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Studies on static-cosmological model

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Abstract

The Cosmological principle is the hypothesis that the universe is spatially homogeneous and isotropic. In the study of cosmological model, the Einstein field equations and construct a static model of the universe. A method has proposed by Berman 1991 [3] on the basis of the conservation law, to incorporate the variable Gravitational constant 'G' and Cosmological constant 'Λ' in the Einstein field equations. The solutions are obtained by choosing suitable values for the constant α and β, a number of cosmological models may be constructed and the validity of these models may be tested on the theoretical and observational grounds.

Key words : Cosmological model; perfect fluid; static model; variable cosmological and gravitational constants.

MSC Classification number: 83C05, 83F05

Introduction

The speculations about the nature of the universe are as old as man himself. Newton's gravitational theory meets with serious difficulties when applied to the universe as a whole. The three crucial tests of the general theory of relativity show the modification the Newtonian theory and gives solution to the problem of the field of a star in the empty space surrounding it at least to the distance of the order of the dimensions of the solar system. Now it is of great interest to apply the general theory of relativity to the universe as a whole. Einstein took it shortly after developing the general theory of relativity. Since them it has been a matter of interest of many investigators. It seems to be very much interesting because certain large-scale properties of the universe are experimentally known and capable of comparison with such a model of the universe. The Einstein's modified field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{1}$$

The constant Λ is such that its effect is negligible for phenomenon in the solar system or even in our own galaxy; but gains importance when the Universe as a whole is considered. By considering various values of Λ with various T_{ij} different cosmological models can be constructed. The static solution of Eq. (1) represent the "Static Cosmological Models" and in the same way for non-static ones for time dependent physical parameters.

The cosmological constant is a parameter describing the energy density of the vacuum and a potentially important contributor to the dynamical history of the universe. A sufficiently large cosmological constant will force galaxies to accelerate away from us, in contrast to the tendency of ordinary form of energy to slow down the recession of distant objects. The value of Λ in our present universe is not known and may be zero, although there is some evidence for a nonzero value of cosmological constant precise determination of this number will be one of the primary goals of observational cosmology in the near future.

In General Relativity, any form of energy affects the gravitational field, so the vacuum energy becomes a potentially crucial ingredient. To a good approximation, we believe that the vacuum is the some everywhere in the universe, so that the vacuum energy density is a universal number, which we call the cosmological constant. More precisely, the conventionally defined cosmological constant Λ is proportional to the vacuum energy density $\rho\lambda$; they are related by $\Lambda = \left(8\pi G/3c^2\right)\rho\lambda$, where '$G$' is Newton's constant of gravitation and 'c' is the speed of light.

The cosmological term Λg_{ij} has been introduced by Einstein in to his field equations to construct a static model of the universe. Later on, it has been observed that the universe is expanding and therefore the cosmological term has not been of much significance, recently it has been shown that 'Λ' is not a constant but it is a variable quantity (Eqs. (1), (3), (4), (5)). The same is true for the gravitational constant 'G' (Eqs .(5) and (6)). To incorporate the variable gravitational constant G and the variable Cosmological term Λ in to the Einstein field equations, a method has proposed by Berman[3] on the basis of the conservation law, where the field equations are written as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G T_{ij} + \Lambda g_{ij} \tag{2}$$

and the conservation law is expressed as : $T^{ij}{}_{;j} = 0$ (3)

Using Eq. (3) in Eq. (2), we have

$$8\pi G_{;i} T^{ij} + \Lambda_{;i} g^{ij} = 0 \tag{4}$$

These shows that Λ and G both are simultaneously. Berman and Rahman[1,8] have constructed homogeneous and isotropic cosmological models with the help of the Eqs. (2), (3) and (4) by assuming the variations of $G\rho$ and Λ as:

$$G\rho = At^{-2}, \qquad \Lambda = Bt^{-2} \tag{5}$$

Here we study the cosmological models by assuming

$$G\rho = \frac{\alpha\Lambda}{8\pi} \tag{6}$$

where α is a constant and therefore the variations of $G\rho$ and Λ given in Eq. (5) is included in Eq. (6), we write,

$$At^{-2} = \frac{\alpha B t^{-2}}{8\pi} \Rightarrow \alpha = 8\pi \frac{A}{B}$$

The constant 'α' from the present day observational data.

