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Stiff Fluid Distribution with Variable G and Λ Assuming Power Law Relation in Bianchi type V Viscous Cosmological Model

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Abstract

We have investigated Bianchi Type V stiff fluid cosmological model with variable gravitational and cosmological constant. To get the exact solution it is assumed that equation of state is $P = p - 3 \eta H$, $\rho = p$, where $\eta = \eta_0 \rho^h$, p is the isotropic pressure, η is the coefficient of shear viscosity, H the Hubble constant, η_0 and h are taken as constants. A , B and C are assumed as the functions of cosmic time t using power law relation. Some other physical and geometrical aspects of the model are also discussed in terms of cosmic time t .

Key words : Bianchi type V universe, viscosity, stiff fluid, Hubble parameter variation, variable G and Λ .

Introduction

Bianchi Type V cosmological models are interesting in the study because these models contain better structure both in geometric and physical aspects than the FRW model. This model represents the open FRW model with negative curvature. In early stage of the universe when neutrino decoupling occurred, the matter behaved like viscous fluid¹. The coefficient of viscosity decreases as universe expands. The nature of viscosity in cosmology in Bianchi Type V cosmological model has been examined by number of authors^{2,3,4,5} and with heat flow by Banerjee and Sanyal⁶. Ram⁷ has investigated perfect fluid distribution in general relativity in Bianchi type V cosmological model. Coley⁸ has investigated equation of state $p = \gamma \rho$ for Bianchi Type V imperfect fluid cosmological models, where p is the isotropic pressure, ρ the energy density of matter, where $0 \leq \gamma \leq 1$. Caunto and Narlikar⁹ have shown that the Gravitational varying cosmology is consistent with whatsoever cosmological observations are available. Abdel Rehman¹⁰ has taken a cosmological model, in

which G varies with time and density is equal to critical density but energy is conserved. The bulk viscosity is associated with grand unified theory phase transition when inflationary scenario exists. A wide range of observations^{11,12,13} suggest that the universe possesses a non-zero cosmological constant. The possibility that this parameter is dynamical, theories are increasingly exploring. A number of authors viz. Beesham^{14,15}, Arbab^{16,17}, Bali and Tinker¹⁸, Singh and Baghel¹⁹ has investigated a cosmological model consisting of variable cosmological ($\Lambda \sim t^2$) and Gravitational constant with bulk viscosity, where the bulk viscosity is taken as a power function of density. Also some authors examined various cosmological models for stiff fluid distribution as bulk viscous LRS Bianchi type V tilted stiff fluid cosmological model in general relativity by Bali and Kumawat²⁰ and new exact solution of Bianchi type V cosmological stiff fluid model in Lyra's geometry by Yadav *et.al.*²¹. Bhaskara Rao *et.al.*²² investigated that Bianchi type-V bulk viscous string cosmological model in a self-creation theory of gravitation. Also Kohli²³ searched for Dynamics of a Vacuum Bianchi Type V Uni-verse with an Arbitrary Cosmological Constant.

The relevance to the study of Bianchi type V cosmological model for bulk viscosity has been discussed for barotropic fluid distribution in our earlier papers (Bali and Tinker¹⁸) where we have taken A, B, C are functions of t -alone. Here I have examined time varying G and Λ terms in the Bianchi type V bulk viscous models for Stiff fluid distribution in which A, B and C are assumed the functions of t using power law relation, it is also assumed that $P = p - 3\eta H$, $p = \rho$, $\eta = \eta_0 \rho^h$ where p is the isotropic pressure, η is the coefficient of viscosity, H the Hubble constant, η_0 and h are constants. The model represents a decelerating phase if $n < 1$ and accelerating phase if $n > 1$. Some other physical and geometrical aspects of the model are also discussed.

Metric and field equations :

We consider the Bianchi Type V metric in the form of line element

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{2x} dz^2 \quad (1)$$

where A, B, C are functions of cosmic time t alone.

The Einstein's field equation with varying G and Λ terms is given by

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G T_i^j + \Lambda g_i^j \quad (2)$$

Where, G is the gravitational constant and Λ the cosmological constant, G and Λ are functions of t . T_{ij} , energy momentum tensor for viscous fluid distribution is represented by

$$T_i^j = (\rho + P) v_i v^j + P g_i^j \quad (3)$$

$$\text{Where, } P = p - 3\eta H \quad (4)$$

here $p = \gamma \rho, 0 \leq \gamma \leq 1$ is the equilibrium pressure, η the coefficient of viscosity, $3\eta H$ is usually known as bulk viscous pressure, H is Hubble's parameter and ρ is the energy density of matter, v^i is the four velocity of the fluid, together with $v_i v^i = -1$.

