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A Study of Magnetohydrodynamic Flow of Conducting Walter's Visco-Elastic Fluid in a Long Uniform Rectangular Channel

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Abstract

The aim of the present paper is to study the magneto hydrodynamic flow of conducting Walter's visco-elastic fluid in a long uniform straight channel of rectangular cross-section under the influence of time varying pressure gradient and uniform magnetic field applied perpendicularly to the flow of fluid. The exact solution for the velocity of fluid has been obtained by using integral transform technique. Some particular cases of pressure gradient have been discussed in detail. Also we have discussed the case when magnetic field is withdrawn. Besides, the corresponding viscous flow problem has been derived as a limiting case when the relaxation time parameter tends to become zero.

Key words : Magnetohydrodynamic, visco-elastic, pressure gradient, Magnetic field

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Introduction

Kumar, Singh and Sharma⁶; Agrawal, Agrawal and Varshney¹ discussed the flow of visco-elastic fluid under the influence of magnetic field. Cintaginjala, Rao and Rao² studied about Walter's memory flow. Kumar, Kumar and Rao⁵ analyzed the result of radiation effect on an unsteady MHD flow. Tripathi, kumar and Singh¹¹ discussed the unsteady MHD flow of conducting Walter's visco-elastic fluid through porous medium. Garg, Singh and Bansal⁴; Sarkar Das and Jana¹⁰ discussed the Hall Effect on MHD flow. Sarma⁹ studied the MHD flow of visco-elastic fluid through a channel in the presence of magnetic field. Rajput and Kanaujia⁷; Rath and Mohapatra⁸ discussed the problem of the MHD flow of conducting fluids through Channels. Fazuruddin, Sreekanth and Raju³ investigated the problem of magneto hydrodynamic visco plastic flow over a vertical plate with convective heating.

In the present paper, the flow of Walter's visco-elastic fluid in a long uniform rectangular channel under the influence of time dependent pressure gradient has been studied. Various particular cases have also been discussed in detail. We have also derived the case when magnetic field is withdrawn *i.e.* if $M \rightarrow 0$.

Formulation of the problem :

Here we are considering the motion of conducting visco-elastic Walter's fluid inside a long uniform rectangular tube and under transverse uniform magnetic field.

The boundary walls of rectangular tube considered to be the planes $x = \pm a$, $y = \pm b$. The motion is under the influence of time dependent pressure gradient. Let the motion of the fluid along z -axis *i.e.* along the axis of rectangular channel.

According to the Navier-Stokes equation of motion for visco-elastic Walter's fluid under the influence of uniform magnetic field applied perpendicularly to the flow of fluid is given by

$$\frac{\partial W}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial z} + \nu \left(1 + \mu_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} W \quad (1)$$

where $W(x,y,t)$ is the velocity of the fluid in z -direction, μ_1 the kinematical coefficient of visco-elasticity, ρ the density of the fluid, $\nu \left(= \frac{\mu}{\rho} \right)$ the coefficient of viscosity, σ the electrical conductivity and B_0 is the magnetic inductivity.

Introducing the following non-dimensional quantities:

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, z^* = \frac{z}{a}, t^* = \frac{\nu}{a^2} t, p^* = \frac{a^2}{\rho \nu^2} p$$

$$W^* = \frac{a}{\nu} W, \mu_1^* = \frac{\nu}{a^2} \mu_1,$$

in equation (1), we get (after dropping stars)

$$\frac{\partial W}{\partial t} = -\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right) - M^2 W \quad (2)$$

where $M = B_0 a \sqrt{\frac{\sigma}{\mu}}$ (Hartmann number)

Here, the initial and boundary conditions are

$$W(x, y, 0) = 0 \quad (3)$$

$$\left. \begin{aligned} W(1, y, t) = 0, \quad 0 \leq y \leq 1, \quad t > 0 \\ \frac{\partial W}{\partial x} = 0, \quad x = 0 \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} W(x, l, t) = 0, \quad 0 \leq x \leq 1 & \quad t > 0 \\ \frac{\partial W}{\partial x} = 0, & \quad y = 0 \end{aligned} \right\} \quad (5)$$

where $l = \frac{b}{a}$

Solution of the problem :

For solving eqn. (2), we use the following finite Fourier cosine transforms defined as:

$$W_c(i, y, t) = \int_0^1 W(x, y, t) \cos(p_i x) dx \quad (6)$$

$$W_{\bar{c}}(x, j, t) = \int_0^l W_c(x, y, t) \cos(p_j y) dy, \quad (7)$$

where $p_i = (2i + 1) \frac{\pi}{2}$, $p_j = (2j + 1) \frac{\pi}{2l}$

Consequently, we have the following inverse of finite Fourier cosine transforms:

$$W(x, y, t) = 2 \sum_{i=0}^{\infty} W_c(i, y, t) \cos(p_i x) \quad (8)$$

$$W_c(i, y, t) = \frac{2}{l} \sum_{j=0}^{\infty} W_{\bar{c}}(i, j, t) \cos(p_j y) \quad (9)$$

