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Accurate Connected Edge Domination Number in Graphs

VENKANAGOUDA M. GOUDAR[1], VENKATESH S.H. [2], VENKATESHA [3] and TEJASWINI K.M. [4]

[1] Department of Mathematics, Gouthama Research Centre, (Affiliated to Kuvempu University), Sri Siddhartha Institute of Technology, Tumkur, karnataka, India

[2] Research Scholar, Gouthama Research Centre, (Affiliated to Kuvempu University), Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India

[3] Department of Mathematics, Kuvempu University Shankarghatta, Shimoga, Karnataka, India.

[4] Department of Mathematics, Sri Siddhartha Institute of Technology, Tumkur, karnataka, India

Corresponding Author Email : sh1.venkatesh@gmail.com

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Abstract

An accurate edge dominating set D of a graph $G=(V,E)$ is an accurate connected edge dominating set, if $<D>$ is connected. The accurate connected edge domination number $\gamma_{cae}(G)$ is the minimum cardinality of an accurate connected edge dominating set. In this paper, we initiate a study of $\gamma_{cae}(G)$ in terms of vertices, edges, cut vertices and different parameters of G. Further we characterize the accurate connected edge domination number in cartesian product and corona of graphs.

Key words: Edge dominating set, Accurate edge dominating set, Accurate connected edge dominating set.

Mathematics Subject Classification: 05C,05C05, 05C70.

1 Introduction

Let G be a finite, simple, non-trivial, undirected and connected (p,q) graph with vertex set $V(G)$

and edge set $E(G)$. The greatest distance between any two vertices of a connected graph G is called the diameter of G and is denoted by $diam(G)$. For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x. In general $< X >$ to denote the subgraph induced by the set of vertices X. The maximum(minimum) degree of a vertex v is denoted by $\Delta(G)(\delta(G))$.

A nonseparable graph is connected, nontrivial and has no cut points. A block of a graph is a maximal nonseparable subgraph. As usual P_p, C_p, W_p and K_p are respectively the path, cycle, wheel and complete graph.

Spider is a tree with one vertex of degree at least three and all others with degree at most one. Caterpillar tree is a tree in which all the vertices are within distance one of a central path. A polyiamond composed of six equilateral triangle is called Hexiamond.

A set $D \subseteq E(G)$ is said to be an edge dominating set if every edge in $< E(G) - D >$ is adjacent to some edges in D. The Edge domination number of G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma'(G)$. This concept was introduced by Mitchell and Hedetniemi[7].

A edge dominating set D of a graph G is an accurate edge dominating set, if $< E - D >$ has no dominating set of cardinality $|D|$. The accurate edge domination $\gamma_{ae}(G)$ is the minimum cardinality of an accurate dominating set. This concept was introduced by Venkatesh , Kulli, Venkanagouda M Goudar and Venkatesha[10].

In this paper we follow the notations of [4].

2 Preliminary notes :

We need the following results to prove further results.

Theorem 2.1[12] *If G is a* (p,q) *graph without isolated vertex then* $\frac{q}{\Delta(G)+1} \leq \gamma'(G)$.

Theorem 2.2[3] *For any connected graph G of order* $p > 3$, $\gamma_c(G) \leq p-2$.

Theorem 2.3[8] *For any graph G,* $\gamma'(G) \leq q - \Delta(G)$.

In the next section we define and discuss some results on accurate connected edge domination number of a graph.

3 Accurate connected edge domination number of a graph :

We define a new parameter accurate connected edge domination number of a graph. An accurate edge dominating set D of a graph $G = (V, E)$ is an accurate connected edge dominating set, if $< D >$ is connected. The accurate connected edge domination number $\gamma_{cae}(G)$ is the minimum cardinality of an accurate connected edge dominating set.

