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Two warehouses Inventory Model with Stock-dependent demand and Maximum life time

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Abstract

In this paper, a two warehouses inventory model with stock-dependent demand has been studied. In case there is limited capacity of the owned warehouse (OW), another warehouse is used named rented warehouse (RW) for large ordered quantity. The deterioration rate of inventory items is different in both the warehouses. In the case of OW, we allow for shortages and consider the time dependent backlogging rate. To achieve the optimal ordered quantity and the optimal interval for the total inventory cost, a solution procedure is established and it concludes the developed model with further extended environments.

Key words. Inventory, rented warehouse, own warehouse, backlogging rate, maximum life time.

Introduction

Today's, companies have identified that in addition to the maximizing profit; customer satisfaction plays a vital role for acquiring and maintaining a victorious position in the competitive market. Some products/ items such as milk, bread, fish, blood, vegetables, fruits, medicines and radioactive chemicals have finite shelf life and start to deteriorate once they are replenished. During the last so many year mathematical ideas have been widely used in various fields mostly for the controlling and keeping inventory. Finding the minimum total inventory cost associated with the inventory system one must keep in mind when to order and how much to

order or how much to manufacture. First Ghare and Schrader[1] gave an idea of an inventory model with an exponentially decaying inventory and also presented the EOQ inventory model with fixed deterioration rate and without shortages. Covert and Philip[2] extended the Ghare and Schrader's model and discussed an EOQ model with two-parameter Weibull distribution deterioration rate. Misra[3] presented an inventory model under two type of deterioration rate variable and constant both. Agarwal[4], Dave and Patel[5], Dave[7], Aggarwal and Jaggi[8], Wee[9], Dye and Ouyang[13] etc., authors discussed some important inventory models for deteriorating items at a constant deteriorate rate. In these above mentioned models, the demand and deterioration rate was assumed to be constant. The relationship between time and deterioration rate is a general phenomena considered in the inventory models. Later, some models developed including variable deteriorating rate by Wee[10], Bhunia and Maiti[12], Mana and Chaudhary[14], Liao[16], Sarkar[21]. The idea of stock dependent demand are assumed by some authors, such as Sharma[6], Goswami and Choudhary[11], Singh and Malik[19], Sharma *et al*.[24], Sarkar and Sarkar[22], Gupta *et al*.[25], Singh *et al*.[26], Vashisth *et al*.[28], Malik *et al*.[30] and Kumar *et al*.[29] developed the inventory models with stock dependent demand.

Today's daily used items such as milk, bread, vegetables, fruits etc, such items maintain their freshness for some time they not deteriorated as soon as they obtained by the retailer/seller. During their fresh time of items maintained their originality; Ouyang *et al*.[15] named this phenomenon "non-instantaneous deterioration" and prepared a new model with non-instantaneous deteriorating items with the permissible delay in payments. Singh and Malik[18] considered an Optimal ordering policy inventory model with linear deterioration and exponential demand under the two storage capacities. Sana[17] examined optimal selling price and lot size inventory model with time varying deterioration under the partial backlogging. Sett *et al*.[20] investigated an inventory model of two-warehouse considering quadratic increasing demand and time varying deterioration. Sarkar and Sarkar[22] developed an economic quantity model with probabilistic deterioration rate in the production system. Sarkar *et al*.[27] presented an inventory model with quality improvement and backorder price discount under controllable lead time. Vashisth *et al*.[31] presented an inventory model with multivariate demand for non-instantaneous decaying products under the trade credit policy.

Notations and Assumptions

For the proposed inventory model, the following notations used in this paper:

$D(t)=d_1+d_2I(t)$	:	the demand rate per unit time,
W_1	:	the capacity of RW
W_2	:	the capacity of OW
C_o	:	the ordering cost per order,
h_1	:	the holding cost in RW per unit time per unit,
h_2	:	the holding cost in OW per unit time per unit,
k	:	the deteriorating cost in both RW and OW per unit time per unit,
T	:	the total length of the ordering cycle
$TIC(t_2, T)$	:	the total inventory cost per unit time of the proposed system.

The following assumptions are used for proposed model:

1. The deterioration rate are $\theta_1(t)$ and $\theta_2(t)$ at any time t, $\theta_1(t)=\frac{1}{1+m_1-t}$, $\theta_2(t)=\frac{1}{1+m_2-t}$, where

$0\le\theta_1(t)\le1$, $0\le\theta_2(t)\le1$; m_1 and m_2 are maximum life time of the products in RW and OW respectively.

1. The capacity of owned warehouse and rented warehouses are W_1 and W_2 respectively.
2. Shortages are allowed and backlogging rate is time proportional, denoted as $B(x) = \frac{1}{1+\lambda x}$, where λ is positive constant and *x* is the waiting time for the subsequently refill.

Mathematical Model :

For the developed model, assume that during the time-interval [0, t_1], no deterioration occurs in the products, during [t_1, t_1+t_2] deterioration and demand are the factors for decreasing the inventory levels. In RW system, I_{r1} and I_{r2} is the inventory levels in the intervals [0, t_1] and [t_1, t_1+t_2] respectively. During [t_1+t_2, t_1+t_2+t_3] in the OW system inventory level decreases up to zero level due to demand and deterioration. In the time interval [t_1+t_2+t_3, T] shortages are occurs and unsatisfied demands are backlogged. During the time interval [0, T], the following inventory levels are represented by the differential equations:

$$\frac{dI_{r1}(t)}{dt} = -\left[d_1 + d_2 I_{r1}(t)\right], \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_{r2}(t)}{dt} + \left(\frac{1}{1+m_1-t} + d_2\right) I_{r2}(t) = -d_1, \qquad t_1 \le t \le t_1 + t_2 \tag{2}$$

$$\frac{dI_{o1}(t)}{dt} = 0, \qquad 0 \le t \le t_1 \tag{3}$$

$$\frac{dI_{o2}(t)}{dt} + \left(\frac{1}{1+m_2-t}\right) I_{o2}(t) = 0, \quad t_1 \le t \le t_1 + t_2 \tag{4}$$

$$\frac{dI_{o3}(t)}{dt} + \left(\frac{1}{1+m_2-t} + d_2\right) I_{o3}(t) = -d_1, \quad t_1 + t_2 \le t \le t_1 + t_2 + t_3 \tag{5}$$

$$\frac{dI_n(t)}{dt} = -\frac{d_1}{1+\lambda(T-t)}, \quad t_1 + t_2 + t_3 \le t \le T \tag{6}$$

