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Effect of Internal Heat Generation on the Onset of Brinkman-Benard Thermomagnetic Convection in a Saturated Porous Layer

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Abstract

A model for onset of ferroconvection via internal heat generation in a ferrofluid saturated porous layer is explored. The Brinkman–Lapwood extended Darcy equation with fluid viscosity different from effective viscosity is used to describe the flow in the porous medium. The lower boundary of the porous layer is assumed to be rigid- paramagnetic and insulated to temperature perturbations, while at upper stress-free boundary a general convective-radiative exchange condition on perturbed temperature is imposed. The resulting eigenvalue problem is solved numerically using the Galerkin method. It is found that increasing in the dimensionless heat source strength N_s, magnetic number M_1, Darcy number Da and the non-linearity of magnetization parameter M_3 is to hasten, while increase in the ratio of viscosities Λ, Biot number Bi and magnetic susceptibility χ is to delay the onset of ferroconvection. Further, increase in Bi, Da^{-1} and N_s and decrease in Λ, M_1 and M_3 is to diminish the dimension of convection cells.

Key words: ferroconvection, porous layer, Galerkin method, internal heat generation, viscosity ratio.
Mathematics Subject Classification: 58D30, 76E06, 76R10

1. Introduction

Magnetic fluids are stable colloidal suspensions of magnetic nanoparticles in a carrier fluid such as water, hydrocarbon (mineral oil or kerosene), or fluorocarbon. The nanoparticles typically have sizes of about 100 angstroms, or 10 nm and they are coated with surfactants in order to prevent the coagulation.

Magnetic fluids are also called ferrofluids and various kinds of such fluids have been developed and successfully used in many engineering applications. Presently, these fluids are in wide use in seals, bearings, magnetostatic support, jet printers, separation of nonmagnetic particles, flow control and drag reduction, dampers, actuators, sensors, transducers, and medical applications. An authoritative introduction to this fascinating subject is amply provided in the books (see[1-3]). Thus, magnetic fluids have received much attention in the scientific community over the years.

The magnetization of ferrofluids depends on the magnetic field, temperature, and density. Hence, any variations of these quantities induce change of body force distribution in the fluid and eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field. There have been numerous studies on thermal convection in a ferrofluid layer called thermomagnetic convection analogous to Rayleigh-Benard convection in ordinary viscous fluids. The theory of convective instability in a horizontal layer of ferrofluid began with Finlayson[4]. Slavtchev *et al.*[5] have investigated the stability of thermoconvective flows of a ferrofluid in a horizontal channel subjected to a longitudinal temperature gradient and an oblique magnetic field. Recently, Nanjundappa *et al.*[6] have studied the effect of magnetic field dependent viscosity on the onset of thermal convection in a horizontal ferrofluid layer heated from below. Recently, Nanjundappa *et al.*[7] have investigated theoretically the effect of magnetic field dependent viscosity on the onset of Benard-Marangoni ferroconvection in a horizontal ferrofluid layer.

Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment chemicals, and emplacement of geophysically imageble liquids into particular zones for subsequent imaging etc. Rosensweig *et al.*[8] have studied experimentally the penetration of ferrofluids in the Heleshaw cell. The stability of the magnetic fluid penetration through a porous medium in high uniform magnetic field oblique to the interface is studied by Zahn and Rosensweig[9]. The thermal convection of a ferrofluid saturating porous medium in the presence of a vertical magnetic field is studied by Vaidyanathan *et al.*[10]. A detailed study on the onset of ferroconvection in a sparsely packed porous layer for more realistic velocity and thermal boundary conditions has been made by Shivakumara *et al.*[11]. Recently, Nanjundappa *et al.*[12] have performed linear stability analysis to investigate buoyancy driven convection in a ferrofluid saturated porous medium.

