ISSN 2231-346X (Print)

JUSPS-A Vol. 29(7), 264-269 (2017). Periodicity-Monthly

Section A

(Online) ISSN 2319-8044

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Mathematics

website:- www.ultrascientist.org

Estd. 1989

On gspsg-homeomorphisms and sggsp-homeomorphisms in topological spaces

MANOJ GARG

P. G. Department and Research Centre of Mathematics, Nehru Degree College, Chhibramau, Kannauj, U.P., (India)

Corresponding Author Email : garg_manoj1972@yahoo.co.in

http://dx.doi.org/10.22147/jusps-A/290704

Acceptance Date 1st June, 2017, Online Publication Date 2nd July, 2017

Abstract

In the present paper we introduce two new types of mappings called gspsg-homeomorphism and sggsp-homeomorphism and then shown that one of these mapping has a group structure. Further we investigate some properties of these two homeomorphisms.

Key words: Homeomorphism; gspsg-homeomorphism; sggsp-homeomorphism.

1. Introduction

The notion homeomorphisms play a very important role in topology. A homeomorphisms between two topological spaces X and Y is a bijective map $f : X \rightarrow Y$ when both f and f^{-1} are continuous. Devi, Balachandran and Maki[11] in 1995 defined two new classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two new classes of maps called sgc-homeomorphisms and gsc-homeomorphisms. Ahmed and Narli[15] in 2007 defined two classes of maps called gsg-homeomorphisms and sgs-homeomorphisms. Garg, Chauhan and Agarwal[17] in 2007 introduced two new classes of maps namely gsψ-homeomorphisms and ψgs-homeomorphisms. Garg *et al.*[18] again in 2007 introduced two classes of maps called sgψ-homeomorphisms and ψsg- homeomorphisms. In this paper we introduce two new classes of maps called gprsg-homeomorphisms and sggpr-homeomorphisms and then study some of their properties.

Throughout the present paper, (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ) the cl(A), int(A) and A^C denote the closure of A, the interior of A and the complement of A in X respectively.

2. Preliminaries :

In this section we recall the following definitions.

Definition 2.01 : A subset A of a topological space (X, τ) is called semi-open[6] (resp. pre-open, regular open) set if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A= int(cl(A))$). The complement of semi open, pre-open, regular open set is called semi-closed, pre-closed, regular-closed respectively.

Definition 2.02 : A subset A of a topological space (X, τ) is called semi-generalized closed[7] (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open. The complement of sg-closed set is called sg-open set. Every semi-closed set is sg-closed set. The family of all sg-closed sets of any topological space (X, τ) is denoted by sgc (X, τ).

Definition 2.03 : A subset A of a topological space (X, τ) is called generalized semi closed[8] (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The complement of gs-closed set is called gs-open set. Every closed (semi-closed, g-closed and sg-closed) set is gs-closed set. The family of all gs-closed sets of any topological space (X, τ) is denoted by gsc (X, τ).

Definition 2.04 : A subset A of a topological space (X, τ) is called ψ-closed[12] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open. The complement of ψ-closed set is called ψ-open set. Every closed (semi-closed) set is ψ-closed set and every ψ-closed set is sg-closed (gs-closed) set. The family of all ψ-closed sets of any topological space (X, τ) is denoted by ψc (X, τ).

Definition 2.05 : A subset A of a topological space (X, τ) is called generalized pre-regular closed[13] (briefly gpr-closed) set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is reguler-open in (X, τ). The complement of gpr-closed set is called gpr-open set. Every closed set, sg-closed set and gs-closed set is gpr-closed set.The family of all gpr-closed sets of any topological space (X, τ) is denoted by gprc(X, τ).

Definition 2.06 : A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called sg-continuous[14] (resp. gs-continuous[11], ψ-continuous[12], gpr-continuous, sg-irresolute[4], gs-irresolute[11], ψ-irresolute[12], gpr-irresolute[11]) if the inverse image of every closed (resp. closed, closed, closed, sg-closed, gs-closed, ψ-closed, gpr-closed) set in Y is sg-closed (resp. gs-closed, ψ-closed, gpr-closed, sg-closed, gs-closed, ψ-closed, gpr-closed) set in (X, τ).

