ISSN 2231-3478 (Print)
JUSPS-B Vol. 29(7), 149-162 (2017). Periodicity-Monthly
Section B
(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

Heat and Mass transfer on Unsteady MHD Oscillatory flow of Blood through porous arteriole with Hall effects

KAMBOJI JYOTHI[1] and M. VEERA KRISHNA[*2]

[1, 2] Department of Mathematics, Rayalaseema University, Kurnool, Andhra Pradesh (India)
Corresponding author Email:- veerakrishna_maths@yahoo.com
http://dx.doi.org/10.22147/jusps-B/290701

Acceptance Date 2nd June, 2017, Online Publication Date 2nd July, 2017

Abstract

We considered the MHD oscillatory flow of blood in a porous arteriole under the influence of uniform transverse magnetic field in a parallel plate channel taking hall current into account. Heat and mass transfer during arterial blood flow through porous medium are also studied. A mathematical model is developed for unsteady state situations using slip conditions. The unsteady hydromagnetic equations are solved by using regular perturbation method. Analytical expressions for the velocity, temperature and concentration profiles, wall shear stress, rates of heat and mass transfer and volumetric flow rate have been obtained and computationally discussed with respect to the non-dimensional parameters.

Key words: Heat transfer; mass transfer; chemical reaction; oscillatory flow; blood flow; hall current effect.

Subject Classification codes: 80A20, 76Sxx, 76N20, 76A05, 76A10

1. Introduction

Paces of many physiological functions, including the flow through blood vessels are affected by drugs. The rates of different biochemical reactions that are responsible for the contraction muscles, secretion of different materials such as insulin, mucus and stomach acid by the glands and the transmission of massages by the nerves can be accelerated or decelerated by the action of drugs. The rate at which the kidney cells perform the regulation of the volume of water/salts in the body is affected by drugs. The rate at which blood flows through arteries can also be enhanced or slowed down by the application of drugs. Angirasa *et al.*[4] , Sharma[27], Elbashbeshy *et al.*[16] Singh *et al.*[28,29,30], Singh *et al.*[29], Ganga (2010), Reddy and Reddy[25], Sharma[27] has studied the effect of fluctuating thermal and mass diffusion on unsteady free convective flows. The effect of chemical reaction on free convection studied by Muthucumaraswamy and Meenakshi Sundaram[21]. Anjalidevi and Kandasamy[5] have analyzed the effect of chemical reaction on MHD

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flow. Choudhury and Jha[10] have investigated the same on MHD micropolar fluid flow in slip flow regime. Al-Odat and Al-Azab[3] have studied the influence of chemical reaction on transient MHD free convection over a moving vertical plate. Kandasamy *et al.*[17] have investigated the influence of chemical reaction on MHD flow with heat and mass transfer over a vertical stretching sheet. Ahmed [2] has analyzed the effect of chemical reaction on transient MHD free convective flow over a vertical plate. Bala *et al.* [6] have investigated the radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate. Baoku *et al.*[7] have analyzed the influence of thermal radiation on a transient MHD Couette flow through a porous medium. Basu *et al.*[8] have studied the radiation and mass transfer effects on transient free convection flow. Muthucumaraswamy and Chandrakala[20] have analyzed the radiation, heat and mass transfer effects on moving isothermal vertical plate. Rao *et al.*[24] discussed the chemical effects on an unsteady MHD free convection fluid past a semi-infinite vertical plate embedded in a porous medium with heat absorption. El-Aziz[15] investigated the radiation effect on the flow and heat transfer over an unsteady stretching sheet. Sandeep *et al.*[26] investigated the effect of radiation chemical reaction on transient MHD free convective flow. Suneetha *et al.*[33] analyzed the radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. Authors like Kelly *et al.*[18], Subhash *et al.*[32], Sonth *et al.*[31], Abel *et al.*[1], Choudhury and Mahanta[12], Choudhury and Dey[11], Choudhury and Das[13] etc. have analyzed some problems of physical interest in this field. Recently, Krishna and Swarnalathamma (2016), Swarnalathamma and Krishna[34] discussed the peristaltic MHD flows. Krishna and M.G. Reddy (2016) & Krishna and G.S. Reddy (2016) discussed MHD free convective rotating flows. The aim of the present investigation was to study the effect of chemical reaction, as well as heat and mass transfer on the oscillatory MHD flow of blood, under a single framework, treating blood as a second-grade fluid.

2 *Formulation and solution of the Problem :*

The circulatory system mainly consists of three-dimensional cylindrical vessels. However, in some cases, such as in micro vessels of the lungs, motion of blood can be approximately considered as channel flow. The formulation analysis that follows, we use Cartesian coordinates.

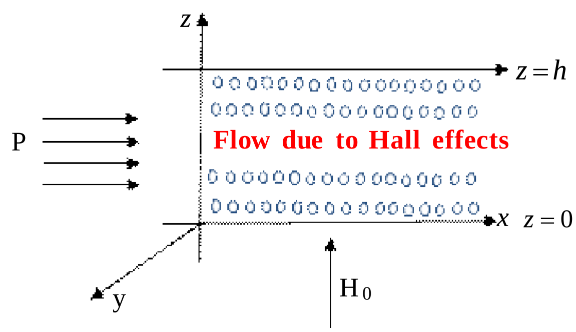

Fig. 2.1 Physical configuration of the Problem

The flow is considered symmetric about the axis of the channel and driven by the stretching of the channel wall, such that the velocity of each wall is proportional to the axial coordinate. In order to study the second-order effects of unsteady MHD flow of blood taking Hall current into account, let us first consider the flow of a second-order fluid between two parallel plates at z = 0 and z = h, where the x-axis is taken parallel length of plates and z-axis along a direction perpendicular to the plates. The model developed here pertains to a pathological state of an arterial segment (as in the case of multiple atherosclerosis), when the lumen has turned porous due to the deposition of different materials such as cholesterol, lipids and fatty substances. The physical sketch of the problem is as shown in Fig. 2.1. A magnetic field of constant intensity B_0 is considered to be applied in the *y*-direction.

