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Unsteady free convective flow of a fluid through a porous medium in a channel with adiabatic under the effect of an inclined magnetic field

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Abstract

In this paper, we investigated the unsteady free convective flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of an inclined magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

Key words: Free convection, adiabatic, porous medium, inclined magnetic field.

Mathematics Subject Classification 2010: 80A20, 80-xx.

1. Introduction

The interaction between the magnetic field and the conduction fluid drastically modifies the flow, with effects on such significant flow properties as heat transfer, the fact character of which is strongly dependent on the orientation of the magnetic field. When fluid flows through a magnetic field, an electric field or consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. The production of this current has led to MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting. The influence of a magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics in the manufacture of rayon, nylon etc. The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field was analyzed by Sreekanth *et al.*[11]. Gourla and Katoch[4] have studied the unsteady free convection MHD flow between two heated vertical plates. Recently, Bhaskar[1] have studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate adiabatic. MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field was analyzed by Manyonge *et al*.[7]. The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu *et al*.[5]. Simon[9] has investigated the effect of heat of transfer on unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field. The effect of inclined magnetic field on unsteady flow past on moving vertical plate with variable temperature were studied by Navneet Kumar Singh *et al*.[8]. Siva Shankara Rao and Siva Prasad[10] have investigated the MHD effects on unsteady free convective flow of a fluid through a porous medium in a channel with adiabatic.

However, flow through a porous medium has been of significant interest in recent years particularly among geophysical fluid dynamicists. Examples of natural porous media are beach sand, stand stone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. Influence of MHD and radiation effects on oscillatory flow through a porous medium with constant suction velocity has been studied by El-Hakeem[3]. Makinde and Mhone[6] have investigated the heat transfer and MHD effects on an oscillatory flow in a channel filled with porous medium.

In view of these, we studied the unsteady free convective flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of an inclined magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

2. Mathematical formulation :

We consider the free convective unsteady MHD flow of a viscous incompressible electrically conducting fluid between two heated vertical parallel plates filled with porous medium. Let x -axis being taken along the vertically upward direction through the central line of the channel and the y -axis being perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field B_0 acts at an angle $\alpha \left(0 \le \alpha \le \dfrac{\pi}{2} \right)$, to the y -axis u is the velocity in the direction of flow of fluid, along the x -axis and v is the velocity along the y -axis. Consequently, u is a function of y and t, but v is independent of y. The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible. In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $t > 0$, the temperature of the plate at $y = h$ changes according to the temperature function: $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where T_w and T_0 are the temperature at the plates $y = h$ and at $y = -h$ respectively, and $n(\ge 0)$ is a real number, denoting the decay factor.

The equations governing the flow field are given by

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_0)-\left(\frac{\nu}{k}+\frac{\sigma B_0^2}{\rho}\cos^2\alpha\right)u \tag{2.2}$$

$$\frac{\partial T}{\partial t}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

here ρ is the density of the fluid, B_0 is the magnetic field strength, σ is the electrical conductivity of the fluid, ν is the co-efficient of kinematic viscosity, k is the permeability of the porous medium. K is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, β is the co-efficient of thermal expansion, g is the acceleration due to gravity and T' is the temperature of the fluid.

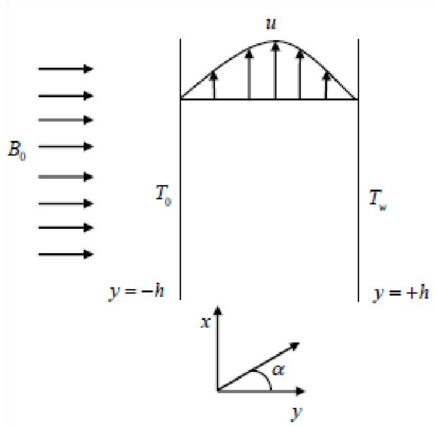

Fig. 1 The physical model

The initial and boundary conditions for the problem are:

