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Hyper B-Ideals in Hyper B-Algebra

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Abstract

Hyperstructure theory has many applications to several areas of pure and applied sciences. This paper investigates some properties of hyper B-algebras which is a generalization of B-algebras. This paper also introduces the notion of hyper B-ideals, weak hyper B-ideals and strong hyper B-ideals in hyper B-algebras and gives some relations among these hyper B-ideals. Relations between hyper B-ideals and subhyper B-algebras of H is also discussed. Moreover, homomorphism on hyper B-algebra is defined and some related properties are given. Finally, proof of the relationship between the hyper B-algebras and hypergroups is provided.

Key words: Hyperstructure, Hyper B-algebra, hyper B-ideals

AMS Mathematics Subject Classiffication: 20N20, 06F35, 03G25

1 Introduction

B-algebras were introduced by J. Neggers and H. S. Kim[4] in 2002. It is a class of algebra which coincides in some sense to the class groups. Several papers were then published on the theory of B-algebras which are parallel to the theory of groups.

Marty's introduction of hypergroups at the 8th Congress of Scandinavian Mathematicians in 1934 paved the way for the development of hyperstructure theory (also called multivalued algebras). Since then, a great deal of literature has been published on its applications to several factors both in applied and pure sciences. Endam et al applied the concept of hyperstructure to B-algebras and proved that hyper B-algebra is a natural extension of B-algebra and presented some basic properties.

2 Preliminaries :

Let H be a non-empty set endowed with a *hyperoperation* " $\circledast$ " that is, " $\circledast$ " is a function from $H \times H$ to $P^*(H)$, where $P^*(H)$ is the set of all nonempty subsets of H. For two nonempty subsets A and B of H, $A \circledast B = \bigcup_{a \in A, b \in B} a \circledast b.$ We shall use $x \circledast y$ instead of $x \circledast \{y\}$, $\{x\} \circledast y$, or $\{x\} \circledast \{y\}$. When A is a nonempty subset of H and $x \in H$, we agree to write $A \circledast x$ instead of $A \circledast \{x\}$. Similarly, we write $x \circledast A$ for $\{x\} \circledast A$. In effect, $A \circledast x =: \bigcup_{a \in A} a \circledast x$ and $x \circledast A =: \bigcup_{a \in A} x \circledast a.$

The following definitions of hyper B-algebra, semi-subhyper B-algebra and subhyper B-algebra were introduced by Endam *et al.* Also, some examples were from Endam *et al.*

Definition 2.1 : A *hyper B-algebra* is a set H with constant 0 and endowed with hyperoperation " $\circledast$ " satisfying the following axioms: for all $x, y, z \in H$, (H1) $x \ll x$; (H2) $x \circledast H = H = H \circledast x$; and (H3) $(x \circledast y) \circledast z = x \circledast (z \circledast (0 \circledast y))$ where $x \ll y$ if and only if $0 \in x \circledast y$, and for every $A, B \subseteq H$, $A \ll B$ if and only if for all $a \in A$, there exists $b \in B$ such that $a \ll b$.

A hyper B-algebra H with constant 0 and hyperoperation " $\circledast$ " is denoted by $(H; \circledast, 0)$.

Example 2.2 : (i) Let $(H; *, 0)$ be a B-algebra and define a hyperoperation $\circledast$ on H by $x \circledast y = \{x * y\}$. Then $(H; \circledast, 0)$ is a hyper B-algebra. We can see that $(\mathbb{Z}, \circledast, 0)$, $(\mathbb{Q}, \circledast, 0)$, $(\mathbb{R}, \circledast, 0)$ and $(\mathbb{C}, \circledast, 0)$, are hyper B-algebras where $x\} \circledast y = \{x - y\}$.

(ii) Let $H = \{0, a, b, c\}$ be a set with hyperoperation defined by the following Cayley table:

$\circledast$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0,a,b\}$	$\{0,a,c\}$
a	$\{0\}$	$\{0\}$	$\{0,a,b\}$	$\{0,a,c\}$
b	$\{0,a,b\}$	$\{0,a,b\}$	$\{0,a,b\}$	$\{b,c\}$
c	$\{0,a,c\}$	$\{0,a,c\}$	$\{b,c\}$	$\{0,a,c\}$

Then $(H, \circledast, 0)$ is a hyper B-algebra.