The unsteady hydromagnetic equations of the momentum, heat transfer and mass transfer for the MHD oscillatory flow of second grade fluid through a porous arteriole in the parallel plate system are considered in the form (cf. Makinde and Mhone[19]).

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2\partial t}+B_0J_y-\frac{\nu}{k}u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \tag{2}$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2\partial t}-B_0J_x-\frac{\nu}{k}v \tag{3}$$

$$\frac{\partial T}{\partial t}=\frac{K_1}{\rho C_p}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} \tag{4}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial z^2}-K_c(C-C_0) \tag{5}$$

Where, the meanings of all the symbols appearing in the equations have their usual meaning. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the hall current so that

$$J+\frac{\omega_e\tau_e}{B_O}\left(J\times B\right)=\sigma\left[E+V\times B+\frac{1}{e\eta_e}\nabla P_e\right] \tag{6}$$

Where ω_e is the cyclotron frequency of the electrons, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge and P_e is the electron pressure. The ion-slip and thermo electric effects are not included in equation (6). Further, it is assumed that $\omega_e\tau_e \sim O(1)$ and $\omega_i\tau_i << 1$, where ω_i and τ_i are the cyclotron frequency and collision time for ions respectively. In equation (6) the electron pressure gradient, the ion-slip and thermoelectric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

$$J_x+m\,J_y=\sigma B_0 v \tag{7}$$

$$J_y-m\,J_x=-\sigma B_0 u \tag{8}$$

Where, $m=\tau_e\omega_e$ is the hall parameter.

On solving equations (7) and (8) we obtain

$$J_x=\frac{\sigma B_0}{1+m^2}(v+mu) \tag{9}$$

$$J_y=\frac{\sigma B_0}{1+m^2}(mv-u) \tag{10}$$

Substituting the equations (9) and (10) in (3) and (2) respectively, we obtain

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2\partial t}-\left(\frac{\sigma B_0^2}{\rho(1+m^2)}+\frac{\nu}{k}\right)u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \tag{11}$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2\partial t}-\left(\frac{\sigma B_0^2}{\rho(1+m^2)}+\frac{\nu}{k}\right)v \tag{12}$$

In presence of red cell-slip at the boundary wall of the blood vessels reported by Brunn[9] and Nubar [22].

The corresponding boundary conditions are

$$u=\lambda\frac{\partial u}{\partial z},\ v=\lambda\frac{\partial v}{\partial z},\ T=T_0+(T_w-T_0)e^{i\omega t},\ C=C_0+(C_w-C_0)e^{i\omega t} \quad \text{at} \quad z=h \tag{13}$$

$$u=\lambda\frac{\partial u}{\partial z},\ v=\lambda\frac{\partial v}{\partial z},\ T=T_0,\ C=C_0 \quad \text{at} \quad z=0 \tag{14}$$

Using Rosseland approximation (C. Perdikis and A. Raptis[23]), the radiative transfer term q_r in Eq. (4) may be expressed as

$$q_r=-\frac{4\sigma^*}{3\alpha_r}\frac{\partial T^4}{\partial z} \tag{15}$$

We assume that the temperature differences within the flow are such that T^4 can be expressed as a linear function of the temperature T. This is accomplished by expanding in a Taylor series about T_0 (which is assumed to be independent of z) and neglecting powers of T higher than the first. Thus we have

$$T^4=4T_0^3T-3T_0^4 \tag{16}$$

Then the heat transfer equation becomes

$$\frac{\partial T}{\partial t}=\frac{K_1}{\rho C_p}\frac{\partial^2 T}{\partial z^2}-\frac{16\sigma^* T_0^3}{3\rho C_p \alpha_r}\frac{\partial^2 T}{\partial z^2} \qquad (17)$$

Combining the equations (11) and (12), $q=u+iv,\ \xi=x-iy$ and we obtain

$$\frac{\partial q}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial \xi}+\nu\frac{\partial^2 q}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t}-\left(\frac{\sigma B_0^2}{\rho}+\frac{\nu}{k}\right)q+$$

$$+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \qquad (18)$$

We now introduce the following non-dimensional variables:

$$x^*=\frac{x}{h}, y^*=\frac{y}{h}, z^*=\frac{z}{h}, q^*=\frac{q}{U_0}, t^*=\frac{tU_0}{h}, \theta=\frac{T-T_0}{T_w-T_0},$$

$$\phi=\frac{C-C_0}{C_w-C_0},\ \omega^*=\frac{\omega h}{U_0},\ t^*=\frac{tw_0^2}{\nu}, \xi^*=\frac{\xi}{h}, p^*=\frac{p}{\rho U_0^2}$$

Making use of non-dimensional quantities (dropping asterisks), the governing equation (18), (3) and (4) can be written as

$$\mathrm{Re}\frac{\partial q}{\partial t}=-\frac{\partial p}{\partial \xi}+\frac{\partial^2 q}{\partial z^2}+\alpha\frac{\partial^3 q}{\partial z^2 \partial t}-\left(\frac{M^2}{1+m^2}+\frac{1}{K}\right)$$

$$q+\mathrm{Gr}\,\theta+\mathrm{Gc}\,\phi \qquad (19)$$

$$\mathrm{Pr}\frac{\partial \theta}{\partial t}=(1+R)\frac{\partial^2 \theta}{\partial z^2} \qquad (20)$$

$$\mathrm{Sc}\frac{\partial \phi}{\partial t}=\frac{\partial^2 \phi}{\partial z^2}-\mathrm{Kc}\,\phi \qquad (21)$$

The corresponding non-dimensional boundary conditions assume the form

$$q=\lambda\frac{\partial q}{\partial z},\ \theta=e^{i\omega t}, \phi=e^{i\omega t} \text{ at } z=1 \qquad (22)$$

$$q=\lambda\frac{\partial q}{\partial z},\ \theta=0,\ \phi=0 \text{ at } z=0 \qquad (23)$$