In the figure 3.1, the accurate connected edge dominating set is $A = \{9,10,11\}$. Therefore

$\gamma_{cae}(G) = |A| = 3.$

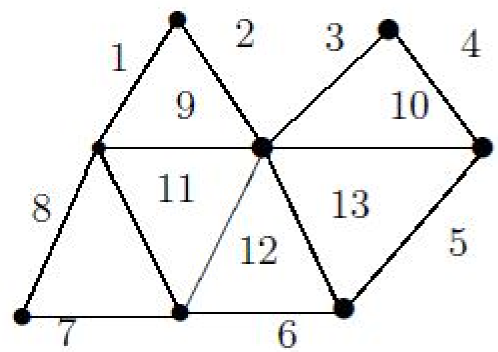

Figure 3.1

4 Accurate connected edge domination number of graphs :

Theorem 4.1

a) For any Wheel W_p, $\gamma_{cae}(W_p) = \lceil \frac{p+1}{2} \rceil$.

b) Let p_2 *be a path and* C_p *be any cycle then* $\gamma_{cae}(P_2 \times C_p) = 2p - 2$.

c) Let p_3 *and* P_{p_1} *be two paths with* $p_1 > 3$ *then* $\gamma_{cae}(P_3 \times P_{p_1}) = 2p_1$.

d) Let K_1 *and* C_p *be the complete graph and cycle respectively with* $p \geq 3$ *then* $\gamma_{cae}(K_1 \circ C_p) = p - 2$.

Theorem 4.2 Let T *be the caterpillar tree with each cut vertex of degree greater than two. Then*

$$\gamma_{cae}(T) = \begin{cases} S+1 & \text{if } s \text{ is odd} \\ S+1 & \text{if } s \text{ is odd} \end{cases}$$

Where s *is the number of cut vertices in* T.

Proof. Let T be the caterpillar tree of (p,q) graph with each cut vertex of degree greater than two. We consider $M = \{e_g / 1 \leq g \leq q\}$ be the set of all non-end edges of T and $N = \{e_j / 1 \leq j \leq q\}$ be the set of all end edges of T. Let $C = \{v_1, v_2, \ldots v_s / 1 \leq s \leq p\}$ be the cut vertices of T and $|C| = s$. Let $D = \{e_1, e_2, \ldots e_d / 1 \leq d \leq q\}$ be the minimum accurate edge dominating set of T. If the induced subgraph $< D >$ is connected then $A = D$ itself forms a minimum accurate connected edge dominating set of T. Otherwise, if the induced subgraph $< D >$ is not connected then consider $D_1 \subseteq E(T) - D$ such that

$A = D \cup D_1$ forms a minimum accurate connected edge dominating set of T.
We have the following cases

Case 1. Suppose $|C|$ is odd.

Consider $N_1 \subseteq N$ and let $D = M \cup N_1$ forms the accurate edge dominating set with minimum cardinality. Clearly, $M \subseteq D$ and the induced subgraph D is connected. Then $A = D$ itself forms the accurate connected edge dominating set of T. Thus, $|A| = |C| + 1$. Therefore $\gamma_{cae}(T) = s + 1$.

Case 2. Suppose $|C|$ is even.

Let $M_1 = \{e_1, e_3, e_5, \ldots e_r / 1 \le r \le g\} \subseteq M$, $N_1 \subseteq N$ and let $D = M_1 \cup N_1$ forms the accurate edge dominating set with minimum cardinality. But the induced subgraph $< D >$ is not connected. Consider $M_2 = \{e_2, e_4, \ldots e_t / 1 \le t \le g\}$ and $A = D \cup M_2$ forms the accurate connected edge dominating set of T with minimum cardinality. Thus, $|A| = |C| - 1$. Therefore $\gamma_{cae}(T) = s - 1$.
Hence the proof.

Theorem 4.3 For any spider tree T and $diam(T) \le 8$, $\gamma_{cae}(T) = \lceil \frac{diam(T)}{2} \rceil + 2\Delta(T)$.