Basic equations and assumptions :

Basic equations :

The Robertson-Walker Metric for homogeneous and isotropic cosmological model space-time is given

$$ds^2 = c^2 dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\} \tag{7}$$

where R(t) is function of 't' only.

By taking large-scale viewpoint the energy momentum tensor of content of the universe takes the same form as for a perfect fluid distribution of matter. The energy momentum tensor is

$$T_{\mu\upsilon} = \left(\frac{p}{c^2} + \rho\right) u_\mu u_\upsilon - p g_{\mu\upsilon} \tag{8}$$

where 'p' is the pressure, 'ρ' is the proper density and u^i are the four velocities of the fluid particles, which are in this case cluster of galaxies. Since the particle is at rest in the coordinate system (r, θ, ϕ), we have $u^\mu = (0,0,0,c)$

we write, u^k=(0,0,0,1), where 'c' is the velocity of light be unity.

Therefore,

$$u_\mu = g_{\mu k} u^k = (0,0,0,c^2) \tag{9}$$

We consider the Einstein field equations as

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4} T_{ij} \tag{10}$$

Using energy momentum tensor in Einstein field equations, we get

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4}\left[\left(\frac{p}{c^2} + \rho\right) u_i u_j - p g_{ij}\right] \tag{11}$$

and the conservation law as: $T^{ij}{}_{;j} = 0$

Substituting the values from metric Eq. (7) and Eq. (9) in Eq. (11), we get the following set of field equations:

$$\frac{3\dot{R}^2}{R^2} + \frac{3kc^2}{R^2} = 8\pi G \rho + \Lambda c^2 \tag{12}$$

$$-\frac{kc^2}{R^2} - \frac{\dot{R}^2}{\dot{R}} - \frac{2\ddot{R}}{R} = \frac{8\pi G p}{c^2} - \Lambda c^2 \tag{13}$$

Differentiating the Eq. (12) w. r. t. 't' we get

$$8\pi\left(\dot{G}\rho+G\dot{\rho}\right)=-\frac{3\dot{R}}{R}\left(-\frac{2\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}+\frac{2kc^2}{R^2}\right)-\dot{\Lambda}c^2 \tag{14}$$

Adding Eq. (12) and Eq. (13), we get

$$-\frac{2\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}+\frac{2kc^2}{R^2}=8\pi G\left(\frac{p}{c^2}+\rho\right) \tag{15}$$

Substituting Eq. (15) in Eq. (14), we get

$$\dot{G}\rho+G\dot{\rho}+\frac{3\dot{R}}{R}G\rho=-\frac{3\dot{R}Gp}{Rc^2}-\frac{\dot{\Lambda}c^2}{8\pi} \tag{16}$$

Multiplying R^3 in Eq. (16), we get

$$c^2R^3\left(\dot{G}\rho+\frac{\dot{\Lambda}c^2}{8\pi}\right)+G\left[c^2\frac{d}{dt}\left(R^3\rho\right)+3\dot{R}R^2p\right]=0 \tag{17}$$

Conservation law :

The conservation law as

$$T^{ij}{}_{;j}=0 \tag{18}$$

and $$T^{ij}{}_{;j}=\frac{\partial T^{ij}}{\partial x^j}+T^{lj}\Gamma^{i}{}_{lj}+T^{il}\Gamma^{j}{}_{lj}$$

Now expanding Eq. (18), we have

$$T^{ij}{}_{;j}=T^{0j}{}_{;j}+T^{1j}{}_{;j}+T^{2j}{}_{;j}+T^{3j}{}_{;j}$$

and $u^i=(0,0,0,1)$

So $T^{00}=\rho$, $T^{11}=\dfrac{p(1-kr^2)}{R^2}$, $T^{22}=\dfrac{p}{R^2r^2}$, $T^{33}=\dfrac{p}{R^2r^2\sin^2\theta}$

and $T^{0j}{}_{;j}=\dfrac{3p\dot{R}+3c^2\dot{R}\rho+c^2R\dot{\rho}}{Rc^2}$, $T^{1j}{}_{;j}=0$, $T^{2j}{}_{;j}=0$, $T^{3j}{}_{;j}=0$

