ISSN 2231-346X (Print)
JUSPS-A Vol. 29(8), 327-338 (2017). Periodicity-Monthly
ISSN 2319-8044 (Online)
Section A

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org
Estd. 1989

Free Convective Flow Past a Vertical Plate Through Porous Medium in Slip-Flow Regime with Periodic Temperature Variations and Viscous Heating

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http://dx.doi.org/10.22147/jusps-A/290804

Acceptance Date 1st June, 2017, Online Publication Date 2nd August, 2017

Abstract

In present paper is study of an unsteady free convective viscous incompressible flow past a vertical porous flat plate through porous medium with periodic temperature has been discussed in slip-flow regime. Terms involving viscous dissipation have been retained in the energy equation. The temperature of the plate oscillates in time about a constant mean. The analytical expressions for flow characteristics are obtained for variable suction at the porous plate through porous medium. The effect of various parameters on the transient velocity, transient temperature, the skin-friction and rate of heat transfer are discussed with the help of graphs.

Key words: Free convection, variable suction, viscous dissipation, rarefaction, porous medium, slip flow.

1. Introduction

The temperature change causes density variation which leads to free convection in the fluid. During the course of the day the change in atmospheric flow is caused by temperature difference. The free convective heat transfer on a vertical semi-infinite plate has been investigated by many researchers. But in all these papers, the plate was assumed to be maintained at a constant temperature, which is also the temperature of the surrounding stationary fluid.

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The transient free-convection flow in the steady flow phenomena in the Cooling Towers is useful.

However, free-convection flow is enhanced by superimposing oscillating temperature on the mean plate-temperature. In the early stage of melting adjacent to heated surface or in transient heating of insulating air gaps by heat input at the start-up of furnaces, transient natural convection is of great interest.

Soundalgekar and Wavre (1997) studied the unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. He has assumed that the plate temperature oscillates in such a way that its amplitude is small. Ramanaiah and Malarvizhi[1] investigated the free convection on a horizontal plate in a saturated medium with prescribed heat transfer coefficient. Vighnesam and Soundalgekar[2] presented the results of the combined free and forced convection flow of water at 4^0C from a vertical plate with variable temperature. Unsteady free convection flow past a vertical porous plate was studied by Anwar[3]. Das *et al.*[4] investigated analytically the transient free convection flow past an infinite vertical plate with periodic temperature variation. The effect of a fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate was given by Hossain *et al*.[5]. Sharma and Chaudhary[6] investigated the effect of variable suction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variation in a slip flow regime. Sharma[7] investigated the influence of periodic temperature and concentration on unsteady free convective viscous incompressible flow and heat transfer past a vertical plate in slip flow regime. Recently, Sharma and Jain[8] have studied the effect of viscous heating on flow on past a vertical plate in slip-flow regime with periodic temperature variations.

This section presents the effects of variable suction on transient free convection flow past an infinite vertical plate through porous medium in a slip flow regime, when the temperature of the plate oscillates in time about a constant mean retaining the terms involving viscous dissipation. The governing equations have been solved for small amplitude oscillation (ε). The solution has been obtained by regular perturbation in terms of ε. The effect of suction and rarefaction parameters has been shown on velocity profiles, skin-friction and temperature distribution.

2. Formulation of the problem:

An unsteady free convective flow of a viscous incompressible fluid past an infinite vertical flat porous plate through porous medium in a slip-flow regime, with variable suction in the form $\left[V^* = -V_0^*\left(1+\varepsilon e^{i\omega^* t^*}\right)\right]$ is considered. It is further assumed that the temperature of the plate oscillates in time about a non zero constant mean. We introduce a co-ordinate system with wall lying vertically in x^*- y^* plane. The x^* – axis is taken in vertically upward direction along the vertical porous plate and y^* – axis is taken normal to it.