(iii) Let $H = \{0, a, b, c, d\}$ be a set with hyperoperation defined by the following Cayley table:

$\circledast$	0	a	b	c	d
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{c,d\}$
a	$\{a\}$	$\{0,a\}$	$\{c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$
b	$\{b\}$	$\{c,d\}$	$\{0,b\}$	$\{a,c,d\}$	$\{a,c,d\}$
c	$\{c\}$	$\{b,c,d\}$	$\{a,c,d\}$	H	H
d	$\{c,d\}$	$\{b,c,d\}$	$\{a,c,d\}$	H	H

Then $(H; \circledast, 0)$ is a hyper B-algebra.

(iv) Let $H = \{0,1,2,3\}$ be a set with hyperoperation defined by the following Cayley table:

$\circledast$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\{1\}$	$\{0,2\}$	$\{1,3\}$	$\{2\}$
2	$\{2\}$	$\{1,3\}$	$\{0,2\}$	$\{1\}$
3	$\{3\}$	$\{2\}$	$\{1\}$	$\{0\}$

Then $(H;\circledast,0)$ is a hyper B -algebra.

(v) Let $H=\{0,a,b\}$ be a set with hyperoperation defined by the following Cayley table:

$\circledast$	0	a	b
0	H	$\{0,a\}$	$\{0,b\}$
a	$\{0,a\}$	H	$\{a,b\}$
b	$\{0,b\}$	$\{a,b\}$	H

Then $(H;\circledast,0)$ is a hyper B -algebra.

Definition 2.3: A nonempty subset K of a hyper B-algebra H is called a *semi-subhyper B-algebra* if $x\circledast y\subseteq K$ for all $x,y\in K$. A semi-subhyper B-algebra K of a hyper B-algebra H is called *subhyper B-algebra* if $x\circledast K=K\circledast x=K$ for all $x\in K$.

3 Hyper B -Ideals of hyper B -algebras :

The following result follows from Definition 2.1

Proposition 3.1: Let H be a hyper B -algebra. Then for every nonempty subsets A,B and C of H (i) $A\ll A$; (ii) $(A\circledast B)\circledast C=A\circledast[C\circledast(0\circledast B)]$; and $A\subseteq B$ implies $A\ll B$.

Definition 3.2: Let I be a subset of a hyper B -algebra H such that $0\in I$.

(i) I is a *hyper B -ideal* of H if for all $x,y\in H$, $x\circledast y\ll I$ and $y\in I$ imply that $x\in I$.

(ii) I is a *weak hyper B -ideal* of H if for all $x,y\in H$, $x\circledast y\subseteq I$ and $y\in I$ imply that $x\in I$.

(iii) I is a *strong hyper B -ideal* of H if for all $x,y\in H$, $x\circledast y\cap I\neq\varnothing$ and $y\in I$ imply that $x\in I$.

Example 3.3 (i) In Example 2.2 (iv), $\{0,1\}$, $\{0,3\}$ and $\{0,2\}$ are hyper B -ideals of H while in Example 2.2 (iii), $\{0\}$ is not a hyper B -ideal of H since $a\circledast\{0\}=\{0\}\ll\{0\}$ but $a\neq 0$.

(ii) In Example 2.2 (iii), $\{0\},\{0,a\},\{0,b\}$ are strong hyper B -ideals of H while $\{0,c\}$ is not since $(d\circledast 0)\cap\{0,c\}=\{c,d\}\cap\{0,c\}\neq\varnothing$ but $d\notin\{0,c\}$.

(iii) In Example 2.2 (ii), $\{0,a,c\}$ and $\{b,c\}$ are weak hyper B -ideals of H while $\{0,b,c\}$ is not since $a\circledast 0=\{0\}\subseteq\{0,b,c\}$ but $a\notin\{0,b,c\}$.