Using conservation law, we find

$$3pR^2\dot{R}+c^2\frac{d}{dt}(\rho R^3)=0 \tag{19}$$

Eq. (19) substituted in Eq. (18), gives

$$\dot{G}\rho+\frac{\dot{\Lambda}c^2}{8\pi}=0 \tag{20}$$

Further adding Eq. (12) and Eq. (13), we get

$$-\frac{\ddot{R}}{R}+\frac{1}{3}\left(\frac{3\dot{R}^2}{R^2}+\frac{3kc^2}{R^2}\right)=\frac{4\pi Gp}{c^2}+4\pi G\rho \tag{21}$$

Substituting the Eq. (12) in Eq. (21), we get

$$3\ddot{R} = -4\pi GR\left[\frac{3p}{c^2} + \rho - \frac{\Lambda c^2}{4\pi G}\right] \tag{22}$$

Eqs. (12), (20), (21) and (22) are the fundamental equations governing a homogeneous and isotropic cosmological model of the universe. Eqs. (12), (20) and (22) are same as the corresponding equations in general relativity, but, here we have additional Eq. (21) due to the variation of G and Λ We consider the dependence of the pressure in a homogeneous and isotropic model of the universe as

$$p = \beta\rho c^2 \tag{23}$$

For $$p = \begin{cases} 0 & \text{if} \quad \beta = 0 \quad \text{then model is pressure - less} \\ \rho/3 & \text{if} \quad \beta = 1/3 \quad \text{then model is filled with radiation} \end{cases}$$

A model is constructed in two ways either considering $\beta = 0$ or $\beta = \frac{1}{3}$. The model constructed with $\beta = 0$ are used to study the universe and present. The model constructed with $\beta = \frac{1}{3}$ are used to study the universe in the past when it was radiation dominated. In this way we do not have a theory how the universe reaches to matter dominated era from the radiation-dominated era. However, for a positive density and positive pressure of the universe, we have

$$\beta \geq 0 \tag{24}$$

The above equality sign implies that the model is pressuerless.

Following Berman[3] and Rahman[1], we consider the dependence of G and Λ as

$$G\rho = \frac{\alpha\Lambda c^2}{8\pi} \tag{25}$$

and thus, the fundamental equations governing the model of the universe are Eqs. (12), (20), (21), (22), (23) and (25).

Variation of ρ, G and Λ

From Eqs. (20) and (23), we get

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{R}}{R}(1+\beta) \tag{26}$$

then from Eqs. (21), (25) and (26), we get

$$\frac{\dot{G}}{G} = \frac{3(1+\beta)}{(1+\alpha)}\frac{\dot{R}}{R} \tag{27}$$

and $$\frac{\dot{\Lambda}}{\Lambda} = \frac{-3\alpha(1+\beta)}{(1+\alpha)}\frac{\dot{R}}{R} \tag{28}$$

Integrating Eqs. (26), (27) and (28) under the conditions

$$t = t_0\,,\ R = R_0\,,\ \rho = \rho_0\,,\ G = G_0\,,\ \text{and}\ \Lambda = \Lambda_0 \tag{29}$$

we get

$$\rho = \rho_0 \left(\frac{R}{R_0} \right)^{-3(1+\beta)} \tag{30}$$

$$G = G_0 \left(\frac{R}{R_0} \right)^{\frac{3(1+\beta)}{1+\alpha}} \tag{31}$$

and $$\Lambda = \Lambda_0 \left(\frac{R}{R_0} \right)^{\frac{-3\alpha(1+\beta)}{1+\alpha}} \tag{32}$$

with $$1+\alpha \neq 0 \tag{33}$$

Values of constants α and β

We may take the initial conditions for the construction of a cosmological model as

$$\begin{aligned} & t = 0, \quad R = R_i = 10^{13}\, cm, \quad \Lambda = \Lambda_f = 10^{28}\, cm^{-2}, \\ & \dot{R} = 0, \quad \ddot{R} > 0, \quad \rho = \rho_i = 10^{17}\, g/cm^3, \\ \text{and} \quad & G = G_f = 6.6 \times 10^{30} \text{ c.g.s. unit} \end{aligned} \tag{34}$$