Where,

$M^2=\frac{\sigma B_0^2 h^2}{\rho\nu}$ is the Hartmann number (Magnetic field parameter), $K=\frac{k}{h^2\rho}$ is the Permeability parameter, $\alpha=\frac{\alpha_1 U_0}{\nu h}$ is the second grade fluid parameter, $\mathrm{Gr}=\frac{g\beta(T_w-T_0)h^2}{\nu U_0}$ is the thermal Grashof number, $\mathrm{Gr}=\frac{g\beta^*(C_w-C_0)h^2}{\nu U_0}$ is the mass Grashof number, $\mathrm{Pr}=\frac{\rho C_p}{K_1}$ is Prandtl parameter, $R=\frac{16\sigma^* T_0^3}{3\alpha_r K_1}$ is the Radiation parameter, $\mathrm{Kc}=DK_c(C_w-C_0)$ chemical reaction parameter and $\mathrm{Sc}=\frac{\nu}{D}$ is the Schmidt number.

From Eq. (19), it follows that, $\partial p/\partial \xi$ is a function of t only. We consider it to be of the form,

$$\frac{\partial p}{\partial \xi}=Pe^{i\omega t} \qquad (24)$$

To solve Eqs. (19), (20) and (21) subject to the boundary conditions (22) and (23), we further write the velocity, temperature and concentration as

$$q(z,t)=q_1 e^{i\omega t} \qquad (25)$$

$$\theta(z,t)=\theta_1 e^{i\omega t} \qquad (26)$$

$$\phi(z,t)=\phi_1 e^{i\omega t} \qquad (27)$$

Substituting these expressions (25), (26) and (27) in (19), (20) and (21) respectively and comparing the co-efficient of like terms we have the equations.

$$(1+\alpha i\omega)\frac{\partial^2 q_1}{\partial z^2}-\left(\mathrm{Re}i\omega+\frac{M^2}{1+m^2}+\frac{1}{K}\right)q_1=-P-\mathrm{Gr}\,\theta_1-\mathrm{Gc}\,\phi_1 \qquad (28)$$

$$(1+R)\frac{\partial^2 \theta_1}{\partial z^2}-\mathrm{Pr}i\omega\theta_1=0 \qquad (29)$$

$$\frac{\partial^2 \phi_1}{\partial z^2}-(\mathrm{Sc}i\omega+\mathrm{Kc})\phi_1=0 \qquad (30)$$

With corresponding boundary conditions

$$q=\lambda\frac{\partial q_1}{\partial z},\ \theta_1=1,\ \phi_1=1 \quad \text{at} \quad z=1 \tag{31}$$

$$q=\lambda\frac{\partial q_1}{\partial z},\ \theta_1=0,\ \phi_1=0 \quad \text{at} \quad z=0 \tag{32}$$

Solving (28) – (30) subject to the conditions (31) and (32), we have velocity field, temperature, concentration respectively, where the expressions for the constants $m_i\,(i=1,2,.....,6)$ and $a_i\,(i=1,2,.....,6)$ are given in Appendix.

$$q(z,t)=\begin{pmatrix} a_1 e^{m_1 z}+a_2 e^{m_2 z}-\dfrac{P}{\mathrm{Re}i\omega+\dfrac{M^2}{1+m^2}+(1/K)}- \\ \dfrac{\mathrm{Gr}}{e^{m_3}-e^{m_4}}\left[\dfrac{e^{m_3 z}}{a_3}-\dfrac{e^{m_4 z}}{a_4}\right]-\dfrac{\mathrm{Gc}}{e^{m_5}-e^{m_6}}\left[\dfrac{e^{m_5 z}}{a_5}-\dfrac{e^{m_6 z}}{a_6}\right]\end{pmatrix}e^{i\omega t} \tag{33}$$

$$\theta(z,t)=\frac{1}{e^{m_3}-e^{m_4}}\left(e^{m_3 z}-e^{m_4 z}\right)e^{i\omega t} \tag{34}$$

$$\phi(z,t)=\frac{1}{e^{m_5}-e^{m_6}}\left(e^{m_5 z}-e^{m_6 z}\right)e^{i\omega t} \tag{35}$$

The volumetric flow rate is calculated as

$$Q=\int_0^1 q\,dz \tag{36}$$

The wall shear stress at the wall of the upper plate representing the upper wall of the blood vessel is found as

$$\tau_w=\left[\frac{\partial q}{\partial z}+\alpha\frac{\partial^2 q}{\partial z\partial t}\right]_{z=1} \tag{37}$$

The rates of heat and mass transfer across the upper plate (upper wall) are calculated as

$$Nu=\left[-\frac{\partial\theta}{\partial z}\right]_{z=1}\ \&\ Sh=\left[-\frac{\partial\phi}{\partial z}\right]_{z=1} \tag{38}$$

3 Results and Discussion

A new mathematical model is accessible here to swot up the effects of chemical reaction as well as heat and mass transfer on the MHD oscillatory flow of blood through porous medium. Deliberation is made of the velocity-slip of erythrocytes. It is significant to note that the pulsatility of blood flow owes its origin to the intermittent ejection of blood into the arterial network by the muscular pumping action (systolic and diastolic) of the heart. The analysis is applicable to pertinent problems of physiological fluids and fluid dynamical problems encountered in various industrial processes. However, the computational study has been carried out by using data which conform to those of blood flow in a diseased blood vessel. On the lower wall of the vessel, both the temperature and concentration of blood mass are maintained constant, while the variation of both of them is of oscillatory nature on the upper wall. These values/ranges of values of the parameters are mostly representative of blood flow, when a chemical reaction sets in. By using these values, the analytical expressions derived in the previous section have been computed by employing suitable software, viz. MATHEMATICA. Variation in the distributions of velocity, temperature, concentration, and volumetric flow rate and wall shear stress has been investigated numerically, with respect to different governing parameters. Role of the same parameters in executing heat and mass transfer in the blood mass has also been investigated.

