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A New Technique on Ranking Trapezoidal Fuzzy Numbers to Solve Fuzzy Assignment Problem

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Abstract

In this paper a ranking procedure based on trapezoidal fuzzy numbers, is applied to a fuzzy valued assignment problem. Using this ranking method we convert any fuzzy assignment problem to a crisp valued assignment problem which then can be solved using the HUNGARIAN method. The proposed method serves as an efficient method in ranking Trapezoidal fuzzy numbers which is illustrated through a numerical example.

Key words : Trapezoidal Fuzzy numbers, Fuzzy Assignment Problem .

AMS Subject Classification : 090BXX

1. Introduction

The assignment problem was originally introduced and developed by Hitch Cock in 1941, in which the parameters such as assignment cost are crisp values. In an assignment problem, n jobs have to be done by n persons depending on their efficiency to do the job. In this problem C_{ij} denotes the cost of empowering the j^{th} job to the i^{th} person. We expect that one person can be empowered strictly one job, also each person can do atmost one job. The problem is to find an optimal assignment so that the total cost of performing all jobs are minimum or the total profit is maximum. Few of the ranking approaches have been reviewed and compared by Bortolan and Degani[2]. Presently Chen and H Wang[4] reviewed the existing method for ranking fuzzy numbers and each approach has drawbacks in some aspects. Measuring normal fuzzy number were first introduced by Jain[5] for decision making in fuzzy situations. Chan(1985) stated that in many situations it is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. Since then remarkable efforts are made on the development of numerous methodologies [Liou and Wang 1992, Chang and Lee 1994, Wang and Lee 2008, Chan and Wang 2009] for the comparison of generalized fuzzy numbers. And it is used in different areas of fuzzy optimization. The development in ordering fuzzy numbers can even be found in[1,3,6,7]. Fuzzy numbers must be graded before a decision is taken by a decision maker.

Michael[9] has discussed the relationship between the algebraic structure of the fuzzy optimum solution of the deterministic problem and its fuzzy equivalent. Fuzzy numbers must be graded before a decision is taken by a decision maker. Ranking normal fuzzy numbers were first introduced by Jain[6] for decision making in fuzzy situations. Ranking fuzzy numbers has attracted research attention due to their vast applications in the theory of fuzzy decision making, risk analysis, data analysis, optimization, etc. Several ranking methods have been proposed by many authors since 1980 [8,10,11].

In this paper a new ranking method is introduced on trapezoidal fuzzy numbers. To illustrate this proposed method, an example is discussed. As the proposed ranking method is very direct and simple it is very easy to understand and using which it is easy to get the fuzzy optimal solution of fuzzy assignment problems occurring in the real life situations . The optimal solution could be got either as a fuzzy number or as a crisp number.

The paper is organized as follows : In section 2, the notion of fuzzy numbers and trapezoidal all fuzzy numbers are recalled. In section 3, the ranking method is proposed for trapezoidal fuzzy numbers. In section 4, a numerical example (FAP) is solved to illustrate the proposed ranking method. Finally the paper ends with a conclusion.

2. Preliminaries :

In this section we define some basic definitions which will be applied in this paper.

Definition 2.1 :

If x is a collection of objects denoted generally by X, then a fuzzy set A in X is explained as a set of ordered pairs A={(x, $\mu_A(x)$/ $x \in$ X} where $\mu_A(x)$ is termed as the function maps each element of X to a membership value between 0 and 1.

Definition 2.2 :

A fuzzy set A is defined on universal set of real numbers is known as a generalized fuzzy number if its membership function has the following attributes

(i) $\mu_A(x)$: R $\rightarrow$ [0,1] is continuous

(ii) $\mu_A(x)$= 0 for all $x \in$ A $(-\infty, a]$ U $[d,\infty)$

(iii) $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$

(iv) $\mu_A(x)$= ω for all $x \in$ $[c, d]$, where $0 < \omega \leq 1$

Definition 2.3 :

A generalized fuzzy number A = (a, b, c, d, ω) is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{\omega(x-a)}{b-a} & , a < x \le b \\ \omega & , b \le x \le c \\ \frac{\omega(x-a)}{d-c} & , c \le x \le d \\ 0 & , \text{otherwise} \end{cases}$$

If $\omega = 1$, then $A = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and A is a generalized or non normal trapezoidal fuzzy number if $0 < \omega < 1$

As a particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $A = (a, b, d; \omega)$