Definition 2.07 : A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called gsg-irresolute[14] (resp. sgs-irresolute[4], gsψ-irresolute[12], ψgs-irresolute[4], sgψ-irresolute[11], ψsg-irresolute[12]) if the inverse image of every gs-closed (resp. sg-closed, gs-closed, ψ-closed, sg-closed,) set in Y is sg-closed (resp. gs-closed, ψ-closed, gs-closed, ψ-closed, sg-closed) set in (X, τ).

Definition 2.08 : A bijective map $f : (X, \tau) \rightarrow (y, \sigma)$ is called sgc-homeomorphism[11] (resp. gsc-homeomorphism[11], gsg-homeomorphism[15], sgs-homeomorphism[15], gsψ-homeomorphism[17], ψgs-homeomorphism[17], sgψ-homeomorphism[18], ψsg-homeomorphism[18]) if f and f^{-1} are sg-irresolute (resp. gs-irresolute, gsg-irresolute, sgs-irresolute, gsψ-irresolute, ψgs-irresolute, sgψ-irresolute, ψsg-irresolte).

3. GSPSG-homeomorphisms :

In this section we introduce gspsg-homeomorphisms and then investigate the group structure of the set of all gspsg-homeomorphisms.

Definition 3.01 : A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a gspsg-irresolute map if the set $f^{-1}(A)$ is sg-closed in (X, τ) for every gsp-closed set A of (Y, σ).

Definition 3.02 : A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a gspsg- homeomorphisms if the function f and the inverse function f^{-1} are both gspsg-irresolute maps. If there exists a gspsg-homeomorphism from X to Y, then the spaces (X, τ) and (Y, σ) are called gspsg-homeomorphic. The family of all gspsg-homeomorphism of any topological space (X, τ) is denoted by gspsgh-(X, τ).

Remark 3.03 : The following examples show that the concepts of homeomorphism and gspsg-homeomorphism are independent of each other.

Example 3.04 : Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, X}. Define f : (X, τ) $\rightarrow$ (X, τ) by identity mapping then f is a homeomorphism but not a gspsg-homeomorphism.

Example 3.05 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is gspsg-homeomorphism but not homeomorphism.

Theorem 3.06 : Every gspsg-homeomorphism is (i) sgc-homeomorphism (ii) gsg-homeomorphism (iii) ψsg-homeomorphism (iv) $\hat{g}sg$ -homeomorphism(v) ψgs-homeomorphism (vi) $\hat{g}gs$ -homeomorphism (vii) sgs-homeomorphism.

The converse of the above proposition is not true as at it can be seen from the following examples.

Example 3.07 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a}, {a, b}, {a, c}, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is sgs-homeomorphism but not gspsg-homeomorphism.

Example 3.08 : Let X ={a, b, c} and τ = {ϕ, {a, b}, X}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is gsg-homeomorphism but not gspsg-homeomorphism.

Example 3.09 : Let X = Y ={a, b, c}, τ = {ϕ, X} and σ = {ϕ, {a, b}, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is ψsg-homeomorphism but not gspsg-homeomorphism.

Example 3.10 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {a, b}, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is -homeomorphism but not gspsg-homeomorphism.

Example 3.11 : In example (3.07), map f is ψgs-homeomorphism but not gspsg-homeomorphism.

Example 3.12 : In example (3.07), map f is sgs-homeomorphism but not gspsg-homeomorphism.

Remark 3.13 : gspsg-homeomorphism is independent form sgψ-homeomorphism and -homeomorphism as it can be seen from the following examples.

Example 3.14 : Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, Y}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is gspsg-homeomorphism but not sgψ-homeomorphism.

Example 3.15 : In example (3.07), map f is sgψ-homeomorphism but not gspsg-homeomorphism.

Example 3.16 : Let X = {a, b, c} and τ = {ϕ, {a}, {c}, {a, c}, X}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is gspsg-homeomorphism but not -homeomorphism.