Proof. Let T be a spider tree with $diam(T) \le 8$. Let $B = \{e_1, e_2, \ldots e_i / 1 \le i \le q\}$ be the set of edges which constitute the diametrical path in T and $|B| = diam(T) \le 8$. Let $u \in \Delta(T)$ be the maximum degree and there exists at least three edges incident to u. That is $\Delta(T) \ge 3$. Let $D = \{e_1, e_2, \ldots e_j / 1 \le j \le q\}$ be the minimum accurate edge dominating set of T.
We have the following cases

Case 1. Suppose $\Delta(T) = 3$.

For every $e_j \in D$ for $1 \le j \le q$ lies in the diametrical path, that is $D \subseteq B$. If $< D >$ is connected then $A = D$ itself forms the accurate connected edge dominating set with minimum cardinality. Otherwise, consider $e_k \in E(T) - D$, $1 \le k \le q$ and $A = \{e_j\} \cup \{e_k\}$ forms minimum accurate connected edge dominating set of I. Also $e_j, e_k \in B$, for all j, k.
Therefore,

$$A \subseteq B,$$
$$|A| \le |B| + 2|u|,$$
$$|A| \le |B| + 2\Delta(T),$$
$$\gamma_{cae}(T) = \lceil \frac{diam(T)}{2} \rceil + 2\Delta(T).$$

Case 2. Suppose $\Delta(T) > 3$.

Let $R = \{e_1, e_2, ...e_m / 1 \le m \le q\}$ be the edge set incident to $u \in \Delta(T)$. There exists at least one edge $e_r \in D \cap B$ and $e_r \in R$, $1 \le r \le m$ such that $\{e_r\} \cup \{e_j\} \subseteq D$ for $1 \le j \le q$. If $< D >$ is connected then $A = D$ itself forms the accurate connected edge dominating set with minimum cardinality. Otherwise, consider $e_f \in E(T) - D$, $1 \le f \le q$ and $A = \{e_f\} \cup \{e_r\} \cup \{e_j\}$ forms minimum accurate connected edge dominating set of T.

Therefore,

$$A \subseteq B,$$
$$|A| \le |B| + 2|u|,$$
$$|A| \le |B| + 2\Delta(T),$$
$$\gamma_{cae}(T) = \lceil \frac{diam(T)}{2} \rceil + 2\Delta(T).$$

Hence the proof.

Theorem 4.4 For any path P_{p_1} *of order* $p_1 \ge 3$, $\gamma_{cae}(P_{p_1}) = q - 2$ *except* $p_1 = 5$.

Proof. Let $v \in \delta(P_{p_1})$ be the minimum degree of P_{p_1} and $|v| = 1$. Let $D = \{e_1, e_2, ...e_s / 1 \le s \le q\}$ be the minimum accurate edge dominating set of P_{p_1}. If $< D >$ is connected then $A = D$ itself forms a accurate connected edge dominating set of P_{p_1} with minimum cardinality. Otherwise, consider $F \subseteq E(P_{p_1}) - D$ and the set $A = D \cup F$ forms the minimum accurate connected edge dominating set of P_{p_1} such that $|A| = \gamma_{cae}(P_{p_1})$. From theorem 2.2,

$$|A| \le p_1 - 2,$$
$$|A| + |v| = p_1 - 2,$$
$$|A| = p_1 - 2 - |v|,$$
$$|A| = p_1 - 2 - 1,$$
$$|A| = (p_1 - 1) - 2,$$
$$|A| = q - 2,$$
$$\gamma_{cae}(P_{p_1}) = q - 2.$$

Hence the proof.

Theorem 4.5 For any complete graph K_{p_1} *of order* $p_1 > 3$,

$$\gamma_{cae}(K_{p_1}) = \begin{cases} 3 & if\ p_1 = 4 and 5 \\ p_1 - 1 & if\ p_1 \geq 6 \end{cases}$$

Proof. Let $G = K_{p_1}$ be a complete graph of order $p_1 > 3$ with maximum degree $\Delta(G) = p_1 - 1$. The distance between any two vertices is exactly one and $diam(G) = 1$.
We have the following cases

Case 1. Suppose $p_1 = 4 and 5$.