The practical problems cited above require a mechanism to control thermomagnetic convection. One of the mechanisms to control (suppress or augment) convection is by maintaining a nonuniform temperature gradient across the layer of ferrofluid. Such a temperature gradient may arise due to (i) uniform distribution of heat sources (ii) transient heating or cooling at a boundary, (iii) temperature modulation at the boundaries and so on. Works have been carried out in this direction but it is still in much-to-be desired state. Rudraiah and Sekhar[13] have investigated convection in a ferrofluid layer in the presence of uniform internal heat source. The effect of non-uniform basic temperature gradients on the onset of ferroconvection has been analyzed by Shivakumara and Nanjundappa[14-16]. Jitender Singh and Renu Bajaj[17] have studied thermal convection of ferrofluids in the presence of uniform vertical magnetic field with boundary temperatures modulated sinusoidally about some reference value. Recently, Idris and Hashim[18] and Ferdows *et al.*[21-24] have investigated the instability of Benard-Marangoni ferroconvection in a horizontal layer of ferrofluid under the influence of a linear feedback control and cubic temperature gradient.

All the aforementioned investigations are limited to classical ferrofluid layer (non-porous domain) and no attempts have been made to understand control of thermomagnetic convection in porous media despite its importance in ferrofluid technology. The intent of the present study is, therefore, to investigate thermomagnetic convection in a ferrofluid-saturated porous layer in the presence of internal heating. The presence of internal heating deviate the basic temperature, magnetic field intensity and magnetization distributions from linear to nonlinear, which in turn play a decisive role in understanding control of thermomagnetic convection. Besides, porous materials used in many technological applications of practical importance possess high permeability values. Accordingly, the flow in the porous medium is described by the Brinkman-Lapwood extended Darcy equation with fluid viscosity different from effective or Brinkman viscosity. The resulting eigenvalue problem is solved by the Galerkin technique with modified Chebyshev polynomials as trial functions. The available results in the literature are obtained as limiting cases from the present study.

2. Mathematical Formulation :

The system considered is an initially quiescent incompressible constant viscosity ferrofluid saturated horizontal porous layer of characteristic thickness d in the presence of a uniform applied magnetic field H_0 in the vertical direction. The horizontal extension of the porous layer is sufficiently large so that edge effects may be neglected. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and z-axis is directed vertically upward. At the lower boundary $z = 0$ a constant heat flux condition of the form

$$-k_1(\partial T / \partial z) = q_T \tag{1}$$

is used, while at the upper boundary $z = d$ a radiative-type of condition of the form

$$-k_1(\partial T / \partial z) = h_t (T - T_\infty) \tag{2}$$

is invoked. In the above equations, T is the temperature, q_T is the conductive thermal flux, k_1 is the overall thermal conductivity, h_t is the heat transfer coefficient, and T_∞ is the temperature in the bulk of the environment. The flow in the porous medium is described by the Brinkman-Lapwood extended Darcy equation with fluid viscosity different from effective viscosity and the Oberbeck-Boussinesq approximation is assumed to be valid. In most of the studies, the Oberbeck-Boussinesq approximation has been quite mistreated, and the justifications that have been given for this approximation are largely incorrect. Rajagopal *et al.*[19] have provided a systematic basis for this approximation and discussed some of the errors in the previous approaches.

It is clear that there exists the following solution for the basic state:

$$T_b(z) = -\frac{Q}{2k_1} z^2 + \frac{Q d}{2k_1} z - \frac{\Delta T}{d} z + T_0 \tag{3}$$

$$\vec{H}_b(z) = \left[H_0 - \frac{K}{1+\chi} \left(\frac{Q z^2}{2k_1} - \frac{Q d z}{2k_1} + \frac{\Delta T}{d} z \right) \right] \hat{k} \tag{4}$$

$$\vec{M}_b(z) = \left[M_0 + \frac{K}{1+\chi} \left(\frac{Q z^2}{2k_1} - \frac{Q d z}{2k_1} + \frac{\Delta T}{d} z \right) \right] \hat{k} \tag{5}$$

