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Section A

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A Theorem on Dislocated Quasi-Metric Space

MANOJ GARG

P.G. Department of Mathematics, Nehru (P.G.) College, Chhibramau, Kannauj U.P. (India)
Corresponding Author Email : garg_manoj1972@yahoo.co.in
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Abstract

In this paper, we introduce a fixed point theorem in generalized form for continuous contracting mappings in dislocated quasi-metric space. Our result generalizes and extends the work of Manoj[6], Isufati[5] and then show that the result of Zeyada[4] is special case of our theorem.

Key words: Fixed point, dislocated quasi-metric space.
Subjict Classification Code: 47H10 and 54H25

1 Introduction

The Banach contraction principle is a remarkable result in metric fixed point theory. Over the years, it has been generalized in different directions and spaces by several mathematicians. It has many applications in various branches of mathematics such as differential equation, integral equation etc. The existence of a fixed point is therefore of paramount importance in several area of mathematics. Manoj[6], Isufati[5] and Zeyada[4] have extended, generalized and improved Banach fixed point theorem in different ways.

The aim of this paper is to obtain a fixed point theorem in the generalized form for continuous contracting mappings in dislocated quasi-metric space.

2 Preliminaries:

Definition 2.1[4] Let X be a non empty set and let $d : X \times X \to [0, \infty)$ be a function satisfying following conditions:

(i) $d(x, y) = d(y, x) = 0$, implies $x = y$,

(ii) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a dislocated quasi-metric on X. If d satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

Definition 2.2[4] A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X, d) is called Cauchy sequence if for, given $\varepsilon > 0$, there exist $n_0 \in N$, such that $\forall$ m, $n \geq n_0$, implies $d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$ i.e. $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$.

Definition 2.3[4] A sequence $\{x_n\}$ dislocated quasi-convergent to x if

$$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$$

In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \to x$.

Definition 2.4[4] A dq-metric space (X, d) is called complete if every Cauchy sequence in it is a dq-convergent.

Definition 2.5[4] Let (X, d) be a dq-metric space. A map $T : X \to X$ is called contraction if there exists $0 \leq \lambda \leq 1$ such that

$$d(Tx, Ty) \leq \lambda\, d(x, y), \text{ for all } x,y \in X.$$

3 Main results

Theorem 3.1 Let (X, d) be a complete dq-metric space and let $T : X \to X$ be a continuous mapping satisfying the following conditions

$$d(Tx, Ty) \leq \alpha \frac{d(y,Ty)[1+d(x,Ty)]}{1+d(x,y)} + \beta\, d(x,y) + \gamma\, d(x, Tx) + \delta\, d(y, Tx)$$

for all $x,y \in X$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$, $\alpha + \beta + \gamma + \delta < 1$. Then T has a unique fixed point.

Proof : Let $x_o \in X$ and define a sequence $\{x_n\}$ in X such that

$$T(x_0) = x_1, T(x_1) = x_2 \ldots\ldots\ldots\ldots, T(x_n) = x_{n+1}\ldots\ldots\ldots\ldots$$

Consider, $d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$

$$\leq \alpha \frac{d(x_n,Tx_n)[1+d(x_{n-1},Tx_{n-1})]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(x_{n-1}, Tx_{n-1}) + \delta\, d(Tx_n, Tx_{n-1})$$

$$\leq \alpha \frac{d(x_n,x_{n+1})[1+d(x_{n-1},x_n)]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(x_{n-1}, x_n) + \delta\, d(x_{n+1}, x_n)$$

Therefore, $d(x_n, x_{n+1}) \leq \dfrac{\beta+\gamma}{1-\alpha-\delta}\, d(x_{n-1}, x_n)$

$$= \lambda\, d(x_{n-1}, x_n)$$

Where $\lambda = \dfrac{\beta+\gamma}{1-\alpha-\delta}$ with $0 \leq \lambda < 1$. In a similar way we will show that

$$d(x_{n-1}, x_n) \leq \lambda\, d(x_{n-2}, x_{n-1})$$

and $\quad d(x_n, x_{n+1}) \leq \lambda^2 d(x_{n-2}, x_{n-1})$

Thus $\quad d(x_n, x_{n+1}) \leq \lambda^n d(x_1, x_0)$

Since $0 \leq \lambda < 1$, as $n \to \infty$, $\lambda^n \to 0$. Hence $\{x_n\}$ is a dq-cauchy sequence in X. Thus $\{x_n\}$ dislocated quasi-convergences to some t_0. Since T is continuous, we have

$$T(t_0) = \lim T(x_n) = \lim x_{n+1} = t_0.$$

Thus $T(t_0) = t_0$. Hence T has a fixed point.

Uniqueness: Let x be a fixed point of T. Then by given condition, we have

$$d(x, x) \le d(Tx, Tx)$$

$$\le \alpha \frac{d(x,Tx)[1+d(x\ ,Tx)]}{1+d(x,x)} + \beta\, d(x, x) + \gamma\, d(x,Tx) + \delta\, d(x, Tx)$$

$\le (\alpha + \beta + \gamma + \delta)\, d(x, x)$. Which gives $d(x, x) = 0$, since $0 \le \alpha + \beta + \gamma + \delta < 1$ and $d(x, x) \ge 0$. Thus $d(x, x) \ge 0$, if x is fixed point of T.

Let $x, y \in X$ be fixed points of T, *i.e*. Tx = x, Ty = y.

Then by given condition,

$$d(x, y) = d(Tx, Ty)$$

$$\le \alpha \frac{d(x,Tx)[1+d(x,Tx)]}{1+d(x,y)} + \beta\, d(x, y) + \gamma\, d(x, Tx) + \delta\, d(y, Tx)$$

$\le (\beta + \delta) d(x, y)$. Which gives $d(x, y) = 0$, since $0 \le \beta + \delta < 1$ and $d(x, y) \ge 0$. Similarly $d(y, x) = 0$ and hence x = y.

Thus fixed point of T is unique.

Remark:

(i) If we put $\gamma = \delta = 0$ we obtained Theorem 3.1 of [5].

(ii) If we put $\alpha = \gamma = \delta = 0$ we obtain Theorem 2.8 of [4].

(iii) If we put $\delta = 0$ we obtain Theorem 3.1 of [6].

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