Let $D = \{x, y, z\} \subseteq E(G)$ be the minimum accurate edge dominating set of G. The induced subgraph $< D >$ is connected. So that $A = D$ itself forms a minimum accurate connected edge dominating set of G such that $|A| = 3$. Clearly, $|A| \leq \Delta(G)$. Obviously the equality holds if $p_1 = 4$. Thus, $|A| = \Delta(G)$.

$\gamma_{cae}(K_{p_1}) = p_1 - 1 = 4 - 1 = 3$.

Suppose $p_1 = 5$,

$$|A| \leq \Delta(G),$$
$$|A| + diam(G) = \Delta(G),$$
$$|A| + 1 = \Delta(G),$$
$$|A| = \Delta(G) - 1,$$
$$\gamma_{cae}(K_{p_1}) = (p_1 - 1) - 1 = p_1 - 2 = 5 - 2 = 3.$$

Case 2. Suppose $p_1 \geq 6$.

Let $D = \{e_1, e_2, ...e_{p_1 - 2}\} \subseteq E(G)$ be the minimum accurate edge dominating set of G. The induced subgraph $< D >$ is not connected. Consider an edge $\{x\} \in N(D)$ and $A = D \cup \{x\}$ forms the minimum accurate connected edge dominating set of G and $|A| = p_1 - 1$.
Clearly,

$$|A| = |V(G)| - diam(G),$$
$$|A| = \Delta(G),$$
$$\gamma_{cae}(K_{p_1}) = p_1 - 1.$$

Hence the Proof.

Theorem 4.6 For any cycle C_{p_1} of order $p_1 \geq 5$, $\gamma_{cae}(C_{p_1}) = q - \Delta(C_{p_1})$.

Proof. Let $G = C_{p_1} : e_1, e_2, ...e_{p_1}$ be the cycle with q edges and $\Delta(C_{p_1}) = 2$. Let

$D=\{e_1,e_2,...e_{\lfloor\frac{p_1}{2}\rfloor+1}\}\subseteq E(G)$ be the minimum accurate edge dominating set of G. The induced subgraph $<D>$ is not connected except C_5 and C_6. Consider $B=\{e_i/1\le i\le q\}\subseteq N(D)$ and $A=D\cup B$ forms the minimum accurate connected edge dominating set of G. Further, edge subset $R=\{y_1,y_2\}\subseteq N(A)$ are connected. By theorem 2.3,

$$|A|=|E(G)|-|R|,$$
$$|A|=q-2,$$
$$\gamma_{cae}(C_{p_1})=q-\Delta(C_{p_1}).$$

Hence the proof.

Theorem 4.7 Let $G=B(p=8,q=13)$ *is a Hexiamond graph,* $\gamma_{cae}(B)\ge diam(B)$.

Proof. Let $E(G)=\{e_1,e_2,...e_{13}\}$ be the edge set of Hexiamond graph G. Let $M=\{e_1,e_2,...e_r/1\le r\le 13\}$ be the minimum set of edges which constitute the longest path between any two distinct vertices $v_1,v_2\in V(G)$ such that $d(v_1,v_2)=diam(G)$.

We have the following cases

Case 1. Suppose $G=B$ contains at least two vertices of maximum degree is five. Let $D=\{e_1,e_2\}$ be the minimum accurate edge dominating set of G and induced subgraph $<D>$ is not connected. Consider $A=D\cup\{e_3\}$ be the minimum accurate connected edge dominating set of G. Clearly, $|A|\ge|M|$. Thus, $\gamma_{cae}(B)\ge diam(B)$.

Case 2. Suppose $G=B$ contains at most one vertex of maximum degree is five. Let $D=\{e_1,e_2,e_3,e_4\}$ be the minimum accurate edge dominating set of G and induced subgraph $<D>$ is connected. Therefore, the set $A=D$ is the minimum accurate connected edge dominating set of G. Clearly, $|A|\ge|M|$. Thus, $\gamma_{cae}(B)\ge diam(B)$.

Hence the proof.

5 Results on corona and cartesian product of graphs :

In this section we discuss the results on Accurate connected edge domination number of corona and cartesian product of two graphs.