All the computational data have been presented in graphical/tabular form. The flow governed by the non-dimensional parameters *M* Hartmann number, *K* permeability parameter, *m* the Hall parameter, Re the Reynolds number, α visco-elastic parameter, R radiation parameter, Gr thermal Grashof number, Gc mass Grashof number, Sc Schmidt number, ω the frequency of oscillation, λ slip velocity parameter, Kc the chemical reaction parameter. The velocity, temperature, concentration, the shear stresses at the boundaries, Nusselt number (Nu), shearwood number (Sh) and the volumetric flow rate (discharge) between the plates are evaluated analytically using regular perturbation technique and computationally discussed for different variations in the governing parameters. The Figs. (3.1-3.11) represent the velocity profiles for *u* and *v*; the Figs (3.13) represent the temperature profiles for θ; the Figs. 3.14 represent the concentration profiles for ϕ. From the Figs. (3.1), we noticed that, both the velocity components u and v reduces with increasing the intensity of the magnetic field or Hartmann number M. Also we have been seen that the resultant velocity is experiences retardation throughout the fluid region. The velocity component u increases and v reduces with increasing permeability parameter K or Hall parameter m. The resultant velocity enhances with increasing K or m in the flow field. We also noticed that lower the permeability lesser the fluid speed is observed the entire fluid region (Figs. 3.2 & 3.6). The similar behaviour is observed for the velocity components with radiation parameter R (Fig. 3.8). This gives an idea of the influence of chemical reaction on the velocity distribution under identical condition of heat radiation. The magnitude of the velocity components u and v as well as resultant velocity reduces in the entire fluid region with increasing second grade fluid parameter α, Pr, Sc and Kc (Figs. 3.3, 3.4, 3.9 & 3.12). Also it indicates that at a particular instant of time, blood velocity reduces as blood visco-elasticity (α) increases. From the Figs (3.5 & 3.7), the velocity components u and v as well as resultant velocity increase with increasing thermal Grashof number Gr, mass Grashof number Gc. The similar behaviour is observed for λ, there is no indication of flow separation in the absence of slip velocity at the wall, but flow separation does take place whenever there is velocity-slip at the boundary. It is important to note that the extent of flow separation increases with the increase in the slip velocity parameter λ. We also find that the magnitude of the velocity component u reduces and v enhances with increasing the frequency of oscillation ω. The resultant velocity reduces throughout the fluid region with increasing the frequency of oscillation (Figs 3.11).

We noticed that from the Fig. (3.13), the magnitude of the temperature reduces with increasing radiation parameter R, where as the reversal behaviour is observed throughout the fluid region with increasing Prandtl number Pr. Also we found that from the Fig. (3.14), the magnitude of the concentration increases with increasing Schmidt number Sc, where as the reversal behaviour is observed throughout the fluid region with increasing chemical reaction parameter Kc. Finally, these reveal that under the purview of the present computational study, at any given distance the temperature/ concentration reduces as the thermal radiation/chemical reaction parameter increases. Further it reveals that for any particular values of thermal radiation/chemical reaction parameter, both the temperature and the concentration increase as we move further and further from the lower wall to the upper one.

The frictional force is determined at the upper wall are presented in the Table. 3.1. This shows that in the absence of any magnetic field, the wall shear stress increases with increase in the value of the Reynolds number, and there occurs a sharp reduction in the wall shear stress, which changes its nature from tensile to compressive. A similar nature of the shear stress is observed, even in the presence of a magnetic field of unit strength; however the change from tensile to compressive is somewhat smooth. The magnitude of the stress components τ_x and τ_y enhances with increasing K, Gr, Gc, m and λ. The opposite nature is observed for the same components with increasing M and Kc. The magnitude of the stress component τ_x reduces and τ_y increases with increasing α, ω and R. The reversal behaviour is for the components τ_x and τ_y with increasing Pr and Sc (Table 3.1). We also noticed that from the table (3.2) the Nusselt number Nu enhances with increasing Radiation parameter R and Prandtl number Pr. Likewise the rate of mass transfer is reduces with increasing Schmidt number Sc and increases with increasing chemical reaction parameter Kc (Table 3.3). From table (3.4) we observed that, volumetric flow rate enhances with increasing K, Gr, Gc, m, λ and R as well as it reduces to M, Pr, Kc, Sc, α, and ω.

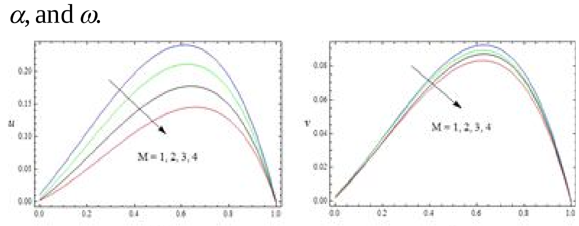

Fig. 3.1 The velocity Profiles for u and v against M with $t = 1$, Re=1

$\mathrm{K} = 1, \alpha = 0.5, \mathrm{Pr} = 0.71, \mathrm{Gr} = 2, \mathrm{Gc} = 5, R = 0.5,$
$\mathrm{Sc} = 0.22, \omega = \pi / 4, \lambda = 0.002, \mathrm{Kc} = 0.5, m = 1$

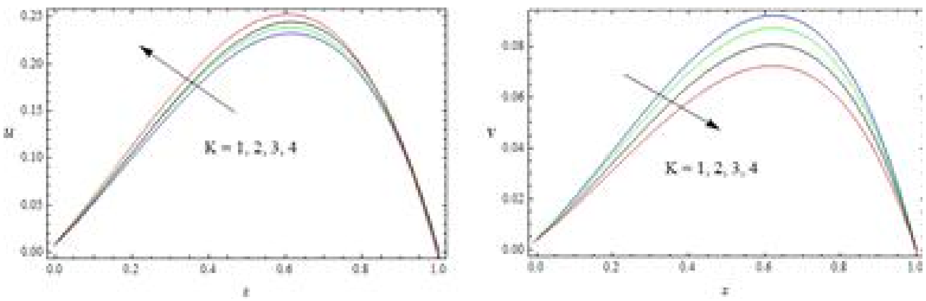

Fig. 3.2 The velocity Profiles for u and v against K with t = 1, Re=1

$\mathrm{M}=1, \alpha=0.5, \mathrm{Pr}=0.71, \mathrm{Gr}=2, \mathrm{Gc}=5, R=0.5, \mathrm{Sc}=0.22, \omega=\pi/4, \lambda=0.002, \mathrm{Kc}=0.5, m=1$