Since the plate is considered infinite in the x^* – direction, all physical quantities will be independent of x^*. Under these assumptions, the physical variables are function of y^* and t^* only. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation) the problem is governed by the following set of equations:

$$\frac{\partial u^*}{\partial t^*} - V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial u^*}{\partial y^*} = g\beta\left(T^* - T_\infty^*\right) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu}{K^*}u^* \tag{1}$$

$$C_p\left[\frac{\partial T^*}{\partial t^*} - V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial T^*}{\partial y^*}\right] = \frac{\kappa}{\rho}\frac{\partial^2 T^*}{\partial y^{*2}} + \nu\left(\frac{\partial u^*}{\partial t^*}\right)^2 \tag{2}$$

The boundary conditions of the problem are

$$\left.\begin{array}{lll} u^* = L^*\left(\dfrac{\partial u^*}{\partial t^*}\right), & T^* = T^*_\infty + \left(1+\varepsilon A e^{i\omega^* t^*}\right) & \text{at } y^* = 0 \\ u^* \to 0, & T^* \to T^*_\infty & \text{as } y^* \to \infty \end{array}\right\} \tag{3}$$

We now introduce the following non dimensional quantities into equations (1) to (3)

$$y = \frac{y^* V_0^*}{\nu}, \quad t = \frac{t^* V_0^{*2}}{4\nu},$$

$$\omega = \frac{4\nu\omega^*}{V_0^{*2}}, \quad u = \frac{u^*}{V_0^*}, \quad \theta = \frac{T^* - T^*_\infty}{T^*_\omega - T^*_\infty}$$

Defining the following quantities

Gr (Grashof number) = $g\beta\nu \dfrac{T^*_\omega - T^*_\infty}{V_0^{*3}}$

Pr (Prandtl number) = $\dfrac{\nu}{(\kappa / \rho\, C_p)}$

E_c (Eckert number) = $\dfrac{V_0^{*2}}{C_p(T^*_\omega - T^*_\infty)}$

H (Rarefaction parameter) = $\dfrac{V_0^* L^*}{\nu}$

K (Porosity parameter) = $\dfrac{V_0^{*2} K^*}{\nu^2}$

Where A is the suction parameter, ν is kinematics viscosity, ω is the frequency. ρ is the density, g is the acceleration due to gravity, β is coefficient of thermal expansion, T^* is the temperature, κ is thermal conductivity and C_p is the specific heat at constant pressure. L^* is a characteristic dimension in the flow field. The (*) stands for dimension quantities. The subscript (∞) denotes the free stream condition. Equations (1) and (2) reduce to the following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = \theta\ Gr + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K} \tag{4}$$

$$\frac{1}{4}\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

The boundary conditions to the problem in the dimensionless form are

$$\left.\begin{aligned} &u=h\left(\frac{\partial u}{\partial y}\right), \qquad \theta=1+\varepsilon e^{i\omega t}, \qquad \text{at} \quad y=0 \\ &u\to 0, \qquad \theta\to 0 \qquad \text{as} \quad y\to\infty \end{aligned}\right\} \tag{6}$$

3. Solution of the problem:

Assuming the small amplitude oscillation ($\varepsilon << 1$), we represent the velocity u and temperature θ, near the plate as

$$\left.\begin{aligned} u(y,t)&=u_0(y)+\varepsilon e^{i\omega t}u_1(y) \\ \theta(y,t)&=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y) \end{aligned}\right\} \tag{7}$$

Substituting (7) in (4) and (5) and comparing the coefficients of identical powers of ε, we obtain

$$\left.\begin{aligned} &u_0''+u_0'-\frac{1}{K}u_0=-Gr\theta_0 \\ &u_1''+u_1'-\left(\frac{i\omega}{4}+\frac{1}{K}\right)u_0=-Gr\theta_1+Au_0' \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} &\theta_0''+\Pr\theta_0'=-Ec\Pr u_0'^2 \\ &\theta_1''+\Pr\theta_1'-\frac{i\omega}{4}\Pr\theta_1=-A\Pr\theta_0'-2Ec\Pr u_0'u_1' \end{aligned}\right\} \tag{9}$$