The cartesian product of the graphs G and H, written as $G\times H$, is the graph with vertex set $V(G)\times V(H)$, two vertices (u_1,u_2) and (v_1,v_2) being adjacent in $G\times H$ if and only if either $u_1=v_1$ and $u_2v_2\in E(H)$, or $u_2=v_2$ and $u_1v_1\in E(G)$.

Theorem 5.1 Let G *be the cartesian product of* $K_p\times K_p$ *of order* $p\ge 2$,

$$\gamma_{cae}(K_p \times K_p) = \begin{cases} \mathrm{p}+1 & \text{if } p = 2 \\ \mathrm{p}+2 & \text{if } p = 3 \\ \mathrm{p}^2 - p - 1 & \text{if } p \geq 4 \end{cases}$$

Proof. Let K_p be the complete graph with p vertices and q edges. Let $G = K_p \times K_p$ be the cartesian product of order $p \geq 2$. Consider

$$V(G) = \{(u_1,v_1),(u_1,v_2),\ldots,(u_1,v_p),$$
$$(u_2,v_1),(u_2,v_2),\ldots,(u_2,v_p),$$
$$\vdots$$
$$(u_p,v_1),(u_p,v_2),\ldots,(u_p,v_p)\}$$

and the edge set $E(G) = \{e_1, e_2, .. e_{2pq}\}$.

We have the following cases

Case 1. Suppose $p = 2$ then $D = \{e_1, e_2, e_3\}$ be the minimum accurate edge dominating set of G and the induced subgraph $< D >$ is connected. Therefore $A = D$ itself forms a minimum accurate connected dominating set of G. Clearly, $|A| = |V(K_p)| + 1$. Hence $\gamma_{cae}(G) = p + 1$.

Case 2. Suppose $p = 3$ then $D = \{e_1, e_2, e_3\}$ be the minimum accurate edge dominating set of G. But the induced subgraph $< D >$ is not connected. Consider $D_1 = \{e_4, e_6\} \subseteq E(G) - D$ such that $A = D \cup D_1$ forms a minimum accurate connected dominating set of G. Clearly, $|A| = |D| + 2$. It implies, $|A| = |V(K_p)| + 2$. Hence $\gamma_{cae}(G) = p + 2$.

Case 3. Suppose $p \geq 4$ then $D = \{e_1, e_2, .. e_t / 1 \leq t \leq q\}$ be the minimum accurate edge dominating set of G. But the induced subgraph $< D >$ is not connected. Consider $D_1 = \{e_1, e_2, \ldots e_r / 1 \leq r \leq q\} \subseteq E(G) - D$ such that $A = D \cup D_1$ forms a minimum accurate connected dominating set of G. Clearly, $|A| = |V(K_p)|^2 - |V(K_p)| + 1$. Hence $\gamma_{cae}(G) = p^2 - p + 1$.

Hence the proof.

Theorem 5.2 Let For any path P_{p_1} *with* $p_1 \geq 2$,

$$\gamma_{cae}(P_{p_1} \times P_{p_1}) = \begin{cases} \mathrm{r}+1 & if\ p_1 = 2 \\ \mathrm{r}-1 & if\ p_1 = 3 \\ \mathrm{r}+2 & if\ p_1 > 3 \end{cases}$$

Where is the number of regions in $P_{p_1} \times P_{p_1}$.

Proof. Let $G = P_{p_1} \times P_{p_1}$ be the (p,q) graph. Let $R = \{r_1, r_2, ...r_{(p_1-1)^2+1}\}$ be the region set of G and $D=\{e_1,e_2,..e_l/1\leq l \leq q\}$ be the minimum accurate edge dominating set of G.

We have the following cases

Case1. Suppose $p_1 = 2$.