Proposition 3.4 : Let H *be a hyper* B *-algebra. Then*

(i) *every hyper B -ideal of H is a weak hyper B -ideal of H, and*

(ii) *every strong hyper* B *-ideal of* H *is a hyper* B *-ideal of* H .

Proof:

(i) Let I be a hyper B -ideal of H . Suppose that $x, y \in H$ such that $x \circledast y \subseteq I$ and $y \in I$. By Proposition 3.1 (iii), $x \circledast y \ll I$. Since I is a hyper B -ideal, it follows that $x \in I$. Thus, I is a weak hyper B -ideal.

(ii) Let I be a strong hyper B -ideal of H . Suppose that $x, y \in I$ such that $x \circledast y \ll I$ and $y \in I$. Then for each $a \in x \circledast y$ there exists $b \in I$ such that $a \ll b$, that is, $0 \in a \circledast b$. Since $0 \in I$, $(a \circledast b) \cap I \neq \varnothing$. Thus, $a \in I$ which means that $x \circledast y \subseteq I$. Hence, $(x \circledast y) \cap I \neq \varnothing$ and so we have $x \in I$. Therefore, I is a hyper B -ideal of H . ■

The converse of Proposition 3.4 (i) may not be true. In fact, the weak hyper B -ideal $\{0, a, c\}$ in Example 2.2 (ii) is not a hyper B -ideal since $b \circledast a = \{0, a, b\} \ll \{a\} \subseteq \{0, a, c\}$ but $b \notin \{0, a, c\}$. Also, the converse of Proposition 3.4 (ii) may not be true. Consider hyper B -ideal $\{0,1\}$ in Example 2.2 (iv) of H but is not a strong hyper B -ideal since $(2 \circledast 1) \cap \{0,1\} = \{1,3\} \cap \{0,1\} \neq \varnothing$ but $2 \notin \{0,1\}$.

Remark 3.5 : $\{0\}$ may not be a (strong or weak) hyper B -ideal.

Consider the hyper B -algebra in Example 2.2 (ii), observe that $\{0\}$ is not a hyper B -ideal since $a \circledast 0 = \{0\} \ll \{0\}$ but $a \neq 0$. It is also not a weak hyper B -ideal since $a \circledast 0 = \{0\}$ but $a \neq 0$. Moreover, it is not a strong hyper B -ideal since $(a \circledast 0) \cap \{0\} = \{0\} \cap \{0\} \neq \varnothing$ but $a \neq 0$. Take notice that $a \ll 0$.

Definition 3.6: Let H be a hyper B -algebra. We say that H satisfies (H4) if $x \ll 0$ implies $x = 0$ for any $x \in H$.

Definition 3.7: If a hyper B *-algebra* H *satisfies* (H4), *then* $\{0\}$ *is a strong hyper* B *-ideal. Moreover, it is a hyper* B *-ideal and a weak hyper* B *-ideal.*

Proof: Let $x \in H$ and suppose that $y \in \{0\}$ and $(x \circledast y) \cap \{0\} \neq \varnothing$. Then $y = 0$ and $(x \circledast 0) \cap \{0\} \neq \varnothing$. So, $0 \in x \circledast 0$, that is, $x << 0$. By assumption, $x = 0$ and so $x \in \{0\}$. Thus, $\{0\}$ is a strong hyper B -ideal. By Propositions 3.4 (i) and (ii), $\{0\}$ is also a hyper B -ideal and a weak hyper B -ideal of H .■

Observe Example 2.2 (iv), $\{0,1,5\}$ is a hyper B -ideal but not a subhyper B -algebra since $1 \circledast \{0,1,5\} = \{0,1,2\} \neq \{0,1,5\}$ while in Example 2.2 (ii), $\{0, a, c\}$ is a subhyper B -algebra but not a hyper B -ideal since $b \circledast c = \{b, c\} \ll \{0, a, c\}$ but $b \notin \{0, a, c\}$.

Remark 3.8 : In general, a subhyper B -algebra may not be a hyper B -ideal and a hyper B -ideal may not be a subhyper B -algebra of a hyper B -algebra H .