$$t=0:\quad u=0, T=T_0 \quad \text{for all } -h\le y\le h$$

$$t>0: u=0, T=T_0+(T_w-T_0)(1-e^{-nt}) \qquad \text{for} \qquad y=h$$

$$u=0, \frac{\partial T}{\partial y}=0 \qquad \text{for} \qquad y=-h \tag{2.4}$$

The non-dimensional variables are

$$\bar{u}=\frac{\nu u}{\beta g h^2(T_w-T_0)}, \bar{y}=\frac{y}{h}, \bar{T}=\frac{T-T_0}{T_w-T_0}, \bar{t}=\frac{\nu t}{h^2}, P_r=\frac{\mu C_p}{K}, \bar{n}=\frac{h^2 n}{\nu}, M=B_0 h\sqrt{\frac{\sigma}{\mu}}$$

$$Da=\frac{k}{h^2} \tag{2.5}$$

in which $\Pr$ is the Prandtl number, Da is the Darcy number and M is the Hartmann number.

Using the non-dimensional variables (2.5) in to the equations (2.2) and (2.3), we obtain

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-N^2u+T \tag{2.6}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \tag{2.7}$$

here $N=\sqrt{\frac{1}{Da}+M^2\cos^2\alpha}$.

Under the above non-dimensional quantities, the corresponding boundary conditions reduces to

$$t=0:\quad u=0, T=0 \qquad \text{for} \qquad -1\le y\le 1$$

$$t>0: u=0, T=(1-e^{-nt}) \qquad \text{for} \qquad y=1$$

$$u=0, \frac{\partial T}{\partial y}=0 \qquad \text{for} \qquad y=-1 \tag{2.8}$$

3. Solution of the problem:

We seek for a regular perturbation series solution to solve the Equations (2.6) and (2.7) of the form

$$u=u_o\left(y\right)+e^{-nt}u_1\left(y\right) \tag{3.1}$$

$$T=T_0\left(y\right)+e^{-nt}T_1\left(y\right) \tag{3.2}$$

Substituting Equations (3.1) and (3.2) into the Equations (2.6) – (2.8) and solving the resultant Equations, we obtain

$$u=\frac{1}{N^2}(1-\frac{\cosh Ny}{\cosh N})+\left(\begin{array}{l}\left(\frac{1+\cos 2m_1}{2\left(m_1^2+m_2^2\right)\cosh m_2\cos 2m_1}\right)\cosh m_2 y\\ +\left(\frac{-1+\cos 2m_1}{2\left(m_1^2+m_2^2\right)\sinh m_2\cos 2m_1}\right)\sinh m_2 y\\ -\left(\frac{1}{m_1^2+m_2^2}\right)\frac{\cos m_1(1+y)}{\cos 2m_1}\end{array}\right)e^{-nt} \tag{3.3}$$

and

$$T=1-\frac{\cos m_1(1+y)}{\cos 2m_1}e^{-nt} \tag{3.4}$$

Here $m_1=\sqrt{n\Pr}$ and $m_2=\sqrt{N^2-n}$.

As $\alpha\to 0$, our results coincides with the results of Siva Shankara Rao and Siva Prasad[10].

4. Results and Discussions

In order to see the effect of various physical parameters on the velocity and temperature, we plotted Figs. 2-8.

The variation of velocity u with inclination angle α for $\Pr = 0.71$, $M = 1$, $Da = 0.1$, $n = 1$ and $t = 1$ is shown in Fig. 2. It is found that, the velocity increases with an increase in α.

Fig. 3 depicts the variation of velocity u with Hartmann number M for $\Pr = 0.71$, $Da = 0.1$, $\alpha = \frac{\pi}{6}$, $n = 1$ and $t = 1$. It is noticed that, the velocity u decreases on increasing M.

The variation of velocity u with Darcy number Da for $\Pr = 0.71$, $\alpha = \frac{\pi}{6}$, $n = 1$ and $t = 1$ is shown in Fig. 4. It is noticed that the velocity u increases with increasing Da.