We use transforms (6) and (7) to initial condition (3), we get

$$W_{\bar{c}}(i, j, 0) = 0 \quad (10)$$

Also taking finite Fourier cosine transform to boundary conditions, we have

$$\left. \begin{aligned} W_c(i, l, t) = 0 \\ \frac{\partial W_c}{\partial y}(i, 0, t) = 0 \end{aligned} \right\} \quad (11)$$

Applying transforms (6) and (7) to the equation of motion (2) and using initial and boundary conditions (10) and (11), we get

$$\zeta_1 \frac{\partial W_c}{\partial t} + \xi_1 W_c = \frac{(-1)^{i+j} F(t)}{\rho_i \rho_j} \quad (12)$$

$$\text{where } W_c = \int_0^1 \int_0^1 W(x, y, t) \cos(p_i x) \cos(p_j y) dx dy$$

$$\frac{\partial p}{\partial z} = -F(t)$$

$$\zeta_1 = 1 - \mu_1 (p_i^2 + p_j^2)$$

$$\text{and } \xi_1 = M^2 + p_i^2 + p_j^2$$

Then using the Laplace transform defined as:

$$\left. \begin{aligned} \bar{W}_c(s) &= \int_0^\infty W_c e^{-st} dt \\ \bar{F}(s) &= \int_0^\infty F(t) e^{-st} dt \end{aligned} \right\} \quad (13)$$

and by condition (11) on equation (12), we get

$$\zeta_1 \bar{W}_c + \xi_1 \bar{W}_c = \frac{(-1)^{i+j} \bar{F}(s)}{\rho_i \rho_j} \quad (14)$$

Now, by Laplace inversion formula and using convolution theorem, we get

$$\bar{W}_c = \frac{(-1)^{i+j}}{\rho_i \rho_j \zeta_1} \int_0^t F(t - \lambda) e^{-(\xi_1/\zeta_1)\lambda} d\lambda \quad (15)$$

Thus, by Fourier cosine inversion formula as in equation (8) and (9), the expression of velocity becomes

$$W(x, y, t) = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j}}{\rho_i \rho_j \eta} \left\{ \int_0^t F(t - \lambda) e^{-c_1 \lambda} d\lambda \right\} \cos(p_i x) \cos(p_j y) \right] \quad (16)$$

$$\text{where } c_1 = \frac{\xi_1}{\zeta_1}, \quad p_i = (2i + 1) \frac{\pi}{2l}, \quad p_j = (2j + 1) \frac{\pi}{2l}$$

We discuss the nature of velocity for following different particular cases:

Case I: Flow under constant pressure gradient:

$$\text{Let, } F(t) = F_0 \quad (\text{a constant})$$

From equation (16) the velocity will be

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j} F_0}{\rho_i \rho_j \xi_1} (1 - e^{-c_1 t}) \cos(p_i x) \cos(p_j y) \right] \quad (17)$$

Case II: Flow under impulsive pressure gradient:

$$\text{Let, } F(t) = f_0 \delta(t)$$

Where $\delta(t)$ is the unit impulse function defined as

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

So, from equation (16), we get the velocity

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j} f_0}{p_i p_j \zeta_1} e^{-c_1 t} \cos(p_i x) \cos(p_j y) \right] \quad (18)$$

Case III: Flow under transient pressure gradient:

$$\text{Let, } F(t) = f_1 e^{-Nt}, \quad (N > 0),$$

Where f_1 is a constant.

So from equation (16), the velocity takes form

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j} f_1 e^{-Nt}}{p_i p_j (\xi_1 - N\zeta_1)} \{1 - e^{-(c_1 - N)t}\} \cos(p_i x) \cos(p_j y) \right] \quad (19)$$

Case IV: Flow under periodic pressure gradient:

$$\text{Let, } F(t) = \text{Re} (F_1 e^{i\omega t}),$$

Where F_1 is a constant,

From equation (16), the velocity becomes

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j} F_1}{p_i p_j (\xi_1^2 - \omega^2 \zeta_1^2)} \{ \omega \zeta_1 \sin \omega t + \xi_1 (\cos \omega t - 1) \} \right] \quad \text{xcos}(p_i x) (p_j y) \quad (20)$$

Case V: When the fluid is purely viscous:

For purely viscous fluid the kinematical co-efficient of visco-elasticity $\mu_1 = 0$ and we get

$$W = \frac{4}{l} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[\frac{(-1)^{i+j}}{p_i p_j} \int_0^t F(t - \theta) e^{-\xi_1 \theta} d\theta \right] \cos(p_i x) (p_j y) \quad (21)$$

$$\text{where } \xi_1 = M^2 + p_i^2 + p_j^2, \quad c_1 = \xi_1$$

Case VI: when magnetic field is withdrawn *i.e.* $M \rightarrow 0$

We get all results for Walter's fluid motion in the absence of magnetic field

The values of ξ_1 and ζ_1 are given by

$$\xi_1 = p_i^2 + P_j^2 \text{ and } \zeta_1 = 1 - \mu_1(p_i^2 + p_j^2) \quad (22)$$

Conclusion

In this paper we discussed the MHD flow of conducting Walter's visco-elastic fluid in a long uniform rectangular channel. Also we discussed the nature of velocity for different cases as flow under constant pressure gradient, impulsive pressure gradient, transient pressure gradient and when fluid is purely viscous and when the magnetic field is withdrawn.

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