Definition 3.9: Let H *be a hyper* B *-algebra and* S *be a subhyper* B *-algebra.*

(i) *S is a weak hyper B-ideal if and only if for all $x \in H \setminus S$ and $y \in S$, $x \circledast y \nsubseteq S$.*

(ii) *S is a hyper B-ideal if and only if for all $x \in H \setminus S$ and $y \in S$, $x \circledast y \not\ll S$.*

(iii) *S is a strong hyper B-ideal if and only if for all $x \in H \setminus S$ and $y \in S$, $(x \circledast y) \cap S = \varnothing$.*

Proof: Let S be a weak hyper B-ideal, $x \in H \setminus S$ and $y \in S$. Suppose that $x \circledast y \subseteq S$. Since $y \in S$ and S is a weak hyper B-ideal, $x \in S$, which is a contradiction to the assumption. Thus, $x \circledast y \nsubseteq S$. Conversely, let $x, y \in H$ such that $x \circledast y \subseteq S$ and $y \in S$. If $x \notin S$, then by assumption, we have $x \circledast y \nsubseteq S$, a contradiction. Thus, $x \in S$ and so S is a weak hyper B-ideal. This proves (i). By similar argument, (ii) and (iii) follow. ■

From the definition of the hyperorder $\ll$, we get the following result.

Lemma 3.10 : Let A, B and C be nonempty subsets of a hyper B-algebra H. Then $A \ll B$ and $B \subseteq C$ imply $A \ll C$.

Theorem 3.11: Let $\{A_i : i \in \mathscr{I}\}$ *be a nonempty collection of subsets of a hyper B-algebra H such that $0 \in A_i$ for all $i \in \mathscr{I}$. If A_i is a (weak/ strong) hyper B-ideal of H for all $i \in \mathscr{I}$, then so is $\bigcap_{i \in \mathscr{I}} A_i$.*

Proof: Let $\{A_i : i \in \mathscr{I}\}$ be a nonempty collection of subsets of H. Suppose that A_i is a hyper B-ideal of H for all $i \in \mathscr{I}$. Since $0 \in A_i$ for all $i \in \mathscr{I}$, $0 \in \bigcap_{i \in \mathscr{I}} A_i$ and so $\bigcap_{i \in \mathscr{I}} A_i \neq \varnothing$. Suppose that $x, y \in H$ such that $x \circledast y \ll \bigcap_{i \in \mathscr{I}} A_i$ and $y \in \bigcap_{i \in \mathscr{I}} A_i$. Since $\bigcap_{i \in \mathscr{I}} A_i \subseteq A_i$ for all $i \in \mathscr{I}$, it follows by Lemma 3.10 that $x \circledast y \ll A_i$ for all $i \in \mathscr{I}$. Also, $y \in A_i$ for all $i \in \mathscr{I}$. Hence, A_i hyper B-ideals for all $i \in \mathscr{I}$ imply $x \in A_i$ for all $i \in \mathscr{I}$. Therefore, $x \in \bigcap_{i \in \mathscr{I}} A_i$ and so $\bigcap_{i \in \mathscr{I}} A_i$ is a hyper B-ideal of H. The proof is similar for weak and strong hyper B-ideals. ■

4 Homomorphism in Hyper B-algebras :

Definition 4.1 : Let $(G; \circledast_G, 0_G)$, $(H; \circledast_H, 0_H)$ be hyper B-algebras and $f : G \to H$ be a function. f is a *homomorphism* if $\forall a, b \in G$, $f(a \circledast_G b) \subseteq f(a) \circledast_H f(b)$. f is a *good homomorphism* if $\forall a, b \in G$, $f(a \circledast_G b) = f(a) \circledast_H f(b)$; f is a *hyper B-homomorphism* if f is a good homomorphism and $f(0_G) = 0_H$.

Definition 4.2 : Let G and H be hyper B-algebras, $f : G \to H$ be a homomorphism and let I be a nonempty subset of H. The set $\ker f = \{x \in G : f(x) = 0_H\}$ is called the *kernel of f*. The set $f(A) = \{f(a) : a \in A\}$ is called the *image of A under* f. If $A = G$, then the set is called the *image of f*.