Example 3.17 : Let X = {a, b, c} and τ = {ϕ, {a, b}, X}. Define f : (X, τ) $\rightarrow$ (Y, σ) by identity mapping then f is -homeomorphism but not gspsg-homeomorphism.

Theorem 3.18 : Every ψgs-homeomorphism and so gsg-homeomorphism and sgc-homeomorphism from a topological space X onto itself is gspsg-homeomorphism if every gsp-closed set is ψ-closed in X.

Theorem 3.19 : Every $\hat{g}sg$ -homeomorphism and $\hat{g}gs$ -homeomorphism from a topological space X onto itself is gspsg-homeomorphism if every gsp-closed set is $\hat{g}$ -closed in X.

Theorem 3.20 : If f : (X, τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, η) are gspsg-homeomorphism then their composition gof : (X, τ) $\rightarrow$ (Z, η) is also gspsg-homeomorphism.

Theorem 3.21 : If gspsgh(X, τ) is non-empty then the set gspsgh(X, τ) is a group under the composition of maps.

Theorem 3.22 : If f : (X, τ) $\rightarrow$ (Y, σ) be a gspsg-homeomorphism then f induces an isomorphism from the group gspsgh(X, τ) onto the group gspsgh(Y, σ).

Proof : Define θ_f : gspsgh(X, τ) $\rightarrow$ gspsgh(Y, σ) by $\theta_f(h) = fohof^{-1}$ for every h $\in$ gspsgh((X, τ). Then θ_f is a bijection. Further, for all $h_1, h_2 \in$ gspsgh (X, τ), $\theta_f(h_1 o h_2) = fo(h_1 o h_2)of^{-1} = (foh_1of^{-1}) o (fo h_2 of^{-1}) = \theta_f(h_1)$

o $\theta_f(h_2)$. So θ_f is a homeomorphism and so it is an isomorphism induced by f.

4. SGGSP-homeomorphisms :

In this section we introduce sggsp-homeomorphism and investigate its properties.

Definition 4.01 : A map $f : (X, \tau) \to (Y, \sigma)$ is called sggsp-irresolute map if the set $f^{-1}(A)$ is gsp-closed in (X, τ) for every sg-closed set A of (Y, σ).

Definition 4.02 : A bijection $f : (X, \tau) \to (Y, \sigma)$ is called a sggsp-homeomorphism if the function f and the inverse function f^{-1} are both sggsp-irresolute maps. If there exists a sggsp-homeomorphism from X to Y, then the spaces (X, τ) and (Y, σ) are called sggsp-homeomorphic.

The family of all sggsp-homeomorphism of any topological space is denoted by sggsph (X, τ).

Theorem 4.03 : Every (i) homeomorphism (ii) sgc-homeomorphism (iii) sgs-homeomorphism (iv) gsc-homeomorphism (v) gsg-homemorphism (vi) sgψ-homeomorphism (vii) $gs\hat{g}$ -homeomorphism (viii) $sg\hat{g}$ -homeorphism (ix) gspsg-homeomorphism (x) gsψ-homeomorphism (xi) gsc-homeomorphism is sggsp-homeomorphism.

The following examples show that the converse of the above proposition is not true.

Example 4.04 : In example (3.14), map f is sggsp-homeomorphism but not homeomorphism.

Example 4.05 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not sgc-homeomorphism.

Example 4.06 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not sgs-homeomorphism.

Example 4.07 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not gsc-homeomorphism.

Example 4.08 : In example (3.10), map f is sggsp-homeomorphism but not gsg-homeomorphism.

Example 4.09 : Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Then $sgc(X, \tau) = \{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, X\} = \psi gs(X, \tau)$, $gspc(X, \tau) = \{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ and $sgc(Y, \sigma) = \psi s(Y, \sigma) = gspc(Y, \sigma) = \{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not -homeomorphism.

Example 4.10 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not $gs\hat{g}$ -homeomorphism.

Example 4.11 : Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not $sg\hat{g}$ -homeomorphism.