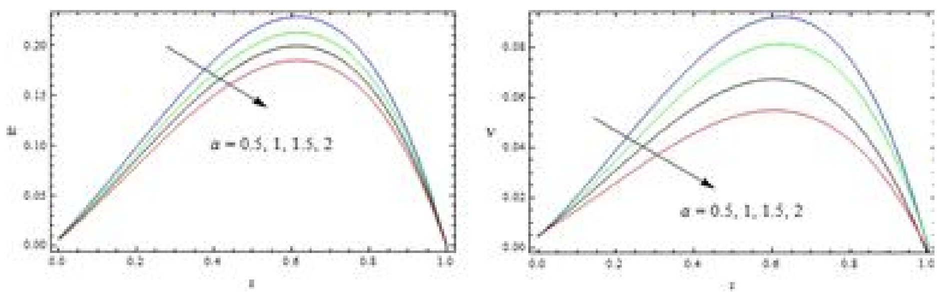

Fig. 3.3 The velocity Profiles for u and v against α with t = 1, Re=1

$\mathrm{K}=1, \mathrm{M}=1, \mathrm{Pr}=0.71, \mathrm{Gr}=2, \mathrm{Gc}=5, R=0.5, \mathrm{Sc}=0.22, \omega=\pi/4, \lambda=0.002, \mathrm{Kc}=0.5, m=1$

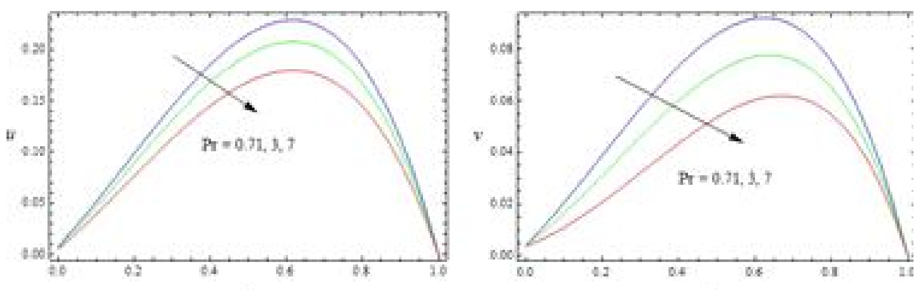

Fig. 3.4 The velocity Profile for u and v against Pr with t = 1, Re=1

$\mathrm{K}=1, \alpha=0.5, \mathrm{M}=1, \mathrm{Gr}=2, \mathrm{Gc}=5, R=0.5, \mathrm{Sc}=0.22, \omega=\pi/4, \lambda=0.002, \mathrm{Kc}=0.5, m=1$

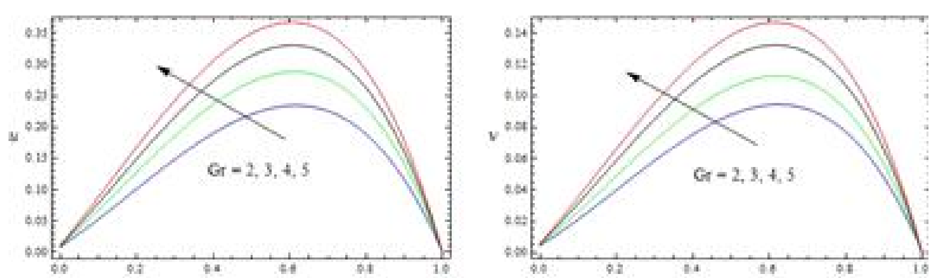

Fig. 3.5 The velocity Profiles for u and v against Gr with t = 1, Re=1

$\mathrm{K}=1, \alpha=0.5, \mathrm{Pr}=0.71, \mathrm{M}=1, \mathrm{Gc}=5, R=0.5, \mathrm{Sc}=0.22, \omega=\pi/4, \lambda=0.002, \mathrm{Kc}=0.5, m=1$

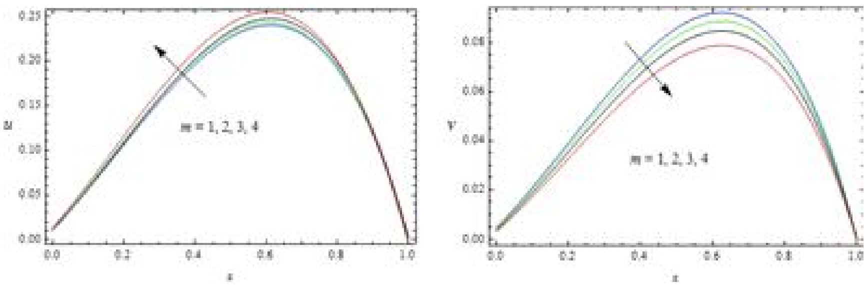

Fig. 3.6 The velocity Profiles for u and v against m with $t = 1$, Re=1

$\mathrm{K} = 1, \alpha = 0.5, \mathrm{Pr} = 0.71, \mathrm{M} = 1, \mathrm{Gc} = 5, R = 0.5, \mathrm{Sc} = 0.22, \omega = \pi / 4, \lambda = 0.002, \mathrm{Kc} = 0.5, \mathrm{Gr} = 2$