The set $f^{-1}(I)=\{x\in G: f(x)\in I\}$ is called the *inverse image of I* under f.

Example 4.3 : Consider the hyper B-algebra $H=\{0,a,b\}$ in Example 2(v). Define $f:H\to H$ by $f(0)=a$, $f(a)=b$ and $f(b)=0$. Then by routine calculations, f is a good homomorphism.`

Definition 4.4 : Let H be a hyper B-algebra. We say that H satisfies (H5) if $0\circledast 0=\{0\}$.

Consider the good homomorphism defined in Example 4.3 Note that $f(0)=a\neq 0$.

Lemma 4.5 : Let $f:G\to H$ be a good homomorphism, where G satisfies (H5). *Then* $f(0_G)=0_H$.

Proof: By (H1) *in* H *and* (H5) *in* G, $0_H\in f(0_G)\circledast_H f(0_G)=f(0_G\circledast_G 0_G)=f(0_G)$. *Thus,* $0_H=f(0_G)$. ■

Remark 4.6: If $f:G\to H$ be a good homomorphism of hyper B-algebras G and H such that G satisfies (H5), then f is a hyper B-homomorphism.

Consider the good homomorphism defined in Example 4.3. $0\ll a$ in H but $f(0)=a\not\ll b=f(a)$.

Lemma 4.5 : Let $f:G\to H$ be a hyper B-homomophism. Then

(i) $x\ll y$ *implies* $f(x)\ll f(y)$ *and*

(ii) $A\ll B$ *implies* $f(A)\ll f(B)$.

Proof:

(i) Let $x,y\in H$ such that $x\ll y$. Then $0\in x\circledast y$ and so $f(0)\in f(x\circledast y)=f(x)\circledast f(y)$. By definition, $0=f(0)\in f(x)\circledast f(y)$. Thus, $f(x)\ll f(y)$.

(ii) Let $A,B\subseteq H$ such that $A\ll B$ and $c\in f(A)$. Then there exists $a\in A$ such that $c=f(a)$. Since $A\ll B$, there exists $b\in B$ such that $a\ll b$. By (i), $c=f(a)\ll f(b)\in f(B)$, that is, there exists $f(b)\in f(B)$ such that $c\ll f(b)$. Hence, $f(A)\ll f(B)$. ■

Theorem 4.8: Let $f:G\to H$ be a good homomorphism.

(i) *If I is a weak hyper B-ideal of H, then $f^{-1}(I)$ is a weak hyper B-ideal of G.*

(ii) *If I is a strong hyper B-ideal of H, then $f^{-1}(I)$ is a strong hyper B-ideal of G.*

Proof:

(i) Let $a,b\in H$ such that $a\circledast b\subseteq f^{-1}(I)$ and $b\in f^{-1}(I)$. Then $f(a)\circledast f(b)=f(a\circledast b)\subseteq f(f^{-1}(I))\subseteq I$ and $f(b)\in I$. Since I is a weak hyper B-ideal of G, it follows that $f(a)\in I$ and so $a\in f^{-1}(I)$. Thus, $f^{-1}(I)$ is a weak hyper B-ideal of G.

(ii) Let $a,b\in H$ such that $(a\circledast b)\cap f^{-1}(I)\neq\varnothing$ and $b\in f^{-1}(I)$. Since $(a\circledast b)\cap f^{-1}(I)\neq\varnothing$, we have $\varnothing \neq f((a \circledast b) \cap f^{-1}(I)) \subseteq f(a \circledast b) \cap f(f^{-1}(I)) \subseteq f(c \circledast b) \cap I = f(a) \circledast f(b) \cap I$. This means that $f(a) \circledast f(b) \cap I \neq \varnothing$. Since $b \in f^{-1}(I)$, $f(b) \in I$. Consequently, $a \in f^{-1}(I)$ since I is a strong hyper B-ideal. Hence, $f^{-1}(I)$ is a strong hyper B-ideal of G. ■

Theorem 4.9: Let $f : G \rightarrow H$ be a hyper B-homomorphism of hyper B-algebras G and H. If H satisfies (H4), *then* $\ker f$ *is a strong hyper B-ideal of G. Consequently,* $\ker f$ *is also a hyper B-ideal and a weak hyper B-ideal of G.*