Taking $(\theta_0, u_0, \theta_1, u_1) = (\theta_{01}, u_{01}, \theta_{11}, u_{11}) + Ec\,(\theta_{02}, u_{02}, \theta_{12}, u_1)$ and using this in (8) and (9) and comparing the coefficients of identical powers of Ec, we get

$$\theta_{01}'' + \Pr \theta_{01}' = 0 \tag{10}$$

$$\theta_{02}'' + \Pr \theta_{02}' = -\Pr u_{01}'^2 \tag{11}$$

$$\theta_{11}'' + \Pr \theta_{11}' - \frac{i\omega \Pr}{4}\theta_{11} = -A\Pr \theta_{01}'^2 \tag{12}$$

$$\theta_{12}'' + \Pr \theta_{12}' - \frac{i\omega \Pr}{4}\theta_{12} = -A\Pr \theta_{02}'^2 - 2\Pr u_{01}'^2 u_{11}'^2 \tag{13}$$

$$u_{01}'' + u_{01}' - \frac{1}{K}u_{01} = -Gr\theta_{01} \tag{14}$$

$$u_{02}'' + \Pr u_{02}' - \frac{1}{K}u_{02} = -Gr\theta_{02} \tag{15}$$

$$u_{11}'' + u_{11}' - \left(\frac{i\omega}{4} + \frac{1}{K}\right)u_{11} = -Gr\theta_{11} + Au_{01}' \tag{16}$$

$$u_{12}'' + u_{12}' - \left(\frac{i\omega}{4} + \frac{1}{K}\right)u_{12} = -Gr\theta_{12} + Au_{02}' \tag{17}$$

The corresponding boundary conditions at $y = 0$ are

$$\left.\begin{aligned} &u_{01} = h\left(\frac{\partial u_{01}}{\partial y}\right), \qquad u_{11} = h\left(\frac{\partial u_{11}}{\partial y}\right), \\ &u_{02} = h\left(\frac{\partial u_{02}}{\partial y}\right), \qquad u_{12} = h\left(\frac{\partial u_{12}}{\partial y}\right) \\ &\theta_{01} = 1, \quad \theta_{11} = 1, \quad \theta_{02} = 0, \quad \theta_{12} = 0, \end{aligned}\right\} \tag{18}$$

and as $y \to \infty$ are

$$\left.\begin{aligned} &u_{01}=0, \quad u_{11}=0, \quad u_{02}=0, \quad u_{12}=0 \\ &\theta_{01}=0, \quad \theta_{11}=0, \quad \theta_{02}=0, \quad \theta_{12}=0 \end{aligned}\right\} \tag{19}$$

where primes denote differentiation with respect to *y*.

Solving the equation (10) to (17) under boundary conditions (18) and (19) we get

$$\theta_{01}(y)=e^{-\Pr y} \tag{20}$$

$$u_{01}(y)=B_3e^{-B_1y}-B_2e^{-\Pr y} \tag{21}$$

$$\theta_{02}(y)=B_8e^{-\Pr y}-B_4e^{-2B_1y}-B_5e^{-2\Pr y}+B_6e^{-B_7y} \tag{22}$$

$$u_{02}(y)=B_{13}e^{-B_1y}-B_9e^{-\Pr y}+B_{10}e^{-2B_1y}+B_{11}e^{-2\Pr y}-B_{12}e^{-B_7y} \tag{23}$$

$$\theta_{11}(y)=C_2e^{-C_1y}+C_3e^{-\Pr y} \tag{24}$$

$$u_{11}(y)=C_5e^{-C_4y}-C_6e^{-C_1y}-C_7e^{-B_1y}-C_8e^{-\Pr y} \tag{25}$$

$$\begin{aligned}\theta_{11}(y)=&C_9e^{-C_1y}+C_{10}e^{-\Pr y}+C_{11}e^{-2B_1y}+C_{12}e^{-2\Pr y}+C_{13}e^{-B_7y}+C_{14}e^{-D_1y}\\&-C_{15}e^{-D_2y}-C_{16}e^{-D_3y}+C_{17}e^{-D_4y}-C_{18}e^{-D_5y}\end{aligned} \tag{26}$$

$$\begin{aligned}u_{12}(y)=&C_{30}e^{-C_4y}-C_{19}e^{-C_1y}-C_{20}e^{-\Pr y}+C_{21}e^{-2B_1y}+C_{22}e^{-2\Pr y}-C_{23}e^{-B_7y}\\&-C_{24}e^{-D_1y}+C_{25}e^{-D_2y}+C_{26}e^{-D_3y}-C_{27}e^{-D_4y}+C_{28}e^{-D_5y}+C_{29}e^{-B_1y}\end{aligned} \tag{27}$$