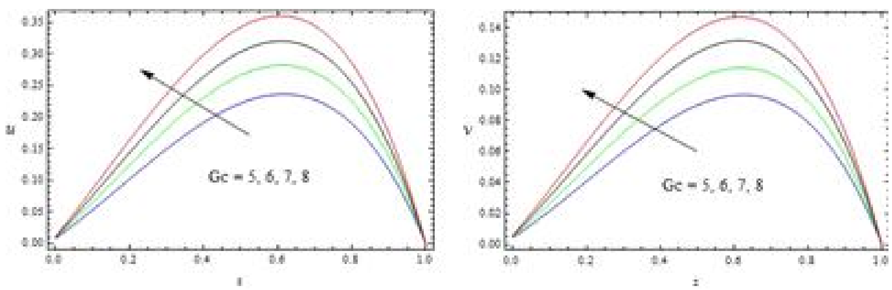

Fig. 3.7 The velocity Profiles for u and v against Gc with $t = 1$, Re=1

$\mathrm{K} = 1, \alpha = 0.5, \mathrm{Pr} = 0.71, \mathrm{Gr} = 2, \mathrm{M} = 1, R = 0.5, \mathrm{Sc} = 0.22, \omega = \pi / 4, \lambda = 0.002, \mathrm{Kc} = 0.5, m = 1$

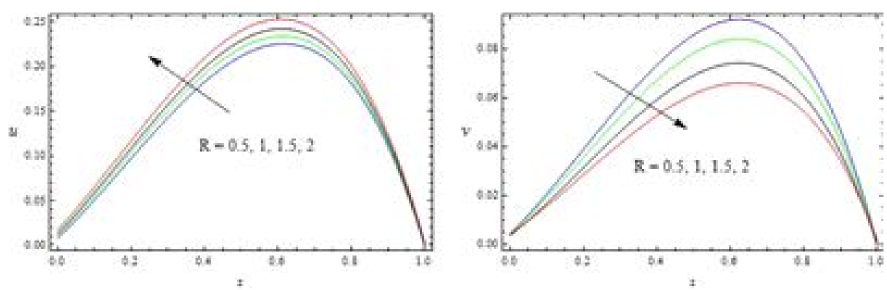

Fig. 3.8 The velocity Profiles for u and v against R with $t = 1$, Re=1

$\mathrm{K} = 1, \alpha = 0.5, \mathrm{Pr} = 0.71, \mathrm{Gr} = 2, \mathrm{Gc} = 5, \mathrm{M} = 1, \mathrm{Sc} = 0.22, \omega = \pi / 4, \lambda = 0.002, \mathrm{Kc} = 0.5, m = 1$

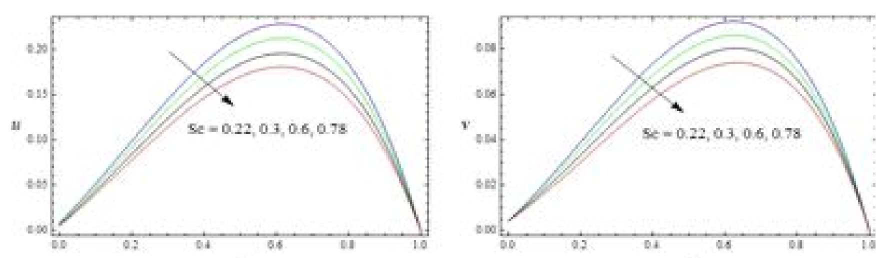

Fig. 3.9 The velocity Profiles for u and v against Sc with $t = 1$, Re=1

$\mathrm{K} = 1, \alpha = 0.5, \mathrm{Pr} = 0.71, \mathrm{Gr} = 2, \mathrm{Gc} = 5, R = 0.5, \mathrm{M} = 1, \omega = \pi / 4, \lambda = 0.002, \mathrm{Kc} = 0.5, m = 1$

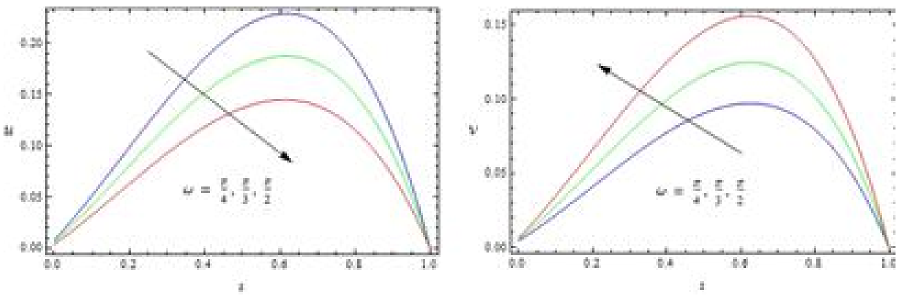

Fig. 3.10 The velocity Profiles for u and v against ω with $t = 1$, Re=1

$$K=1, \alpha=0.5, Pr=0.71, Gr=2, Gc=5, R=0.5, Sc=0.22, M=1, \lambda=0.002, Kc=0.5, m=1$$

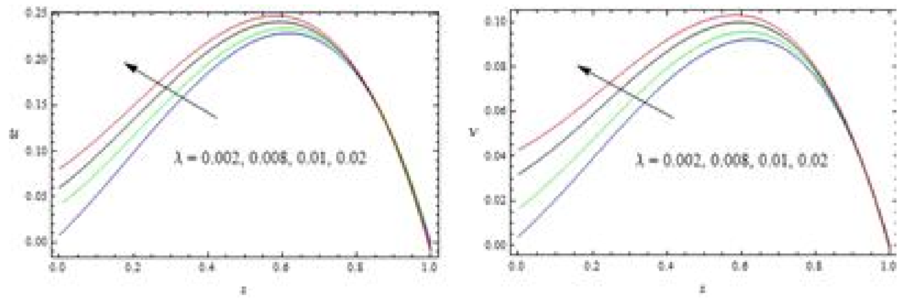

Fig. 3.11 The velocity Profiles for u and v against λ with $t = 1$, Re=1

$$K=1, \alpha=0.5, Pr=0.71, Gr=2, Gc=5, R=0.5, Sc=0.22, \omega=\pi/4, M=1, Kc=0.5, m=1$$