Proof: Since $f(0_G) = 0_H$, it follows that $0_G \in \ker f$. Let $x, y \in G$ be such that $(x \circledast_G y) \cap \ker f \neq \varnothing$ and $y \in \ker f$. Then $f(y) = 0_H$. Now,

$$\varnothing \neq f((x \circledast_G y) \cap \ker f) \subseteq f(x \circledast_G y) \cap f(\ker f) = f(x \circledast_G y) \cap \{0_H\}.$$

Thus $0_H \in f(x \circledast_G y) = f(x) \circledast_H f(y) = f(x) \circledast_H 0_H$, that is, $f(x) \ll_H 0_H$. By (H4), $f(x) = 0_H$ and so $x \in \ker f$. Hence, $\ker f$ is a strong hyper B-ideal of G. By Propositions 3.4 (i) and (ii), $\ker f$ is also a hyper B-ideal and a weak hyper B-ideal of G, respectively. ■

Theorem 4.10: *Let $f : G \rightarrow H$ be a hyper B-homomorphism of hyper B-algebras G and H.*

(i) *If I is a weak hyper B-ideal of H, then $f^{-1}(I)$ is a weak hyper B-ideal of G.*

(ii) *If I is a hyper B-ideal of H, then $f^{-1}(I)$ is a hyper B-ideal of G.*

(iii) *If I is a strong hyper B-ideal of H, then $f^{-1}(I)$ is a strong hyper B-ideal of G.*

Proof: Let $f : G \rightarrow H$ be a hyper B-homomorphism of hyper B-algebras G and H. We only prove (ii). Since $0_H \in I$ and $f(0_G) = 0_H$, it follows that $0_G \in f^{-1}(I)$. Let $a, b \in G$ such that $a \circledast_G b \ll f^{-1}(I)$ and $b \in f^{-1}(I)$. Suppose that $x \in f(a \circledast_G b)$. Then there exists $c \in a \circledast_G b$ such that $x = f(c)$. Since $a \circledast_G b \ll_G f^{-1}(I)$, there exists $d \in f^{-1}(I)$ such that $c \ll_G d$, that is, $0_G \in c \circledast_G d$. Thus, $0_H = f(0_G) \in f(c \circledast_G d) = f(c) \circledast_H f(d) = x \circledast_H f(d)$, that is, $x \ll_H f(d)$. Since $d \in f^{-1}(I)$, $f(d) \in I$. Hence, $f(a) \circledast_H f(b) = f(a \circledast_G b) \ll_H I$. Since I is a hyper B-ideal and $f(b) \in I$, we have $f(a) \in I$ and so $a \in f^{-1}(I)$. Therefore, $f^{-1}(I)$ is a hyper B-ideal of G. The proof is similar for (i) and (ii). ■

Theorem 4.11: Let $f : G \rightarrow H$ be a hyper B-homomorphism. If f is onto and I is a strong hyper B-ideal of G which contains $\ker f$, *then $f(I)$ is a strong hyper B-ideal of H.*

Proof: Let I be a strong hyper B-ideal of H. Since $0_G \in \ker f \subseteq I$, $0_H \in f(I)$. Let $x, y \in H$ such that $(x \circledast_H y) \cap f(I) \neq \varnothing$ and $y \in f(I)$. This means that $y = f(b)$ for some $b \in I$. Since f is onto,

$x = f(a)$ for some $a \in H$. Thus, $\varnothing \neq (x \circledast_{H} y) \cap f(I) = (f(a) \circledast_{H} f(b)) \cap f(I) = f(a \circledast_{G} b) \cap f(I)$. This implies that there exists $z \in H$ such that $z \in f(a \circledast_{G} b)$ and $z \in f(I)$. Thus, there are $r \in a \circledast_{G} b$ and $s \in I$ such that $z = f(r)$ and $z = f(s)$. By (H1) in H, $0_H \in z \circledast_{H} z = f(r) \circledast_{H} f(s) = f(r \circledast_{G} s)$. Thus, $r \circledast_{G} s$ contains an element, say w, such that $f(w) = 0_H$. Hence, $w \in \ker f \subseteq I$ and thus, $(r \circledast_{G} s) \cap I \neq \varnothing$. Since I is a strong hyper B-ideal and $s \in I$, it follows that $r \in I$. Note that $r \in a \circledast_{G} b$. This means that $(a \circledast_{G} b) \cap I \neq \varnothing$. Again, since I is a strong hyper B-ideal and $b \in I$, we have $a \in I$. Therefore, $x = f(a) \in f(I)$ and so $f(I)$ is a strong hyper B-ideal of H. ■