Here, $\vec{M}$ is the magnetization, $\vec{H}$ the magnetic intensity of the fluid, $\hat{k}$ the unit vector in the z-direction, k_1 the thermal conductivity, $\chi = (\partial M / \partial H)_{H_0, T_0}$ the magnetic susceptibility, $K = -(\partial M / \partial T)_{H_0, T_0}$ the pyromagnetic co-efficient, H_0 imposed uniform vertical magnetic field and $M_0 = M(H_0, T_0)$. To investigate the conditions under which the quiescent solution is stable against small disturbances, we consider a perturbed state such that

$$\vec{V} = \vec{V}', \quad p = p_b(z) + p', \quad T = T_b(z) + T',$$
$$\vec{H} = \vec{H}_b(z) + \vec{H}', \quad \vec{M} = \vec{M}_b(z) + \vec{M}' \tag{6}$$

where the primed quantities represent the perturbed variables. Following the standard linear stability analysis procedure[4] and noting that the principle of exchange of stability is valid, the resulting dimensionless equations are then found to be

$$\left[\Lambda \left(D^2 - a^2 \right) - Da^{-1} \right] \left(D^2 - a^2 \right) W = a^2 R_t M_1$$
$$\left[N_s (1 - 2z) - 1 \right] (D\Phi - \Theta) + a^2 R_t \Theta \tag{7}$$

$$\left(D^2 - a^2 \right) \Theta = [N_s (1 - 2z) - 1](1 - M_2 A) W \tag{8}$$

$$\left(D^2 - a^2 M_3 \right) \Phi - D\Theta = 0 \,. \tag{9}$$

Here, $R_t = g \alpha_t \beta d^4 / \nu \kappa A$ is the thermal Rayleigh number, $N_s = Q d / 2\kappa \beta$ the dimensionless heat source strength, $\Lambda = \tilde{\mu}_f / \mu_f$ the ratio of viscosities,

$Da = k/d^2$ the Darcy number, $M_1 = \mu_0 K^2 \beta /(1+\chi)\alpha_t \rho_0 g$ the magnetic number, $M_2 = \mu_0 T_0 K^2 /(1+\chi)\,(\rho_0 C)_1$ the magnetic parameter, $M_3 = (1+M_0/H_0)/(1+\chi)$ the measure of nonlinearity of the magnetization and $A = (\rho_0 C)_1/(\rho_0 C)_2$. The typical value of M_2 for magnetic fluids with different carrier liquids turns out to be of the order of 10^{-6} and hence its effect is neglected as compared to unity.

The boundary conditions, now take the following form: $W = DW = D\Theta = (1+\chi)D\Phi - a\,\Phi = 0$ at $z = 0$ (on the lower rigid boundary)

$W = D^2W = D\Theta + Bi\,\Theta = (1+\chi)D\Phi + a\,\Phi = 0$ at $z = 1$ (on the upper free boundary)

where $Bi = h_t d/k_1$ is the Biot number. The case $Bi = 0$ and $Bi \to \infty$ respectively correspond to constant heat flux and isothermal conditions at the upper boundary.

3. Method of Solution :

Equations (7)-(9) together with boundary conditions given by Eqs.(10a,b) constitute an eigenvalue problem with thermal Rayleigh number R_t being an eigenvlaue. Accordingly, W, Θ and Φ are written as

$$W(z) = \sum_{i=1}^{n} A_i\, W_i(z),\ \ \Theta(z) = \sum_{i=1}^{n} B_i\ \Theta_i(z),$$

$$\Phi(z) = \sum_{i=1}^{n} C_i\ \Phi_i(z) \tag{11}$$

where A_i, B_i and C_i are unknown constants to be determined. The basis functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are generally chosen such that they satisfy the corresponding boundary conditions but not the differential equations. For lower boundary rigid and upper boundary free boundaries, they are chosen respectively as