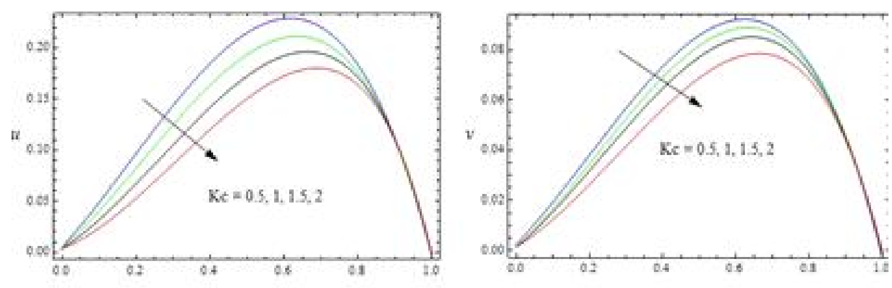

Fig. 3.12 The velocity Profiles for u and v against Kc with $t = 1$, Re=1

$$K=1, \alpha=0.5, Pr=0.71, Gr=2, Gc=5, R=0.5, Sc=0.22, \omega=\pi/4, \lambda=0.002, M=1, m=1$$

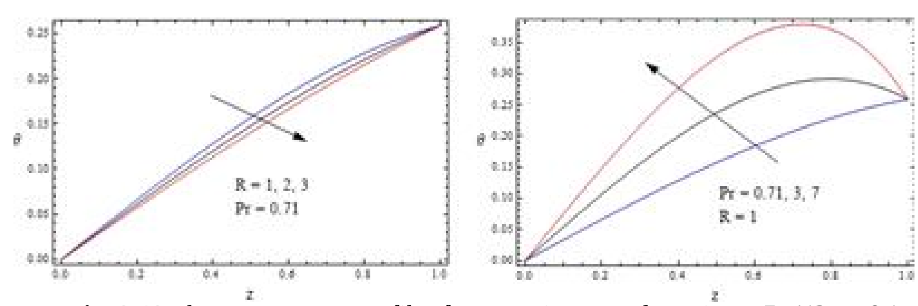

Fig. 3.13 The temperature Profiles for θ against R and Pr wit $\omega=5\pi/12$, $t=0.1$

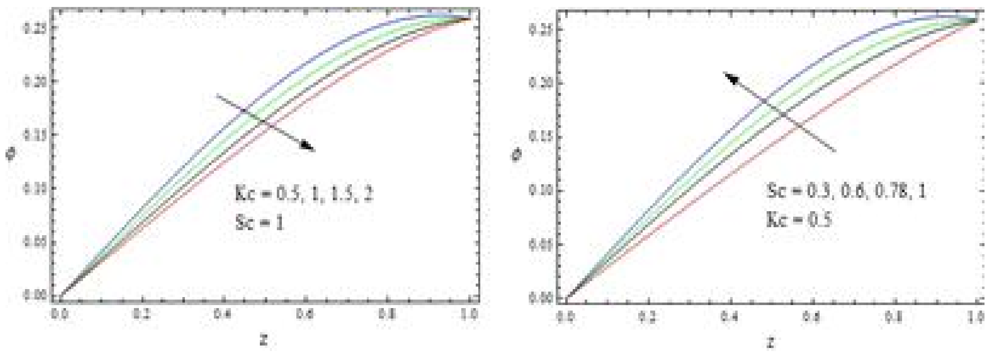

Fig. 3.14 The concentration Profiles for ϕ against Kc and Sc with $\omega = 5\pi/12, t = 0.1$

Table 3.1. Skin Friction

M	K	α	R	Pr	Gr	Gc	Sc	ω	λ	Kc	m	τ_x	τ_y
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.194155	-1.124449
2	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.095123	-1.083292
3	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-0.972012	-1.019974
1	2	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.234540	-1.138433
1	3	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.249641	-1.142705
1	1	0.8	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.185716	-1.150637
1	1	1	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.184588	-1.166308
1	1	0.5	1	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.191836	-1.127161
1	1	0.5	1.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-1.190420	-1.128767
1	1	0.5	0.5	3	2	5	0.3	π/4	0.002	0.5	1	-1.219310	-1.086195
1	1	0.5	0.5	7	2	5	0.3	π/4	0.002	0.5	1	-1.240968	-1.013843
1	1	0.5	0.5	0.71	3	5	0.3	π/4	0.002	0.5	1	-1.416274	-1.332370
1	1	0.5	0.5	0.71	4	5	0.3	π/4	0.002	0.5	1	-1.638393	-1.540291
1	1	0.5	0.5	0.71	2	6	0.3	π/4	0.002	0.5	1	-1.407755	-1.328302
1	1	0.5	0.5	0.71	2	7	0.3	π/4	0.002	0.5	1	-1.621355	-1.532154
1	1	0.5	0.5	0.71	2	5	0.6	π/4	0.002	0.5	1	-1.207231	-1.108872
1	1	0.5	0.5	0.71	2	5	0.78	π/4	0.002	0.5	1	-1.214729	-1.099162
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-0.872493	-1.384037
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	-0.088971	-1.624961
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.008	0.5	1	-1.215191	-1.145498
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.01	0.5	1	-1.222270	-1.152589
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	1.0	1	-1.163961	-1.097990
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	1.5	1	-1.133286	-1.088130
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	2	-1.217668	-1.132895
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	3	-1.225958	-1.135689

Table 3.2. Nusselt number

R	Pr	ω	Nu
0.5	0.71	$5\pi/12$	-0.062010
1	0.71	$5\pi/12$	-0.110643
1.5	0.71	$5\pi/12$	-0.140021
2	0.71	$5\pi/12$	-0.159684
0.5	3	$5\pi/12$	0.512785
0.5	7	$5\pi/12$	1.209667