Substituting equations (20) to (27) in equation (7) we get the expression for the velocity and temperature profiles. The velocity and the temperature are expressed in terms of the fluctuating parts as

$$u(y,t)=u_0(y)+\varepsilon(U_r\cos\omega t-U_i\sin\omega t) \tag{28}$$

$$\theta(y,t)=\theta_0(y)+\varepsilon(T_r\cos\omega t-T_i\sin\omega t) \tag{29}$$

$$u(y,t)=u_{01}+\mathrm{Ec}\,u_{02}+\varepsilon(u_{11}+\mathrm{Ec}\,u_{12})(\cos\omega t+i\sin\omega t) \tag{30}$$

$$\theta(y,t)=\theta_{01}+\mathrm{Ec}\,\theta_{02}+\varepsilon(\theta_{11}+\mathrm{Ec}\,\theta_{12})(\cos\omega t+i\sin\omega t) \tag{31}$$

4. Results and Discussion

The effect of variable suction and oscillating plate temperature on the transient velocity and temperature in slip flow regime is presented in the subsequent sub-section.

(a) Velocity profiles:

In Figure – [1], the transient velocity profiles of boundary layer flow is plotted against *y* for $\varepsilon = 0.2$, $Ec = 0.02$, $K = 0.2$, $Gr = 3$ and $\Pr = 0.71$, and different values of rarefaction parameter *h*, frequency ω and suction parameter *A*. It is observed that the transient velocity increases sharply till $y = 0.8$, after its transient velocity decreases continuously with increasing in y. It is concluded that the transient velocity decreases with increasing suction parameter *A*, but the transient velocity increases with increasing rarefaction parameter *h* and frequency ω.

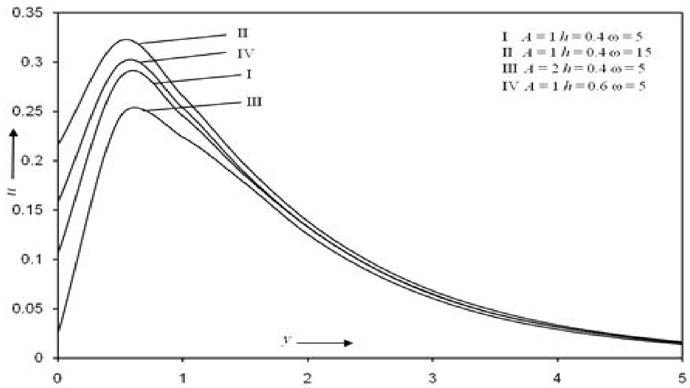

Fig.- [1] , The transient velocity profile for different vlaue of *A*, *h* and ω.

In Figure –[2] the transient velocity profiles of boundary layer flow is plotted against *y* for $\varepsilon = 0.2$, $Ec = 0.02$, $A = 1$, $h = 0.4$ and $\omega = 5$, and different values of porosity parameter *K*, Grashof number *Gr* and Prandtl number Pr. It is observed that the transient velocity increases sharply till $y = 0.8$, after its transient velocity decreases continuously with increasing in y. It is concluded that the transient velocity increases with increasing porosity parameter *K* and Grashof number *Gr*, but the transient velocity decreases with increasing Prandtl number Pr.

