ISSN 2231-346X (Print)

JUSPS-A Vol. 29(8), 348-351 (2017). Periodicity-Monthly

ISSN 2319-8044 (Online)

Section A

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Mathematics

website:- www.ultrascientist.org

Estd. 1989

A New Concept of Pairwise Quasi-H-Closed Spaces Via Ideals

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http://dx.doi.org/10.22147/jusps-A/290806

Acceptance Date 28 June, 2017, Online Publication Date 2nd August, 2017

Abstract

In this paper, we define pair wise quasi-H-closed modulo an ideal spaces and study some of its properties.

Key words and phrases: Topological ideals, (I) QHC-space.

2000 Mathematics Subject Classification: 54E55.

1 Introduction

In 1963, the concept of bitopological space was introduced by Kelly[4] when he was studying properties of asymmetric distance function on a non empty set X. The pair wise QHC-spaces were first introduced by Mukherjee[5]. Subsequently such spaces have further been studied in detail by Kariofillis[3] and Sen *et al*.[8]. In 2002, the concept of pairwise compactness modulo an ideal in bitopological space was introduced by Lal and Gupta[10] inspired by the concept of compactness modulo an ideal introduced by Newcomb[6] in general topology.

In this paper, we define pair wise quasi-H-closed modulo an ideal spaces and study some of its properties.

2 Preliminaries :

2.1 Definition[3]. A point x in space (X, τ_1, τ_2) is said to be ij-θ-contact point of a subset A of X if for any τ_i-open neighborhood U of x, $\tau_j\text{-cl}(U) \cap A \neq \phi$. The set of all ij-$\theta$-contact points of A is said to be ij-θ-closure of A and denoted by ij-θ-cl(A).

2.2 Definition[3]. A point x in a space (X, τ_1, τ_2) is said to be ij-θ-adherent point of filter base **B** on

X [3] if it is ij-θ-contact point of every member of $\mathbf{B}$ or point x in a space (X, τ_1, τ_2) is said to be ij-θ-adherent point of filter base $\mathbf{B}$ on X if $x \in \bigcap_{B \in \mathbf{B}} \text{ij-}\theta\text{-cl}(B)$. The set of all ij-$\theta$-adherent points of $\mathbf{B}$ is called ij-θ-adherence of $\mathbf{B}$ and denoted by ij-θ-adh $\mathbf{B}$.

2.3 Definition[9]. A subset A in a space (X, τ_1, τ_2) is said to be ij-regularly open if $A = \tau_i\text{-int}(\tau_j\text{-cl}(A))$. The complements of ij-regularly open sets are called ij-regularly closed sets.

2.4 Definition[5]. A bitopological space (X, τ_1, τ_2) is said to be pair wise quasi-H-closed if for every τ_i-open cover U of X, there exists a finite subfamily $\{U_1, U_2, U_3, \ldots U_n\}$ of U such that $X = \bigcup_{k=1}^{n} \tau_j\text{-cl}(U_k)$.

2.5. Definition[10]. A bitopological space (X, τ_1, τ_2, I) is said to be pair wise compact modulo an ideal or just (I) FHP-compact[10] if every pair wise open cover U of X has a finite subfamily $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that $X - \bigcup_{k=1}^{n} U_k \in I$.

3.0 Pair wise quasi-H-closed modulo an ideal space :

3.1 Definition. A bitopological space (X, τ_1, τ_2, I) is said to be pair wise quasi-H-closed modulo an ideal or just pair wise (I) QHC if for every τ_i-open cover U of X, there exists a finite subfamily $\{U_1, U_2, U_3, \ldots U_n\}$ of U such that

$$X - \bigcup_{k=1}^{n} \tau_j\text{-cl}(U_k) \in I, \text{ where } i \neq j \text{ and } i, j = 1, 2.$$

It is evident that every pair wise quasi-H-closed space is pair wise quasi-H-closed modulo an ideal and bitopological space (X, τ_1, τ_2) is pair wise quasi-H-closed if and only if it is $(\{\phi\})$QHC.