Example 4.12 : Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not gspsg-homeomorphism.

Example 4.13 : In example (3.05), map f is sggsp-homeomorphism but not gsψ-homeomorphism.

Remark 4.14 : sggsp-homeomorphims is independent from ψsg-homeomorphism, $\hat{g}gs$ -homeomorphism, $\hat{g}sg$ -homeomorphism, ψgs-homeomorphism as it can be seen from the following examples.

Example 4.15 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, X\}$ and $\sigma = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is ψsg-homeomorphism but not sggsp-homeomorphism.

Example 4.16 : In example (4.05), map f is sggsp-homeomorphism but not ψgs-homeomorphism.

Example 4.17 : Let $X = Y = \{a, b, c,\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not $\hat{g}gs$ -homeomorphism.

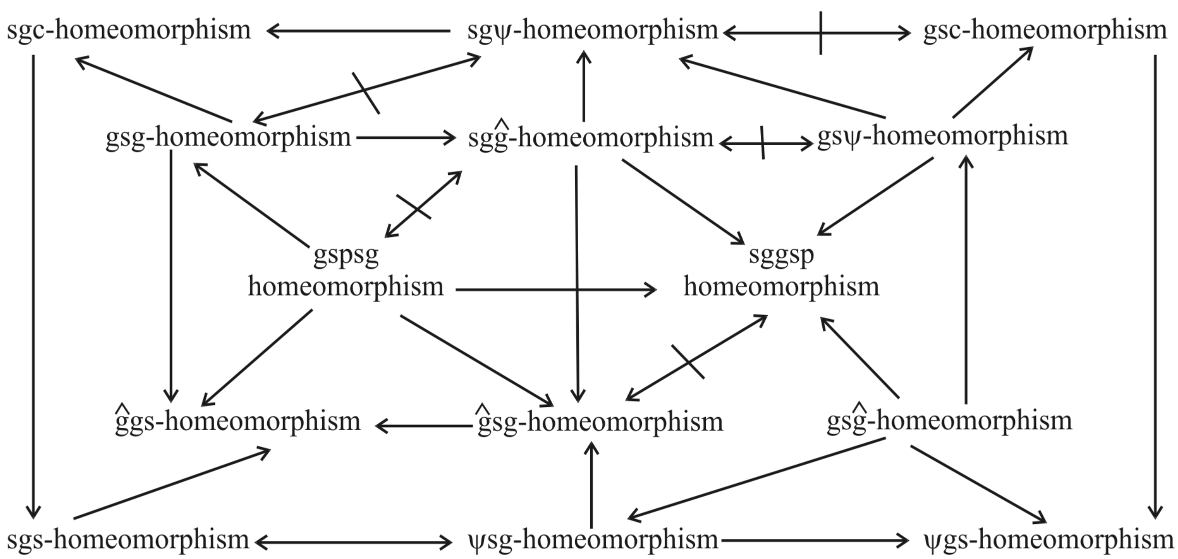
Example 4.18 : In example (4.11), map f is $\hat{g}gs$-homeomorphism but not sggsp-homeomorphism.

Example 4.19 : Let X = Y = {a, b, c,}, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define f : $(X, \tau) \to (Y, \sigma)$ by identity mapping then f is sggsp-homeomorphism but not $\hat{g}sg$-homeomorphism and ψgs-homeomorphism.

Example 4.20 : In example (4.11), map f is $\hat{g}sg$-homeomorphism but not sggsp-homeomorphism.

Example 4.21 : Let X = Y = {a, b, c,}, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, c\}, Y\}$. Define f : $(X, \tau) \to (Y, \sigma)$ by identity mapping then f is ψgs-homeomorphism but not sggsp-homeomorphism.

All the above discussions can be represented by the following diagram.



Where A $\to$ B (resp. A $\leftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent

Theorem 4.22 : Every sggsp-homeomorphism from X onto itself is gspsg-homeomorphism if every gsp-closed set is sg-closed in X.