5 Hyper B-algebras and hypergroups :

This section presents some relationships between hyper B-algebras and hypergroups.

A *hypergroup* is a nonempty set H with a hyperoperation " $\cdot$ " satisfying the following axioms: for all $x, y, z \in H$, (I) $x \cdot H = H = H \cdot x$ and (II) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$. If $x \cdot y = y \cdot x$ for any $x, y \in H$, then H is called a *commutative* hypergroup.

Lemma 5.1: If a hyper B-algebra H satisfies the condition $0 \circledast x = \{x\}$ for any $x \in H$, then $0 \circledast A = A$ for any nonempty subset A of H.

Proof: Let A be a nonempty subset of H and $a \in A$. Then $\{a\} = 0 \circledast a \subseteq 0 \circledast A$ which implies that $A \subseteq 0 \circledast A$. Also, $0 \circledast a = \{a\} \subseteq A$. Thus, $0 \circledast A \subseteq A$ and so equality of the two sets follows. ■

Theorem 5.2: If a hyper B-algebra H satisfies the condition $0 \circledast x = \{x\}$, then H is a hypergroup.

Proof: Let $x, y, x \in H$. By (H2), $x \circledast H = H = H \circledast x$. We only need to show that $\circledast$ is associative. By (H3) and Lemma 5.1,

$$
\begin{aligned}
(x \circledast y) \circledast z &= x \circledast [z \circledast (0 \circledast y)] \\
&= x \circledast [z \circledast y] \\
&= x \circledast [(0 \circledast z) \circledast y] \\
&= x \circledast [0 \circledast (y \circledast (0 \circledast z))] \\
&= x \circledast [0 \circledast (y \circledast z)] \\
&= x \circledast (y \circledast z).
\end{aligned}
$$

Thus, $(H, \circledast, 0)$ is a hypergroup. ■

Definition 5.3: A hyper B-algebra H is said to be *commutative* if $x \circledast (0 \circledast y) = y \circledast (0 \circledast x)$ for any $x, y \in H$.

From the definition of commutativity of hyper B-algebra, if $0 \circledast x = \{x\}$ for any $x \in H$, then $x \circledast y = x \circledast (0 \circledast y) = y \circledast (0 \circledast x) = y \circledast x$.

Remark 5.4: Every commutative hyper B -algebra which satisfies the condition $0 \circledast x = \{x\}$ for any $x \in H$ is a commutative hypergroup.

6 Conclusion

In this paper, the concept of hyper B -ideals, weak hyper B -ideals and strong hyper B -ideals in hyper B -algebras are introduced and relations among these hyper B -ideals are established. Moreover, homomorphism and hyper B -homomorphism are defined and some related properties are given. In the future, we hope to give more characterizations of hyper B -algebras and also, provide an example of hyper B -algebras and its applications in other disciplines.

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References

1. Corsini, P. and Leoreanu, V., *Applications of Hyperstructure Theory*, Advances in Mathematics, Kluwer Academic Publishers, Dordrecht (2003).
2. Endam, J. C. and Teves, R. C., *Some Properties of Cyclic B -algebras*, International Mathematical Forum, Vol. *11*, no. 8, 387-394 (2016).
3. Gonzaga, N. C. and Vilela, J. P. *On Cyclic B -algebras*, Applied Mathemat- ical Sciences, *9,* no. 111, 5507-5522 (2015).
4. Neggers, J. and Kim, H. S., *On B -algebras*, Mat. Vesnik, *54,* 21-29 (2002).