The Einstein's field equation (2) for the space-time metric (1) leads to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \Lambda - 8\pi G P \quad (5)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \Lambda - 8\pi G P \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \Lambda - 8\pi G P \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} = \Lambda + 8\pi G \rho \quad (8)$$

$$\frac{2\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (9)$$

Here dot denotes differentiation with respect to cosmic time t .

The additional equation for divergence of Einstein tensor, for time changes of G and Λ is obtained due to the

contracted Bianchi identity, i.e. $(R_i^j - \frac{1}{2}R g_i^j)_{;j} = 0$ This leads to $(8\pi G T_i^j - \Lambda g_i^j)_{;j} = 0$ It again leads to

$$8\pi G \left[\dot{\rho} + (\rho + P) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + 8\pi \dot{G} \rho + \dot{\Lambda} = 0 \quad (10)$$

The usual conservation of energy after using (4) splits the above equation (10) into two equations

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (11)$$

and

$$8\pi G (-3\eta H) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + 8\pi \dot{G} \rho + \dot{\Lambda} = 0 \quad (12)$$

Now taking the relation

$$\eta = \eta_0 \rho^h \quad (13)$$

Here η_0 is a positive number and h is a constant.

Solution of field equations :

To get the deterministic model, we have assumed the stiff fluid condition i.e.

$$\rho = p \quad (14)$$

Also the solution of the system is assumed in the following form

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C} = H = \frac{n}{t + \alpha} \quad (15)$$

On integration it leads to

$$A = B = C = m(t + \alpha)^n \quad (16)$$

Here, m is constant of integration.

Using equations (14) and (15) in equation (11), we have

$$\rho + \frac{6n\rho}{t + \alpha} = 0 \quad (17)$$

Integrating it, we get

$$\rho = \frac{a}{(t + \alpha)^{6n}} = a(t + \alpha)^{-6n} \quad (18)$$

Now, with the help of equations (15) and (16), equation (8) becomes

$$8\pi G\rho + \Lambda = \frac{3n^2}{(t + \alpha)^2} - \frac{3}{m^2(t + \alpha)^{2n}} \quad (19)$$

On Differentiation of equation (19), we get

$$8\pi \dot{G}\rho + 8\pi G\dot{\rho} + \dot{\Lambda} = -\frac{6n^2}{(t + \alpha)^3} + \frac{6n}{m^2(t + \alpha)^{2n+1}} \quad (20)$$

With the help of Equations (12) and (15), equation (20) becomes

$$G = \frac{(t + \alpha)^{6n-2} \left[\frac{(t + \alpha)^{2(1-n)}}{m^2} - n \right]}{4\pi a \left[3\eta_0 n a^{h-1} (t + \alpha)^{-6n(h-1)-1} - 2 \right]} \quad (21)$$

Now using equations (18) and (21) in equation (19), we get

$$\Lambda = \frac{3n^2}{(t + \alpha)^2} - \frac{3}{m^2(t + \alpha)^{2n}} - \frac{2}{(t + \alpha)^2} \left[\frac{(t + \alpha)^{2(1-n)}}{m^2} - n \right] \quad (22)$$

Again equations (13) and (18) give us

$$\eta = \eta_0 a^h t^{-6nh} \quad (23)$$

The gravitational constant (G), cosmological constant (Λ), energy density (ρ), coefficient of viscosity (η) and the Hubble parameter (H) are given by

$$H = \frac{n}{t + \alpha}$$

$$\rho = a(t + \alpha)^{-6n}$$

$$\eta = \eta_0 a^h t^{-6nh}$$

$$G = \frac{(t + \alpha)^{6n-2} \left[\frac{(t + \alpha)^{2(1-n)}}{m^2} - n \right]}{4\pi a \left[3\eta_0 n a^{h-1} (t + \alpha)^{-6n(h-1)-1} - 2 \right]}$$

$$\Lambda = \frac{3n^2}{(t+\alpha)^2} - \frac{3}{m^2(t+\alpha)^{2n}} - \frac{2}{(t+\alpha)^2} \left[\frac{(t+\alpha)^{2(1-n)}}{m^2} - n \right] \left[3\eta_0 n a^{h-1} (t+\alpha)^{-6n(h-1)-1} - 2 \right]$$