Theorem 4.23 : Every sggsp-homeomorphism from X onto itself is homeomorphism and so gsc-homeomorphism, sgc-homeomorphism, sgs-homeomorphism gsg-homeomorphism, sgψ-homeomorphism, $gs\hat{g}$-homeomorphism and $sg\hat{g}$-homeomorphism if every gsp-closed set is closed in X.

Remark 4.24 : The composition of two sggsp-homeomorphisms is not necessarily sggsp-homeomrphisms as it can be seen from the following example.

Example 4.25 : Let X = Y = Z = {a, b, c,}, $\tau = \{\phi, \{a\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Define f : $(X, \tau) \to (Y, \sigma)$ and g : $(Y, \sigma) \to (Z, \eta)$ by identity mapping then f and g are sggsp-homeomorphism but gof : $(X, \tau) \to (Z, \eta)$ is not sggsp-homeomorphism.

References

1. N. Biswas, On some mappings in topological spaces, Bull. Calcutta Math. Soc. *61,* 127-135 (1969).

2. N. Biswas, On characterization of semi continuous function, *Atti. Accad. Zaz Lincci. Rend Cl. Sci. Fis Math. Natur. 48(8)*, 399-402 (1970).
3. N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo, 19,* 89-96 (1970).
4. S. G. Crossley and S. K. Hilbebrand : Semi Closure, Texar J. Sci., *22,* 99-112 (1971).
5. S. G. Crossley and S. K. Hilbebrand : Semi Topological Properties, Fund. Math., *74,* 233-254 (1972).
6. T. Noiri, A generalization of closed mappings, *Atti. Accad. Zaz Lincci. Rend Cl. Sci. Fis Math. Natur. 54(8),* 412-415 (1973).
7. P. Bhattacharya and B. K. Lahiri, Semi generalized closed sets in topology, *Indian J. Math. 29,* 376-382 (1987).
8. S. P. Arya and N. Tour, Characterizations of s-normal spaces, *Indian J. Pure Applied Math., 21(8),* 717-719 (1990).
9. P. Sundaram, H. Maki, K. Balachandran., Semi generalized continuous maps and semi-$T_{1/2}$ spaces, *Bull. Fukuoka Univ. Ed. Part-III, 40,* 33-40 (1991).
10. R. Devi, H. Maki and K. Balachandran, Semi generalized closed maps and generalized semi closed maps, *Mem, Fac. Sci. Kochi Univ. Ser. A. Math. 14,* 41-54 (1993).
11. R. Devi, K. Balachandran and H. Maki, Semi Generalized homomorphism and generalized semi homomorphism in topological spaces, *Indian J. Pure Appl. Math., 26(3),* 271-284 (1995).
12. M.K.R.S. Veera Kumar, Between semi closed sets and semi pre-closed sets, *Rend. Instint. Math. Univ. Trieste (Italy)XXXII*, 25-41 (2000).
13. M.K.R.S. Veera Kumar, $\hat{g}$-closed sets and $\hat{G}LC$-function *Indian J. Math.. 43 (2)*, 231-247 (2001).
14. M.K.R.S. Veera Kumar, On $\hat{g}$-closed sets in topological spaces, *Bull. of Allahabad Math. Soc. 18,* 99-112 (2003).
15. Z.O. Ahmet and N. Sarkan, De-composition of homeomorphisms in topological spaces, *International Journal of Math. Sci. 1,* 72-75 (2007).
16. Garg M., Agarwal S. and Goel C.K., On ψ-homeomorphism in topological spaces, *Reflection De Era, 4,* 9-24 (2010).
17. Garg M., Chauhan A. and Agarwal S., New generalization of homeomorphism in topological spaces, *Ultra Scientist, 20(2)M*, 455-462 (2008).
18. Garg M. and Agarwal S., Some new types of homeomorphism in topological spaces, *Antarktika J. Math. 5(1),* 75-85 (2008).
19. Garg M. and Agarwal S., $\hat{G}GS$- homeomorphism and $GS\hat{G}$- homeomorphism in topological spaces, *Jour. Int. Acd. Phy. Sci, 12,* 89-96 (2008).