Fig. 5 illustrates the variation of velocity u with decay parameter n for $\Pr = 0.71$, $Da = 0.01$, $\alpha = \frac{\pi}{6}$, $M = 2$ and $t = 1$. It is observed that, the velocity u decreases with an increase in n.

The variation of velocity u with Prandtl number Pr for $n = 1$, $\alpha = \frac{\pi}{6}$, $Da = 0.01$, $M = 2$ and $t = 1$ is presented in Fig. 6. It is found that, the velocity u decreases with increasing Pr.

Fig. 7 shows the variation of temperature T with decay parameter n for $\Pr = 0.71$ and $t = 1$. It is found that the temperature T decreases with increasing n.

The variation of temperature T with Prandtl number Pr for $n = 1$ and $t = 1$ is shown in Fig. 8. It is observed that the temperature T decreases with increasing Prandtl number Pr.

5. Conclusions

In this paper, we studied the unsteady free convective flow of an incompressible viscous electrically conducting fluid through a porous medium under the action of an inclined magnetic field between two heated vertical plates by keeping one plate is adiabatic. The expressions for velocity and temperature are obtained by using regular perturbation technique. It is found that the velocity increases with increasing Darcy number Da and α, while it decreases with increasing M, Pr and n and the temperature decreases with increasing Pr and n.

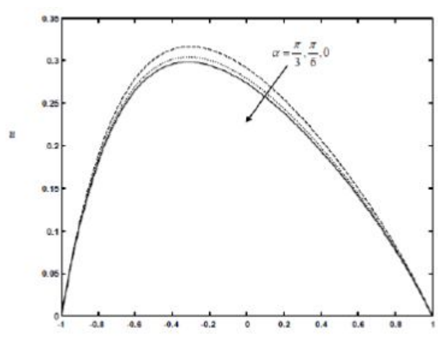

Fig. 2 The variation of velocity u with inclination angle α for $\Pr = 0.71$, $M = 1$, $Da = 0.1$, $n = 1$ and $t = 1$.

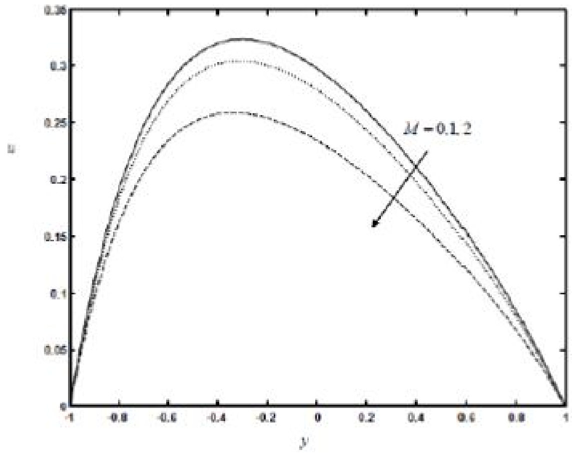

Fig. 3 The variation of velocity u with Hartmann number M for $\Pr = 0.71$, $Da = 0.1$, $\alpha = \dfrac{\pi}{6}$, $n = 1$ and $t = 1$.

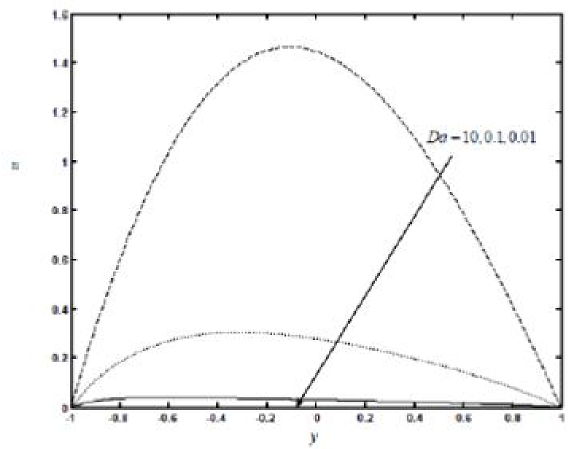

Fig. 4 The variation of velocity u with Darcy number Da for $\Pr = 0.71$, $\alpha = \dfrac{\pi}{6}$, $M = 1$, $n = 1$ and $t = 1$.