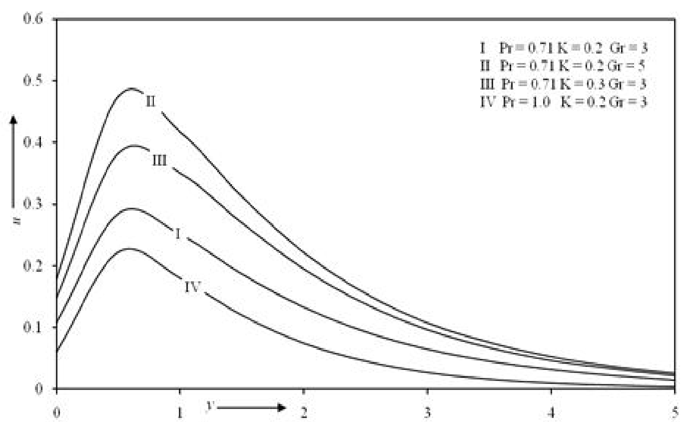

Fig.- [2] , The transient velocity profile for different value of *K*, Pr and *Gr*.

(b) Temperature profiles:

The transient temperature profiles are given in Figure – [3]. It is observed that transient temperature decreases with increasing *A*, while reverse effect is observed for ω. The transient temperature decreases rapidly for $\Pr = 7$ (water) than $\Pr = 0.71$ (air) in the vicinity of the plate.

5. Skin-friction:

The skin-friction on the plate is defined as

$$\tau_x^* = \mu \frac{\partial u^*}{\partial y^*}$$

Where μ is viscosity.

The skin-friction in the non-dimensional form on the plate $y = 0$ is given by

$$\tau_x = \frac{\tau_x^*}{\rho V_0^{*2}} = \tau_m + \varepsilon \left|B\right| \cos(\omega t + \alpha) \tag{32}$$

Where τ_m (Mean skin-friction) $= -B_1B_3 + PrB_2 + Ec\,[-B_1C_1 + B_9Pr - 2B_1B_{10} - 2B_{11}Pr + B_7B_{12}]$

$\boldsymbol{B} = -C_4C_5 + C_1C_6 - B_1C_7 + PrC_8 + Ec\ [-C_4C_{30} + C_{19}C_1 + PrC_{20} - 2B_1C_{21} - 2PrC_{22} + B_7C_{23} + D_1C_{24} - D_2C_{25} - D_3C_{26} + C_{27}D_4 - C_{28}D_5 - B_1C_{29}]$

$$\left|B\right| = \left(B_r^2 + B_i^2\right)^{1/2}, \qquad \tan\alpha = \frac{B_i}{B_r}$$

Such that B_r and B_i represent real and imaginary parts of B

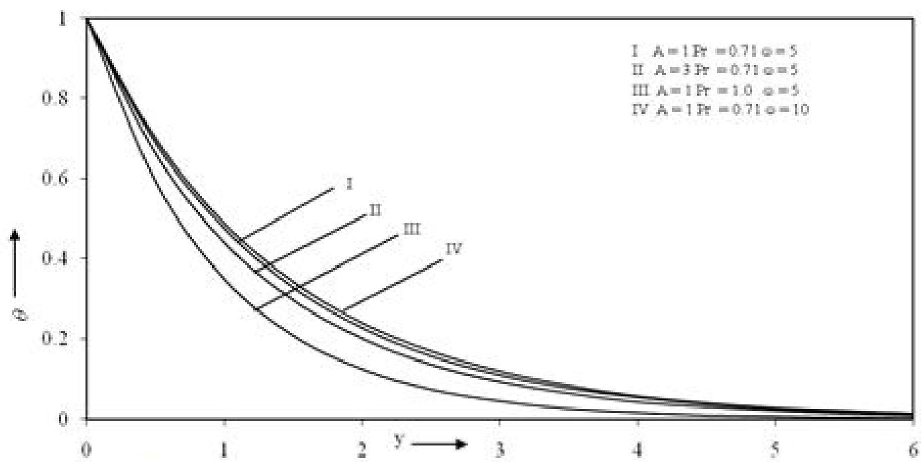

Fig.-[3] The transient temprature profile for different value of A, Pr and ω.