From Eq. (12), we have

$$3\frac{\dot{R}^2}{R^2} - 8\pi G\rho = \Lambda c^2 - \frac{3kc^2}{R^2} \tag{35}$$

According to the initial conditions Eq. (34) at t = 0, the numerical value of $\frac{kc^2}{R^2}(k \neq 0)$ is very small as compared to the numerical values of $8\pi G\rho$ and Λc^2. Thus, for a non-singular model of the universe satisfying the initial conditions Eq. (34), we have a condition

$$8\pi G_i \rho_i + \Lambda_i c^2 = 0 \tag{36}$$

which is same as proposed by Sivaram *et al.*[12] Eq. (25), which is our proposition for the dependence of $G\rho$ on Λ then implies

$$8\pi G_i \rho_i + \alpha \Lambda_i c^2 = 0 \tag{37}$$

at $t = 0$ comparing Eq. (36) and Eq. (29), we have

$$\alpha = -1 \tag{38}$$

For $\alpha = -1$, Eq. (21) and Eq. (25) together imply $G\dot{\rho} = 0$ for $G \neq 0$ this gives $\dot{\rho} = 0,$ which on substitution in $\frac{\dot{\rho}}{\rho} = -3\frac{\dot{R}}{R}(1+\beta)$ gives R = 0 and hence we have a static model of the universe, which is unphysical.

By choosing suitable values for the constant α and β one may have the variation of ρ, G and Λ and then from Eq. (35) one may get the corresponding dependence of R on t. In this way, a number of cosmological models may be constructed and the validity of these models may be tested on the theoretical and observational grounds. Instead of approaching in this way, we propose to first determine the constants α and β by considering two variables out of the three variables ρ, G and Λ. We have three types cosmological models of the universe discussed in the next section.

Cosmological models :

Substituting Eq. (25) in Eq. (12), we get

$$\dot{R} = c\sqrt{\left(\frac{1+\alpha}{3}\right)\Lambda R^2 - k} \tag{39}$$

Also, from Eqs. (22), (23) and (25), we get

$$\ddot{R} = \left[1 - \frac{\alpha(1+3\beta)}{2}\right]\Lambda c^2 R \tag{40}$$

As discussed in the previous section, we have the following three types of the cosmological models depending on the numerical values of the physical quantities considered for the determination of the constants α and β as follows:

Type I

Let us consider Eqs. (30) and (31) and the corresponding numerical values as follows

$$\begin{aligned}
\text{at} \quad & R = R_i = 10^{13}\, cm, \quad \rho = \rho_i = 10^{17}\ g/cm^3 \\
& G = G_i = G_f = 6.6\times 10^{30}\, c.g.s.unit \\
\text{and} \quad & R = R_0 = 10^{28}\, cm \\
& \rho = \rho_0 = 3\times 10^{-31}\ g/cm^3 \\
& G = G_0 = 6.6\times 10^{-8}\, c.g.s.unit
\end{aligned} \tag{41}$$

The numerical values given in Eq. (41) when substituted in $\rho = \rho_0\left(\frac{R}{R_0}\right)^{-3(1+\beta)}$ give

$$1+\beta = \frac{16}{15},\ \frac{1+\beta}{1+\alpha} = -\frac{38}{45} \tag{42}$$

and hence $\alpha = -\frac{43}{19}, \quad \beta = \frac{1}{15}$ (43)

The Eq. (25) then gives

$$\Lambda_i = \frac{8\pi G_i \rho_i}{\alpha c^2} = -8\times 10^{28}\, cm^{-2}$$

and (44)

$$\Lambda_0 = \frac{8\pi G_0 \rho_0}{\alpha c^2} = -2\times 10^{-57}\, cm^{-2}$$

Therefore, in the cosmological model Λ is negative and increasing from $-8\times 10^{28}\, cm^{-2}$ to $-2\times 10^{-57}\, cm^{-2}$ as R increases from $10^{13}\, cm$ to $10^{28}\, cm.$ For $\Lambda < 0, \alpha < 0$ and $\beta > 0,$ Eq. (40) implies that $\ddot{R} < 0,$ i.e., the model is deaccelerating.