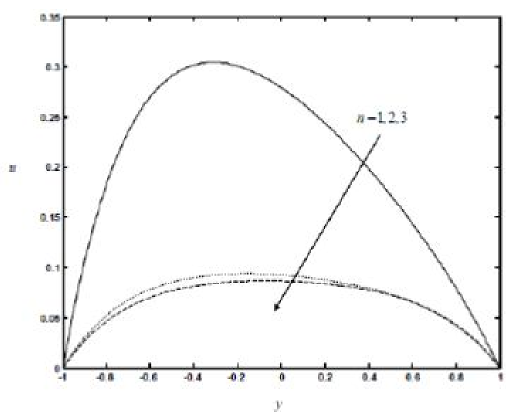

Fig. 5 The variation of velocity u with n for $\Pr = 0.71$, $\alpha = \dfrac{\pi}{6}$, $Da = 0.1$, $M = 1$ and $t = 1$.

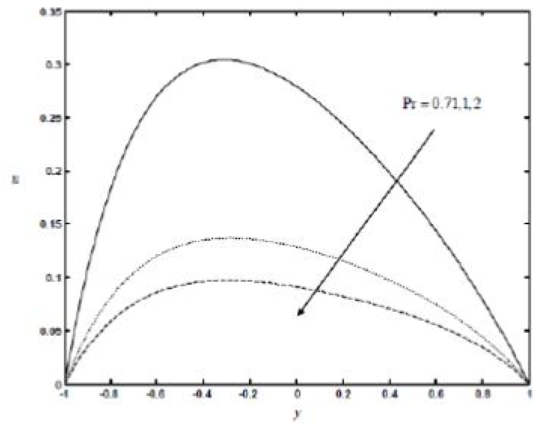

Fig. 6 The variation of velocity u with Prandtl number Pr for $M = 1$, $\alpha = \frac{\pi}{6}$, $Da = 0.1$, $n = 1$ and $t = 1$.

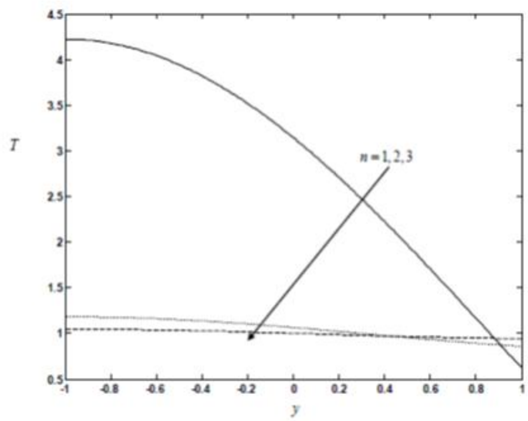

Fig. 7 The variation of temperature T with n for Pr = 0.71 and $t = 1$.

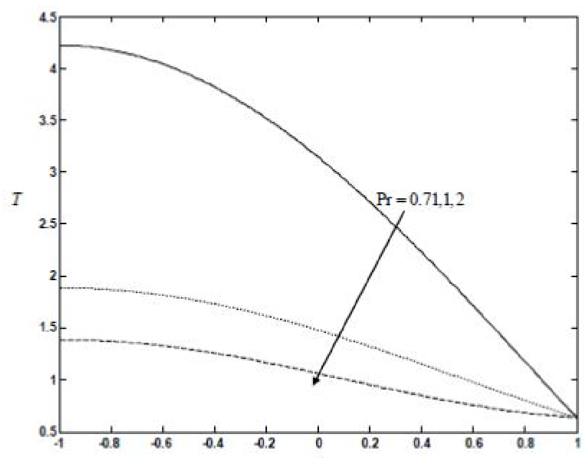

Fig. 8 The variation of temperature T with Prandtl number Pr for $n = 1$ and $t = 1$.

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