Table 3.3. Sherwood number

Sc	Kc	ω	Sh
0.3	0.5	$5\pi/12$	-0.182881
0.6	0.5	$5\pi/12$	-0.067315
0.78	0.5	$5\pi/12$	0.000764
1	0.5	$5\pi/12$	0.082496
0.3	1	$5\pi/12$	-0.228894
0.3	1.5	$5\pi/12$	-0.271903

Table 3.4. Volumetric flow rate

M	K	α	R	Pr	Gr	Gc	Sc	ω	λ	Kc	m	Q
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	8.864866
2	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	4.217359
3	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	2.271674
1	2	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	13.54590
1	3	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	15.72839
1	1	0.8	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	8.636261
1	1	1	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	8.396904
1	1	0.5	1	0.71	2	5	0.3	π/4	0.002	0.5	1	9.196639
1	1	0.5	1.5	0.71	2	5	0.3	π/4	0.002	0.5	1	9.520414
1	1	0.5	0.5	3	2	5	0.3	π/4	0.002	0.5	1	7.839794
1	1	0.5	0.5	7	2	5	0.3	π/4	0.002	0.5	1	7.510754
1	1	0.5	0.5	0.71	3	5	0.3	π/4	0.002	0.5	1	9.624051
1	1	0.5	0.5	0.71	4	5	0.3	π/4	0.002	0.5	1	10.38323
1	1	0.5	0.5	0.71	2	6	0.3	π/4	0.002	0.5	1	10.34629
1	1	0.5	0.5	0.71	2	7	0.3	π/4	0.002	0.5	1	11.82771
1	1	0.5	0.5	0.71	2	5	0.6	π/4	0.002	0.5	1	7.173915
1	1	0.5	0.5	0.71	2	5	0.78	π/4	0.002	0.5	1	6.219826
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	7.676882
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	1	5.337538
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.008	0.5	1	8.879549
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.01	0.5	1	8.884492
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	1.0	1	8.135850
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	1.5	1	7.865731
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	2	11.27619
1	1	0.5	0.5	0.71	2	5	0.3	π/4	0.002	0.5	3	12.33900

4. Conclusions

The analysis is applicable to pertinent problems of physiological fluids and fluid dynamical problems encountered in various industrial processes. However, the computational study has been carried out by using data which conform to those of blood flow in a diseased blood vessel. On the lower wall of the vessel, both the temperature and concentration of blood mass are maintained constant, while the variation of both of them is of oscillatory nature on the upper wall. The study enables us to conclude the following:

1. The velocity reduces with increasing Hartmann number M and enhances with permeability parameter K or Hall parameter *m*.
2. Blood visco-elasticity lesser flow velocity significantly.
3. The resultant velocity enhance with increasing thermal Grashof number, mass Grashof number and slip parameter
4. The wall shear stress is strongly pretentious by the Reynolds number.
5. At any particular location as the thermal radiation increases, both heat transfer rate and temperature are reduced to an appreciable extent. However, the velocity is not significantly affected by thermal radiation.
6. The rate of Heat transfer boosts with increasing Prandtl number.
7. Concentration and rate of mass transfer are abridged due to chemical reaction. Comparatively, the velocity distribution is less affected due to chemical reaction.
8. The rate of mass transfer is enhanced, as the mass diffusivity reduces (*i.e.* as the Schmidt number increases).

Acknowledgements

The authors are very much thankful to Prof. Y.Narasimhulu, Vice-Chancellor and Dept of Mathematics, Rayalaseema University, Kurnool for giving support to completion of this work.

Appendix:

$$m_1=\sqrt{\frac{\mathrm{Re}i\omega+\dfrac{M^2}{1+m^2}+\dfrac{1}{K}}{1+\alpha i\omega}},\quad m_2=-\sqrt{\frac{\mathrm{Re}i\omega+\dfrac{M^2}{1+m^2}+\dfrac{1}{K}}{1+\alpha i\omega}},\quad m_3=\sqrt{\frac{\mathrm{Pr}i\omega}{1+R}},$$

$$m_4=-\sqrt{\frac{\mathrm{Pr}i\omega}{1+R}},\quad m_5=\sqrt{\mathrm{Sc}i\omega+\mathrm{Kc}},\quad m_6=-\sqrt{\mathrm{Sc}i\omega+\mathrm{Kc}}$$

$$b_1=\left(\begin{array}{l}\dfrac{P}{\mathrm{Re}i\omega+\dfrac{M^2}{1+m^2}+\dfrac{1}{K}}+\dfrac{\mathrm{Gr}}{e^{m_3}-e^{m_4}}\left[\dfrac{1-\lambda m_3}{a_3}-\dfrac{1-\lambda m_4}{a_4}\right]+\\ \dfrac{\mathrm{Gc}}{e^{m_5}-e^{m_6}}\left[\dfrac{1-\lambda m_5}{a_5}-\dfrac{1-\lambda m_6}{a_6}\right]\end{array}\right)$$

$$b_2=\frac{P}{\mathrm{Re}i\omega+\dfrac{M^2}{1+m^2}+\dfrac{1}{K}}+\frac{\mathrm{Gr}}{e^{m_3}-e^{m_4}}\left[\frac{(1-\lambda m_3)e^{m_3}}{a_3}-\frac{(1-\lambda m_4)e^{m_4}}{a_4}\right]+\frac{\mathrm{Gc}}{e^{m_5}-e^{m_6}}\left[\frac{(1-\lambda m_5)e^{m_5}}{a_5}-\frac{(1-\lambda m_6)e^{m_6}}{a_6}\right],$$

$$a_1 = \frac{b_2 - b_1 e^{m_2}}{(e^{m_1} - e^{m_2})(1-\lambda m_1)},$$

$$a_2 = \frac{b_2 - b_1 e^{m_1}}{(e^{m_1} - e^{m_2})(1-\lambda m_2)}$$

$$a_i = (1+\alpha i\omega)m_i^2 - \left(\mathrm{Re}i\omega + \frac{M^2}{1+m^2} + \frac{1}{K}\right),\ i = 3,4,5,6$$

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