The set $D=\{e_1,e_2,...e_{p_1+1}\}$ is accurate edge dominating set of G and also induced subgraph $< D >$ is connected. Therefore $A = D$ itself forms the minimum accurate connected edge dominating set of G. Clearly,

$$|A| = p_1+1,$$

$$|A| = [(p_1-1)^2+1]+1,$$

$$|A| = |R|+1,$$

$$\gamma_{cae}(P_{p_1} \times P_{p_1}) = r+1.$$

Case 2. Suppose $p_1 = 3$.

Clearly, $D=\{e_1,e_2,...e_{p_1+1}\}$ is accurate edge dominating set of G and its induced subgraph $< D >$ is connected. Therefore $A = D$ itself forms the minimum accurate connected edge dominating set of G. Clearly,

$$|A| = p_1+1,$$

$$|A| = [(p_1-1)^2+1]-1,$$

$$|A| = |R|-1,$$

$$\gamma_{cae}(P_{p_1} \times P_{p_1}) = r-1.$$

Case 3. Suppose $p_1 \geq 3$.

The set $D=\{e_1,e_2,..e_l/1\leq l \leq q\}$ is not connected and consider $A = D \cup D_1$ forms a minimum accurate connected dominating set of G, where $D_1 \subseteq E(G) - D$. Clearly,

$$|R| > |D|,$$
$$|R| > |A|,$$
$$|R| = |A| - 2,$$
$$|A| = |R| + 2,$$
$$|A| = [(p_1 - 1)^2 + 1] + 2,$$
$$\gamma_{cae}(P_{p_1} \times P_{p_1}) = r + 2.$$

Hence the proof.

Theorem 5.3 Let P_{p_1} be a path of length greater than or equal to two and C_4 is a cycle then, $\gamma_{cae}(P_{p_1} \times C_4) \geq diam(P_{p_1} \times C_4) + \Delta(P_{p_1} \times C_4)$.

Proof. Let $G = P_{p_1} \times C_4$ be the cartesian product of (p, q) graph. Let $v_1 \in V(G)$ be the maximum degree of G and $|v_1| = 4 = \Delta(P_{p_1} \times C_4)$. Let $\{e_1, e_2, ... e_{p_1+1}\}$ forms the diametrical path of G such that $diam(G) = |V(P_{p_1})| + 1$. Let $D = \{e_i / 1 \leq i \leq q\}$ be the γ_{ae} set of G. We observed that induced subgraph $< D >$ is disconnected. Choose an edge set $F = \{e_j / 1 \leq j \leq q\} \subseteq E(G) - D$ and $A = D \cup F$ forms the minimum accurate connected edge dominating set of G.

Thus,

$$|A| \geq |V(P_{p_1})| + |v_1| + 1,$$
$$|A| \geq (|V(P_{p_1})| + 1) + |v_1|,$$
$$\gamma_{cae}(P_{p_1} \times C_4) \geq diam(P_{p_1} \times C_4) + \Delta(P_{p_1} \times C_4).$$

Hence the proof.

Theorem 5.4 Let $G = K_{p_1} \times K_{p_2}$ be the cartesian product graph with $p_1 < p_2$, $p_1 \geq 2$, $\gamma_{cae}(K_{p_1} \times K_{p_2}) \geq 2p_2 - 3$. *Equality holds if $p_1 = 2$.*

Proof. Let $G = K_{p_1}$ and $H = K_{p_2}$ be the two graph of order p_1 and p_2 respectively. Let $E(G \times H) = \{e_1, e_2, .. e_q\}$ be the edge set of $G \times H$.

We have the following cases

Case 1. Suppose $G = K_2$ and $H = K_{p_2}$ with $p_2 \geq 3$.

The edge set $E(G \times H) = \{e_1, e_2, ... e_{p_2^2}\}$ and let $D = \{e_1, e_2, ... e_{p_2}\}$ be the accurate edge dominating set of $G \times H$. But induced subgraph $< D >$ is disconnected except $K_2 \times K_3$. Consider $\{e_j\} \in E - D$, $1 \le j \le q$ such that $A = D \cup \{e_j\} = \{e_1, e_2, ...e_{2p_2-3}\}$ forms the minimum accurate connected edge dominating set of $G \times H$. Therefore, $|A| = 2|V(H)| - 3$. Thus, $\gamma_{cae}(G \times H) = 2p_2 - 3$.