Type II

Let us consider Eqs. (30) and (31) and the corresponding the numerical values as follows:

$$R = R_i = 10^{13} cm,\ \ \rho = \rho_i = 10^{17}\, g/cm^3,\ \ \Lambda = \Lambda_i = \Lambda_f = -10^{28} cm^{-2}$$

and $$R = R_0 = 10^{28} cm,\ \ \rho = \rho_0 = 3\times 10^{-31}\, g/cm^3, \tag{45}$$

$$\Lambda = \Lambda_0 = \Lambda_f = -10^{-57}\, cm^{-2}$$

The numerical values given in Eq. (45) when substituted in Eqs. (37) and (40) gives

$$1+\beta = \frac{16}{15}, \qquad \frac{\alpha(1+\beta)}{1+\alpha} = \frac{17}{9} \tag{46}$$

and hence, $$\beta = \frac{1}{15} \quad \text{and} \quad \alpha = -\frac{85}{37} \tag{47}$$

Eq. (25) then gives

$$G_i = \frac{\alpha \Lambda_i c^2}{8\pi \rho_i} = 8\times 10^{30}\, c.g.s.units \tag{48}$$

and $$G_0 = \frac{\alpha \Lambda_0 c^2}{8\pi\ \rho_0} = 3\times 10^{-7}\, c.g.s.units \tag{49}$$

In this case also $\ddot{R} < 0$ as may be seen from the Eq. (40). Therefore the model is deaccelerating.

Type III

Let us consider Eqs. (30) and (31) and the corresponding numerical values as follows:

$$R = R_i = 10^{13} cm,\ \ G = G_i = G_f = 6.6\times 10^{30} c.g.s.units$$

$$\Lambda = \Lambda_i = \Lambda_f = -10^{28} cm^{-2}$$

and $$R = R_0 = 10^{28} cm, \qquad G = G_0 = 6.6\times 10^{-8} c.g.s.units, \tag{50}$$

$$\Lambda = \Lambda_0 = -10^{-57}\, cm^{-2}$$

The numerical values given in Eq. (50) when substituted in Eqs. (30) and (31) gives

$$\frac{1+\beta}{1+\alpha} = -\frac{38}{85}, \quad \frac{\alpha(1+\beta)}{1+\alpha} = \frac{17}{9} \tag{50}$$

and hence, $$\alpha = -\frac{85}{38}, \beta = \frac{2}{45} \tag{51}$$

Eq. (25) then gives $\rho_i = \frac{\alpha \Lambda_i c^2}{8\pi G_i} = 10^{17}\ g/cm^3$

and $$\rho_0 = \frac{\alpha \Lambda_0 c^2}{8\pi G_0} 10^{-30}\ g/cm^3 \tag{52}$$

For $\alpha < 0, \beta > 0$ and $\Lambda < 0,$ Eq. (40) implies that $\ddot{R} < 0,$ therefore, the model is deaccelerating.

Conclusion

In this paper, we have seen that the Einstein field equations with variables G and Λ. Such that $G\rho = \frac{\alpha \Lambda c^2}{8\pi}$ and the usual conservation law implies that $\alpha < -1,$ for $G > 0$ and $G < 0,$ $\Lambda < 0$ and $\dot{\Lambda} > 0$. Applying the initial conditions as proposed by Isham *et al*.[6] and Sinha *et al*.[10] for the construction of non-singular cosmological models, we have seen that there are three types of singular cosmological models depending on the constants α and β. On the observational ground, all the cases the model is deaccelerating.

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