3. Ranking of Trapezoidal Fuzzy Numbers :

A number of approaches have been proposed for the ranking of fuzzy numbers. The ranking of A *i.e* R(A) is calculated as follows :

$$R(A) = \frac{w}{2}(b-a) + \frac{w}{2}(d-c) + \omega(c-b)$$

Take $\omega = 1$

$$R(A) = \frac{1}{2}(b-a) + \frac{1}{2}(d-b) + (c-b)$$

$$R(A) = \frac{d-a+c-b}{2}$$

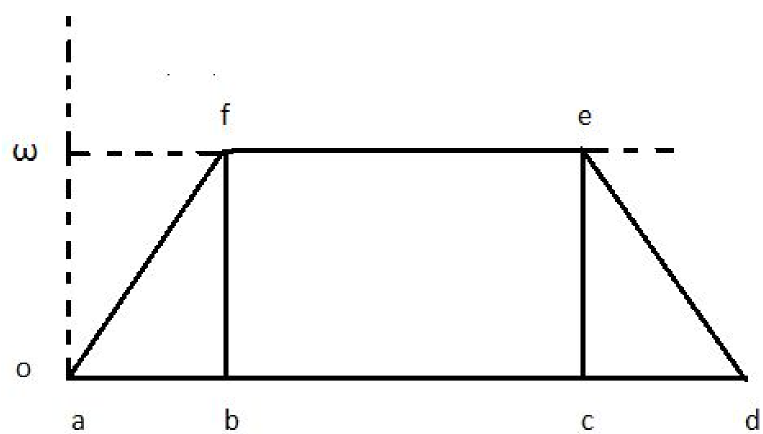

Figure-1: Graphical representation of the Trapezoidal fuzzy number A=(a, b, c, d :1)

4. Numerical Example :

Example :

Consider an Fuzzy Assignment Problem of assigning n jobs to n machines (one job to one machine). Let C_{ij} be the cost matrix whose elements are trapezoidal fuzzy numbers. The problem is to find the minimum assignment so that the total cost of job assignment becomes minimum.

$$C_{ij} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{bmatrix} (3,5,6,7) & (5,8,11,12) & (9,10,11,15) & (5,8,10,11) \\ (7,8,10,11) & (3,5,6,7) & (6,8,10,12) & (5,8,9,10) \\ (2,4,5,6) & (5,7,10,11) & (8,11,13,55) & (4,6,7,10) \\ (6,8,10,12) & (2,5,6,7) & (5,7,10,11) & (2,4,5,7) \end{bmatrix} \end{array}$$

Step 1: Now by using the ranking technique , we convert the given fuzzy problem in to a crisp value problem.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{bmatrix} 2.5 & 5 & 3.5 & 4 \\ 3 & 2.5 & 4 & 3 \\ 2.5 & 4.5 & 24.5 & 3.5 \\ 4 & 3 & 4.5 & 3 \end{bmatrix} \end{array}$$

After applying Assignment Method the optimal solution is

$A \rightarrow J_3$, $B \rightarrow J_2$, $C \rightarrow J_1$, $D \rightarrow J_4$

The fuzzy optimal total cost is $a_{13} + a_{22} + a_{31} + a_{44}$

= (9,10,11,15) +R(3,5,6,7) + (2,4,5,6) + (2,4,5,7)

= (16,23,27,35). And the crisp value of the optimum fuzzy assignment cost is 12.

Conclusion

Ranking fuzzy numbers is a demanding work in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers. This paper proposed a ranking method that ranks fuzzy numbers which is simple and efficient. We find that our method looks more suitable. The proposed ranking procedure can be applied in various decision making problems. Also it gives us the optimum cost which is very much lower. Thus, this method provides an applicable optimal solution which helps the decision maker to solve the real life assignment problems having fuzzy parameters. As a future extension , the proposed ranking procedure can be applied to solve intuitionistic fuzzy asignment problems and other types of problems like game theory, fuzzy transportation problems and network flow problems.

References

1. S. Abbasbandy and T. Hajjari, 'A new approach for ranking of trapezoidal fuzzy numbers', Computers and

Mathematics with Applications, *57*, pp.413-419 (2009).
2. G. Bortolan and R. Degani, 'A review of some methods for ranking fuzzy subsets' , Fuzzy Sets and Systems, *15*, pp.1-19 (1985).
3. C.H. Cheng, 'A new approach for ranking fuzzy numbers by distance method', Fuzzy Sets and Systems, *95*, pp. 307-317 (1998).
4. S.J. Chen and C.L. Hwang, 'Fuzzy multiple attribute decision making' , Springer, Berlin (1992).
5. R. Jain, 'Decision making in the presence of fuzzy variables' , IEEE Transactions on systems, Man and Cybernetics, *6*, pp. 698-703 (1976).
6. T.S. Liou and M.J. Wang, 'Ranking fuzzy numbers with integral value', Fuzzy Sets and Systems, *50*, pp.247-255 (1992).
7. P. Malini and M. Ananthanarayanan, 'Solving fuzzy transportation problem using ranking of octagonal fuzzy numbers', International Journal Pure and Applied Mathematics,*110*, pp.275-282 (2016).
8. P. Malini and M. Ananthanarayanan, 'Solving fuzzy assignment problem using ranking of generalized trapezoidal fuzzy numbers', *Indian Journal Science and Technology, 9,* pp.1- 4 (2016).
9. Y. Yuan, 'Criteria for evaluating fuzzy rankingmethod', Fuzzy Sets and Systems, *43*, pp.139-157 (1991).
10. R.R. Yager, 'A procedure for ordering fuzzy subsets of the unit interval', Information Sciences, *24,* pp.143-161 (1981).
11. H.J. Zimmermann, 'Fuzzy programming and linear programming with several objective Functions', Fuzzy Sets and System, *1*, pp. 45-55 (1978).