3.2 Theorem. Let (X, τ_1, τ_2, I) be pairwise quasi-H-closed modulo an ideal. If σ_i, i = 1, 2, are topologies on X such that $\sigma_i \subset \tau_i$ and I^* is an ideal on X with $I \subset I^*$, then $(X, \sigma_1, \sigma_2, I^*)$ is pair wise (I^*) QHC.

Proof. Let U be any σ_i-open cover of X. Since $\sigma_i \subset \tau_i$, U is also τ_i-open cover of X. Since (X, τ_1, τ_2, I) is pairwise quasi-H-closed modulo an ideal, there exists a finite subfamily $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that $X - \bigcup_{k=1}^{n} \tau_j\text{-cl}(U_k) \in I$. We have

$$X - \bigcup_{k=1}^{n} \sigma_j\text{-cl}(U_k) \subset X - \bigcup_{k=1}^{n} \tau_j\text{-cl}(U_k), \text{because of } \sigma_i \subset \tau_i,$$

$$\Rightarrow X - \bigcup_{k=1}^{n} \sigma_j\text{-cl}(U_k) \in I, \text{because of the heredity property of ideal} I,$$

$$\Rightarrow X - \bigcup_{k=1}^{n} \sigma_j\text{-cl}(U_k) \in I^*, \text{because of } I \subset I^*.$$ Hence $(X, \sigma_1, \sigma_2, I^*)$ is pair wise (I^*) QHC.

3.3 Theorem. A bitopological space (X, τ_1, τ_2, I) is pair wise (I) QHC if and only if every τ_i-open filter base in $\wp(X) - I$ has τ_j-adherent point.

Proof. Let $\mathbf{B}$ be any τ_i-open filter base in $\wp(X) - I$. Suppose, if possible, it has no τ_j-adherent point. Then collection $\{ X - \tau_j\text{-cl}(U) : U \in \mathbf{B} \}$ is τ_j-open cover of X. By the hypothesis, there exists a finite subfamily $\{X - \tau_j\text{-cl}(U_k) : k = 1, 2, 3, \ldots, n\}$ such that

$$X - \bigcup_{k=1}^{n} \{\tau_i\text{-cl}(X - \tau_j\text{-cl}(U_k))\} \in I .$$

Clearly $\bigcap_{k=1}^{n} \{U_k \in \mathbf{B}\} \in I$. Since $\mathbf{B}$ is τ_i-open filter base in $\wp(X) - I$, then $B \in \mathbf{B}$ such that $B \subset \bigcap_{k=1}^{n} \{U_k\}$. But than $B \in I$, which contradicts the fact that $\mathbf{B}$ is τ_i-open filter base in $(X) - I$.

Conversely**,** suppose (X, τ_1, τ_2, I) is not pairwise (I)QHC. Let U be any τ_i-open cover of X, then there exists a finite subfamily $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that $X - \bigcup_{k=}^{n} \{\tau_j\text{-cl}(U_k) : k = 1,2,3,\ldots, n\} \notin I$,

We may assume that U is closed under finite union. Then $\mathbf{B} = \{X - \tau_j\text{-cl}(U) : U \in \mathcal{U}\}$ is τ_j-open filter base in $\wp(X) - I$. So, by hypothesis

$$\bigcap \tau_i\text{-cl}(X - \tau_j\text{-cl}(U) : U \in \mathcal{U}) \neq \phi.$$

Let $x \in X$ be any point in the intersection. Then

$$x \in \tau_i\text{-cl}(X - \tau_j\text{-cl}(U)) = X - \tau_i\text{-int}(\tau_j\text{-cl}(U)), \text{ for each } U \in \mathcal{U} .$$

Since $U \subset \tau_i\text{-int}(\tau_j\text{-cl}(U))$, for each $U \in \mathcal{U}$ therefore $x \in X - U$ for each $U \in \mathcal{U}$. But this contradicts the fact that U be τ_i-open cover of X.

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