Case 2. Suppose $G = K_{p_1}$ with $p_1 > 2$ and $H = K_{p_2}$ with $p_2 \ge 4$.

Let $D = \{e_1, e_2, ..e_t / 1 \le t \le q\}$ be the accurate edge dominating set of $G \times H$. But induced subgraph $< D >$ is disconnected. Consider $\{e_j / 1 \le j \le q\} \subseteq E - D$ such that $A = D \cup \{e_j\}$ forms the minimum accurate connected edge dominating set of $G \times H$. Therefore, $|A| \ge 2|V(H)| - 3$. Thus, $\gamma_{cae}(G \times H) \ge 2p_2 - 3$.

Hence the proof.

Now we define corona, Let G_1 and G_2 be the graphs of order p_1 and p_2 respectively. The corona of two graphs G_1 and G_2 is the graph $G_1 \circ G_2$ obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_1.

Theorem 5.5 For any G_3 and path P_{p_1} path of order p_1, $\gamma_{cae}(C_3 \circ P_{p_1}) \le 2(p_1 + 1)$.

Proof. Let $G = C_3 \circ P_{p_1}$ be the corona of (p, q) graph. Let $V(C_3) = \{u_1, u_2, u_3\}$ and $V(P_{p_1}) = \{v_1, v_2, ...v_p\}$ be the vertex set of G_3 and P_{p_1} respectively. Let $x \in V(G)$ be the maximum degree of G and $\Delta(G) = p_1 + 2$. Let $D \subseteq E(G)$ be the accurate edge dominating set with minimum cardinality. Also the induced subgraph $< D >$ is not connected except $C_3 \circ K_1$. Consider $F \subseteq E(G) - D$ such that $A = D \cup F$ forms minimum accurate connected edge dominating set of G.

Clearly,

$$|A| \le |V(P_{p_1})| + \Delta(G),$$

$$|A| \le p_1 + (p_1 + 2),$$

$$\gamma_{cae}(C_3 \circ P_{p_1}) \le 2(p_1 + 1).$$

Hence the proof.

Theorem 5.6 For any path P_{p_1} of length greater than two and the path P_2, $\gamma_{cae}(P_{p_1} \circ P_2) = 2p_1 - 1$.

Proof. Consider the graph $G = P_{p_1} \circ P_2$ with p vertices and q edges. Let $B_1 = \{b_1, b_2, ...b_{p_1}\}$ be the block set such that $E(b_i(G)) = \{e_{1i}, e_{2i}, e_{3i} / 1 \le i \le p_1\}$ for every block $b_i \in B_1$ and $B_2 = \{b_1', b_2', ...b_{p_1-1}'\}$

be the block set such that $E(b_j^{'}(G))=\{x_{1j}/1\leq j\leq p_1-1\}$ for every block $b_j^{'}\in B_2$. Let $D=\{e_1,e_2,...e_{p_1+1}\}$ be the $\gamma_{ae}(G)$ of G. But D is not accurate connected edge dominating set of G, which is a contradiction. Choose some edges $\{e_f/1\leq f\leq q\}\subseteq(E(G)-D)$ and $A=D\cup\{e_f\}$ forms minimum $\gamma_{cae}(G)$ set of G. We observe that $E(b_j^{'}(G))\subseteq A$ for $1\leq j\leq p_1-1$. Thus, $|A|=|B_2|+|B_1|$. It implies that, $|A|=(p_1-1)+p_1$. Therefore, $\gamma_{cae}(P_{p_1}\circ P_2)=2p_1-1$.
Hence the proof.

6 Conclusion

In this paper, we introduced the new parameter $\gamma_{cae}(G)$ of G, in the field of domination theory of graphs. We obtained many exact values and bounds for $\gamma_{cae}(G)$. We also obtained some results of $\gamma_{cae}(G)$ on the cartesian product and corona of graphs.

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