In Figure – [4], the amplitude of transient skin-friction is shown different values of parameters. It is observed from the figure that the amplitude decrease with the increasing *h* (rarefaction parameter) and frequency ω for air (Pr = 0.71). Physically this is true because the increase in Prandtl number is due to increase in the viscosity of the fluid, which makes the fluid thick and hence a decrease in the velocity of the fluid. The amplitude of skin-friction increase with increasing *A* for air (Pr = 0.71). Phase tan α of skin-friction has been shown in Figure – (5). It is observed that the phase of skin-friction decreases with increasing *h* for air (Pr = 0.71). The phase is observed to be increasing with increase of suction parameter *A* for air (Pr = 0.71). The values of tanα show that there is always a phase lead.

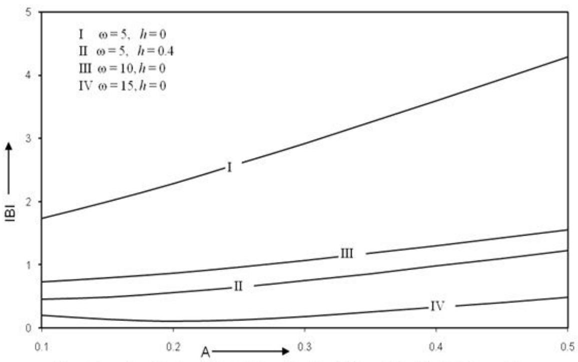

Fig.- [4] , Amplitude of skin friction for Gr =3, Pr = 0.71, K = 0.2 & ε = 0.2.

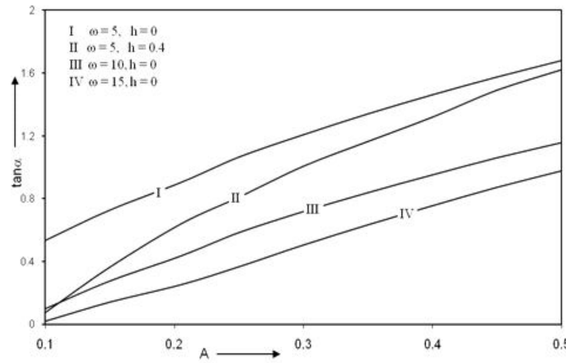

Fig.- [5] Phase of skin friction for Gr =3, Pr = 0.71, K = 0.2 and ε = 0.2.

6. Coefficient of Heat Transfer:

The Nusselt number (Nu) on the vertical plate is defined as follows:

$$Nu = -\frac{\kappa}{\rho V_0^* C_p (T_w^* - T_\infty^*)} = \left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0}$$

$$Nu = \frac{1}{\Pr}\Big[\Pr + Ec(B_8 \Pr - 2B_1B_4 - 2\Pr B_5 + B_6B_7) + \varepsilon\, e^{i\omega t}\big\{C_1C_2 + C_3 \Pr$$

$$+ Ec(C_1C_9 + \Pr C_{10} + 2B_1C_{11} + 2\Pr C_{12} + B_7C_{13} + D_1C_{14} - D_2C_{15}$$

$$- D_3C_{16} + D_4C_{17} - D_5C_{18})\ \big\}\ \Big]$$

In order to find the physical effect we separate real and imaginary parts.

$$Nu = \frac{-\Pr + Ec(-B_8 \Pr + B_1B_4 + 2\Pr B_5 - B_6B_7)}{-\Pr} + \varepsilon|H|\cos(\omega t + \delta)$$

Where $H = \left(H_r^2 + H_i^2\right)^{1/2}$, $\qquad \tan\delta = \dfrac{H_i}{H_r}$

The amplitude $|H|$ and phase tanδ of rate of heat transfer have been shown in Figure – (6) and Figure– (7) respectively. It is observed that the amplitude and phase of heat transfer increases with increasing h (rarefaction parameter) and frequency ω. It is observed that the amplitude of heat transfer increases with increasing A. it is observed for phase of heat transfer. Figure – (7) shows that there always remains a phase lead. The phase of rate of heat transfer increases with increases of h and frequency ω. But