CASE-I For better understanding the results, Taking $n = 1$, $1 < m < \infty$ and $\alpha = 0$, then the above mentioned quantities are given by

$$H = \frac{1}{t} \quad (24)$$

$$\rho = a t^{-6} \quad (25)$$

$$G = \frac{M t^4}{8\pi a \left[1 - \frac{3}{2} \eta_0 a^{h-1} t^{5-6h} \right]} \quad (26)$$

$$\Lambda = \frac{3M}{t^2} - \frac{1}{t^2} \frac{M}{\left[1 - \frac{3}{2} \eta_0 a^{h-1} t^{5-6h} \right]} \quad (27)$$

$$\eta = \eta_0 a^h t^{-6h} \quad (28)$$

Where, $1 - \frac{1}{m^2} = M$, if $1 < m < \infty$ than

$0 < M < 1$,

In the case-I taking $h = 5/6$, then we have

$$G = \frac{N t^4}{8\pi a}, \quad a > 0 \quad (29)$$

$$\Lambda = \frac{3M}{t^2} - \frac{2N}{t^2} = \frac{1}{t^2} [3M - 2N] \quad (30)$$

$$\eta = \eta_0 a^{5/6} t^{-5} \quad (31)$$

Where $N = \frac{M}{1 - \frac{3}{2} a^{-1/6} \eta_0}$

In the case-I taking $h = 1/6$, we get

$$G = \frac{M t^4}{8\pi a [1 - \frac{3}{2} \eta_0 a^{-\frac{5}{6}} t^4]} = \frac{M}{8\pi a [t^4 - \frac{3}{2} \eta_0 a^{-\frac{5}{6}}]} = \frac{M}{8\pi a [t^4 - L_1]} \quad (32)$$

$$\Lambda = \frac{3M}{t^2} - \frac{1}{t^2} \frac{M}{[1 - \frac{3}{2} \eta_0 a^{-\frac{5}{6}} t^4]} = 2M [\frac{1}{t^2} - \frac{1}{2} L_1 t^2 - \frac{1}{2} L_1 t^6 - \frac{1}{2} L_1 t^{10} \dots] \quad (33)$$

$$\eta = \frac{\eta_0 a^{\frac{1}{6}}}{t} \quad (34)$$

Where, $\frac{3}{2} \eta_0 a^{-\frac{5}{6}} = L_1$,

CASE-II Assuming $n = 1$, $m = 1$ and $\alpha = 0$, then

$H = 1/t$, $\rho = at^{-6}$, $\eta = \eta_0 a^h t^{-6h}$, $G=0$, $\Lambda=0$, In this condition, we get

$A = B = C = t$ It shows us, when time increases H , ρ , η decreases but Gravitational and Cosmological constant vanishes in this situation.

For the model (1), The Spatial volume is $R^3 = ABC = m^3(t + \alpha)^{3n}$.

$$\Rightarrow \frac{\dot{R}}{R} = \frac{n}{t + \alpha} \quad \text{and} \quad \frac{\ddot{R}}{R} = \frac{n(n-1)}{(t + \alpha)^2}$$

The generalized deceleration parameter q is defined as

$$q = -\frac{\frac{\ddot{R}}{R}}{\left(\frac{\dot{R}}{R}\right)^2} = \frac{1}{n} - 1 \quad (35)$$

Here deceleration parameter $q < 0$ if $n > 1$ and $q > 0$ if $0 < n < 1$. At $n = 1$, $q = 0$.

Also the Volume expansion θ and shear σ are defined as $\theta = V_{;i}^i$ and $\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}$, with σ_{ij} being the shear

tensor. Here Volume expansion $\theta = 3 \frac{\dot{R}}{R} = 3H = \frac{3n}{(t + \alpha)}$

$$\text{and } \sigma = \frac{\kappa}{R^3} = \frac{\kappa}{m^3(t + \alpha)^{3n}}$$

Conclusion

The matter density, Hubble parameter & coefficient of shear viscosity (η) decrease with time. For case

I, when $n=1$, $1 < m < \infty$ and $\alpha = 0$ then $H \propto \frac{1}{t}$, $\Lambda \propto \frac{1}{t^2}$ which are considered to be fundamental and matches

with the result as obtained by Beesham (1986). The gravitational constant G increases with time. ρ and η are initially very large and tends to zero for large values of t .

Universe exhibits a decelerating phase if $0 < n < 1$ and shows accelerating phase if $n > 1$, i.e. our model is evolving from decelerating phase to accelerating phase. The recent observations of SNe Ia depicts that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$ ^{11, 12}. Also when $n = 1$, then $q = 0$, which shows that every galaxy moves with a constant speed. This

model approaches isotropy for large values of t i.e. $\frac{\sigma}{\theta} \rightarrow 0$ as $t \rightarrow \infty$ when $\alpha = 0$. Here when we take $h=5/6$ and

$h=1/6$, H , ρ , and η are initially very large and tends to zero for large values of t , G is zero initially but it increases with time.

The value of deceleration parameter q depicts a transition from initial decelerating to present accelerating phase and provides the fastest speed at which the Universe is increasing for large times. This is the future situation as is observed by many well-known authors. We also see that the Universe becomes isotropic in future time.

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