 (2007).
4. Jaffrin, M. Y. and Shapiro, A. H., Peristaltic

Pumping, *Ann. Rev. Fluid Mech., 3,* pp. 13-36 (1971).

5. Latham, T. W., *Fluid motions in peristaltic pump*, M.S. Thesis, MIT, Cambridge, Massachussetts (1966).
6. Li, A. A., Nesterov, N. I., Malikova, S. N. and Kilatkin, V. A., The use of an impulse magnetic field in the combined of patients with store fragments in the upper urinary tract. *Vopr kurortol Fizide. Lech Fiz Kult*, *3*, pp. 22-24 (1994).
7. Mekheimer, Kh.S., Peristaltic flow of blood under effect of a magnetic field in a non uniform channel, *Appl. Math. Comput.*, *153*, pp. 763-777 (2004).
8. Prasad, V. V. and Ramacharyulu N. C. P., Unsteady flow of a dusty in compulsive pressure gradient, *Defence Sri. J. 30*, pp. 125-130 (1981).
9. Shapiro, A. H., Jaffrin, M.Y. and Weinberg, S. L., Peristaltic pumping with long wavelengths at low Reynolds number, *J. Fluid Mech. 37,* pp. 799-825 (1969).
10. Srivastava, L. M. and Srivastava, V. P., Peristaltic transport of blood: Casson model II, *J. Biomech*., *17,* pp. 821-829 (1984).
11. Srivastava, L. M., and Agrawal, R. P., Oscillating flow of a conducting fluid with a suspension of spherical particles, *J. Appl. Mech.* 47, pp. 196-199 (1980).
12. Sud, V. K. Sephon, G. S. and Mishra, R. K., Pumping action on blood flow by a magnetic field, *Bull. Math. Biol. 39,* pp. 385 – 390 (1977).