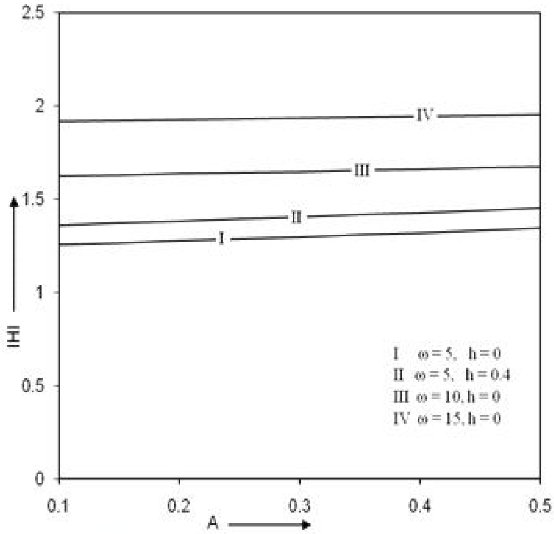

Fig.[6] Amplitude of heat transfer for Gr = 3, Pr = 0.71, K = 0.2 and e = 0.2

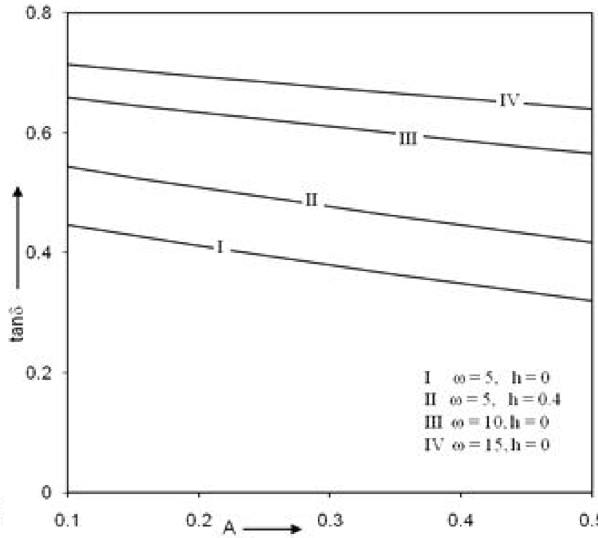

Fig - [7] Phase of heat transfer forfor Gr = 3, Pr = 0.71, K = 0.2 and e = 0.2

7. Conclusion

The theoretical solution of the behavior of unsteady free convective viscous incompressible flow past a vertical porous flat plate through porous medium with periodic temperature has been discussed in slip-flow regime. Terms involving viscous dissipation have been retained in the energy equation. The temperature of the plate oscillates in time about a constant mean. The analytical expressions for flow characteristics are obtained for variable suction at the porous plate through porous medium. The governing equations of motion are solved by perturbation technique. The study concludes the following results.

(i) The transient velocity decreases with increasing suction parameter *A*, but the transient velocity increases with increasing rarefaction parameter *h* and frequency ω.

(ii) The transient velocity increases with increasing porosity parameter *K* and Grashof number *Gr*, but the transient velocity decreases with increasing Prandtl number Pr.

(iii) The transient temperature decreases with increasing *A*, while reverse effect is observed for ω. The transient temperature decreases rapidly for Pr = 7 (water) than Pr = 0.71 (air) in the vicinity of the plate.

(iv) The amplitude decrease with the increasing *h* (rarefaction parameter) and frequency ω for air.

(v) The amplitude of skin-friction increase with increasing *A* for air.

(vi) The phase of skin-friction decreases with increasing *h* for air.

(vii) The phase is observed to be increasing with increase of suction parameter *A* for air. The values of tanα show that there is always a phase lead.

(viii) The amplitude and phase of heat transfer increases with increasing *h* (rarefaction parameter) and frequency ω.

(ix) The amplitude of heat transfer increases with increasing *A*.

(x) The phase of rate of heat transfer increases with increases of *h* and frequency ω. But the amplitude of heat transfer decreases with increasing *A*.

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