$$W_i = (z^4 - 5z^3/2 + 3z^2/2)T^*_{i-1},\ \Theta_i = (z - z^2/2)T^*_{i-1}$$

$$\Phi_i = (z - 1/2)T^*_{i-1} \tag{12}$$

where T_i^*'s are the modified Chebyshev polynomials. This leads to a relation involving the characteristic parameters R_t, N_s, Λ, Da^{-1}, M_1, M_3, Bi, χ and a in the form

$$f(R_t, N_s, \Lambda, Da^{-1}, M_1, M_3, Bi, \chi, a) = 0. \tag{13}$$

The critical value of R_t (i.e., R_{tc}) is determined numerically with respect to a for different values of Ns, Λ, Da^{-1}, M_1, M_3, Bi and χ.

4. Numerical Results and Discussion

The linear stability analysis has been carried out to investigate the effect of internal heat generation on the onset of thermomagnetic convection in a horizontal ferrofluid saturated porous layer. The lower boundary is rigid-insulating, while the upper boundary is free with general thermal convection boundary condition. The critical Rayleigh number R_{tc} and the corresponding critical wave number a_c are obtained numerically using the Galerkin technique. The convergence is obtained by using a eight order Galerkin expansion of the trial functions.

The critical Rayleigh number R_{tc} and the corresponding critical wave number a_c are presented graphically in Figs. 1-5. Figures 1(a) and 1(b) respectively show the variation of R_{tc} and the corresponding a_c as a function of inverse of Darcy number Da^{-1} for different values of heat transfer coefficient Bi. It is seen that the R_{tc} increases with increasing Da^{-1} and hence its effect is to delay the onset of thermomagnetic convection. For a fixed thickness of the porous layer, increase in the value of Da^{-1} leads to decrease in the permeability of the porous medium which in turn retards the flow of thermomagnetic convection in a ferrofluid saturated porous medium. Besides, it is observed that an increase in the value of heat transfer coefficient Bi is to increase the critical Rayleigh number and thus its effect is to delay the onset of thermomagnetic convection. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable. Figure 1(b) represents the corresponding critical wave number a_c and it indicates that increase in the value of Bi and Da^{-1} is to increase a_c and thus its effect is to reduce the size of convection cells.

The model that we have considered to describe the flow in a porous medium rests on an effective viscosity different from fluid viscosity. In the present study, the ratio of these two viscosities is taken as a separate parameter, denoted by Λ, and found that it has a profound influence on the onset of thermomagnetic convection in a ferrofluid saturated porous medium. Figures 2(a) and 2(b), respectively represent the variation of R_{tc} and the corresponding a_c as a function of Da^{-1} for various values of Λ. From Fig. 2(a), it is observed that an increase in the value of viscosity ratio Λ is to delay the onset of thermomagnetic convection. This is because; increase in the value of Λ is related to increase in viscous effect which has the tendency to retard the fluid flow and hence higher heating is required for the onset of thermomagnetic convection. In other words, higher value of Λ is more effective in suppression of thermomagnetic convection in a ferrofluid saturated porous medium. To the contrary, increase in the value of Λ is to decrease a_c and thus its effect is to increase the size of convection cells (see Fig. 2(b)).

The effect of magnetic number M_1 on the onset of thermomagnetic convection is made clear in Fig. 3(a) by presenting the critical Rayleigh number R_{tc} as a function of Da^{-1}. It is seen that an increase in the value of magnetic number M_1 is destabilizing magnetic force on the system. In Fig. 3(b) plotted the critical wave number a_c as a function of inverse of Darcy number Da^{-1} for different values of M_1. It is observed that an increase in the value of M_1 is to increase the value of a_c and thus leads to reduce dimension of the convection cells.

5. Conclusions

The onset of penetrative ferroconvection via internal heating in a ferrofluid saturated Brinkman porous layer is investigated. The lower boundary is considered to be rigid – insulating to temperature perturbations while the upper boundary is free and subject to a general thermal condition on the perturbed temperature. The resulting eigenvalue problem is solved numerically by employing the Galerkin technique. The effect of internal heating is to alter the basic temperature distribution from linear to parabolic with respect to porous layer height and found that it has a profound effect on the stability characteristics of the system.

The following conclusions can be drawn from the present study:

(i) The effect of increase in the internal heat source strength N_s is to lower the critical thermal Rayleigh number R_{tc} and hence to hasten the onset of ferroconvection in a ferrofluid saturated porous layer. The critical stability parameters do not fit the empirical equation $R_{tc}/R_{tc0} + R_{mc}/R_{mc0} = 1$ as $M_3 \to \infty$ in the presence of internal heating but otherwise found to be true in its absence.

(ii) The system becomes more unstable with an increase in the value of magnetic number M_1 and nonlinearity of fluid magnetization parameter M_3.

(iii) The critical thermal Rayleigh number R_{tc} increases with an increase in the value of Biot number Bi, ratio of viscosities Λ, inverse Darcy number Da^{-1} as well as magnetic susceptibility χ and thus their effect is to delay the onset of ferroconvection.

(iv) The effect of increase in Bi, Da^{-1}, M_1 and N_s as well as decrease in Λ and M_3 is to increase the critical wave number a_c and hence their effect is to decrease the dimension of convection cells.

(vi) It is possible to either augment or suppress ferroconvection in a porous medium by tuning the physical parameters of the system.

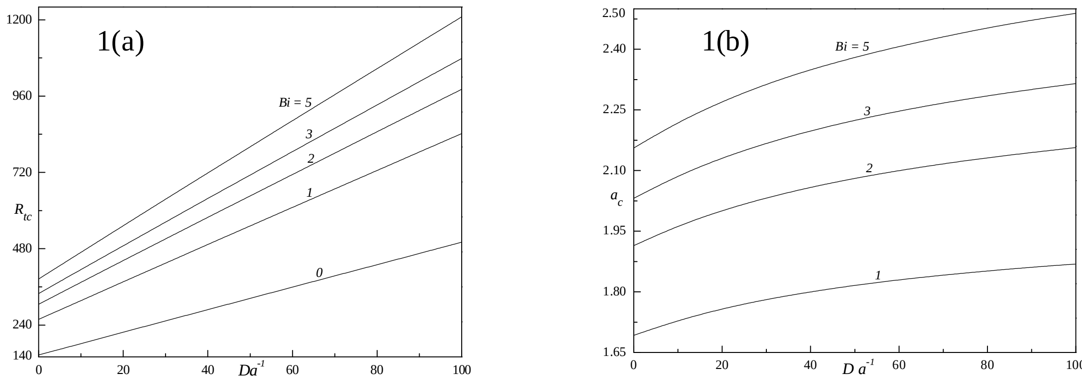

Fig. 1 Variation of (*a*) R_{tc} and (b) a_c as a function of Da^{-1} for different values of Bi when $Ns = 2$, $Bi = 2$, $M_1 = 2$, $M_3 = 1$ and $\chi = 0$

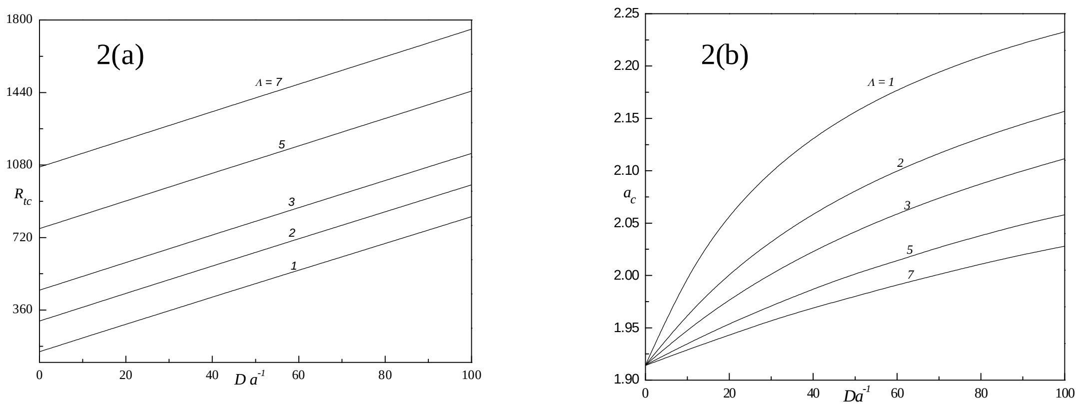

Fig. 2 Variation of (*a*) R_{tc} and (b) a_c as a function of Da^{-1} for different values of Λ when $Ns = 2$, $Bi = 2$, $M_1 = 2$, $M_3 = 1$ and $\chi = 0$

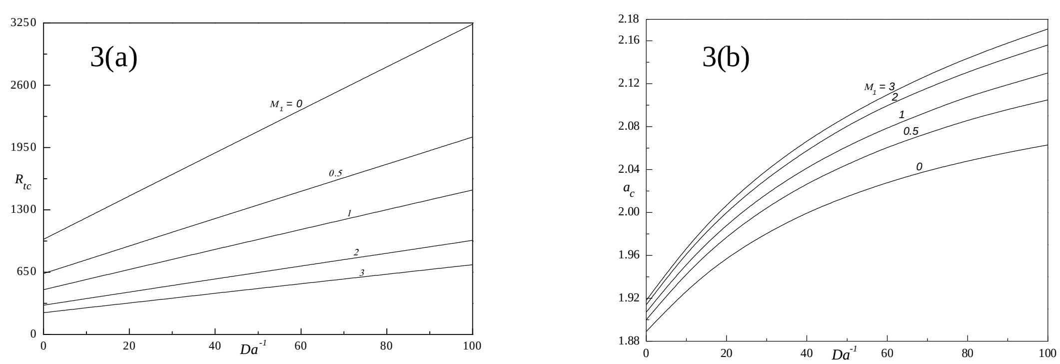

Fig. 3 Variation of (*a*) R_{tc} and (b) a_c as a function of Da^{-1} for different values of M_1 when $N_S = 2$, $Bi = 2$, $M_1 = 2$, $\Lambda = 2$ and $\chi = 0$

Scope/Objective:

- Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment chemicals, and emplacement of geophysically imageble liquids into particular zones for subsequent imaging etc.
- The practical problems cited above require a mechanism to control thermomagnetic convection. One of the mechanisms to control (suppress or augment) convection is by maintaining a nonuniform temperature gradient across the layer of ferrofluid. Such a temperature gradient may arise due to (i) uniform distribution of heat sources (ii) transient heating or cooling at a boundary, (iii) temperature modulation at the boundaries and so on. Works have been carried out in this direction but it is still in much-to-be desired state.
- The presence of internal heating deviate the basic temperature, magnetic field intensity and magnetization distributions from linear to nonlinear, which in turn play a decisive role in understanding control of thermomagnetic convection. Besides, porous materials used in many technological applications of practical importance possess high permeability values. Accordingly, the flow in the porous medium is described by the Brinkman-Lapwood extended Darcy equation with fluid viscosity different from effective or Brinkman viscosity.
- The intent of the futuristic study is to investigate nonlinear ferroconvection, effect of coriolis force and modulation of temperature in a ferrofluid-saturated with/or without porous layer in the presence of internal heating. The presence of internal heating deviate the basic temperature, magnetic field intensity and magnetization distributions from linear to parabolic with respect to porous layer height, which in turn play a decisive role in understanding control of ferroconvection. Besides, porous materials used in many technological applications of practical importance possess high permeability values. For example, permeabilities of compressed foams as high as $8\times10^{-6}m^2$ and for a 1*mm* thick foam layer the equivalent Darcy number is equal to 8 (see Nield *et al.*[20] and references therein[21-24]).

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