Equations (1-6) holding with the following boundary conditions $I_{r1}(0) = W_1$, $I_{r2}(t_1 + t_2) = 0$, $I_{o2}(t_1) = W_2$, $I_{o3}(t_1 + t_2 + t_3) = 0$, $I_n(t_1 + t_2 + t_3) = 0$ respectively. Using the above boundary conditions the solutions of the equations (1-6) are as follows:

$$I_{r1}(t) = \frac{d_1}{d_2}\left(e^{-d_2 t} - 1\right) + W_1 e^{-d_2 t}, \quad 0 \le t \le t_1 \tag{7}$$

$$I_{r2}(t) = d_1(1+m_1-t)\, e^{-d_2 t}\left[\frac{d_2^2}{4}\left\{t^2 - (t_1+t_2)^2\right\} + u_1(t - t_1 - t_2) + u_2\, log\left(\frac{1+m_1-t}{1+m_1-t_1-t_2}\right)\right] \tag{8}$$

$$I_{o1}(t) = W_2 \tag{9}$$

$$I_{o2}(t) = W_2\left(\frac{1+m_2-t}{1+m_2-t_1}\right) \tag{10}$$

$$I_{o3}(t)=d_1(1+m_2-t)\,e^{-d_2 t}\left[\frac{d_2^2}{4}\left\{t^2-(t_1+t_2+t_3)^2\right\}+v_1(t-t_1-t_2-t_3)+v_2\log\left(\frac{1+m_2-t}{1+m_2-t_1-t_2-t_3}\right)\right] \quad (11)$$

$$I_n(t)=\frac{d_1}{\lambda}\log\left(\frac{1+\lambda(T-t)}{1+\lambda t_4}\right) \quad (12)$$

Due to continuity of Inventory levels in this developed model; for RW at t=t_1, from Equations (7) and (8), we have $I_{r1}(t_1)=I_{r2}(t_1)$ (13)

Solving Equation (13), we get

$$W_1=\frac{d_1}{d_2}\left(e^{d_2 t_1}-1\right)-d_1(1+m_1-t_1)\left[\frac{d_2^2}{4}\left(t_2^2+2t_1t_2\right)+u_1t_2-u_2\log\left(\frac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\right] \quad (14)$$

Also for OW at t=t_1+t_2, from Equations (10) and (11), we have $I_{o2}(t_1+t_2)=I_{o3}(t_1+t_2)$ (15)

Solving Equation (15), we get

$$W_2=d_1(1+m_2-t_1)\,e^{-d_2(t_1+t_2)}\left[-\frac{d_2^2}{4}\left(t_3^2+2t_1t_3+2t_2t_3\right)-v_1t_3-v_2\log\left(\frac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\right] \quad (16)$$

The optimum inventory cost per cycle contains the following terms:

The ordering cost per cycle is OC= C_o (17)

The holding cost for RW per cycle is given by

$$HC_{RW}=h_1\left[\int_0^{t_1} I_{r1}(t)dt+\int_{t_1}^{t_1+t_2} I_{r2}(t)dt\right]$$

$$=h_1\left[\frac{d_1}{d_2}t_1+\left(1-e^{-d_2t_1}\right)\left(\frac{d_1+W_1d_2}{d_2^2}\right)\right.$$

$$+x_1\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(t_1(t_1+t_2)^2-\dfrac{t_1^3}{3}-\dfrac{2}{3}(t_1+t_2)^3\right)+u_1\left(t_1(t_1+t_2)-\dfrac{t_1^2}{2}-\dfrac{(t_1+t_2)^2}{2}\right)\\ -u_2\left\{t_2-(1+m_1-t_1)\log\left(\dfrac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\right\}\end{array}\right\}$$

$$-x_2\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{t_1^2}{2}(t_1+t_2)^2-\dfrac{t_1^4}{4}-\dfrac{(t_1+t_2)^4}{4}\right)+u_1\left(\dfrac{t_1^2}{2}(t_1+t_2)-\dfrac{t_1^3}{3}-\dfrac{(t_1+t_2)^3}{6}\right)\\ -u_2\left\{\left(\dfrac{1+m_1}{2}\right)t_2+\left(\dfrac{t_2^2+2t_1t_2}{4}\right)-\left(\dfrac{(1+m_1)^2-t_1^2}{2}\right)\log\left(\dfrac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\right\}\end{array}\right\}$$

$$+x_3\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{t_1^3}{3}(t_1+t_2)^2-\dfrac{t_1^5}{5}-\dfrac{2}{15}(t_1+t_2)^5\right)+u_1\left(\dfrac{t_1^3}{3}(t_1+t_2)-\dfrac{t_1^4}{4}-\dfrac{(t_1+t_2)^4}{12}\right)\\ -u_2\left\{\begin{array}{l}\dfrac{(1+m_1)^2}{3}t_2+\left(\dfrac{t_2^2+2t_1t_2}{6}\right)(1+m_1)-\left(\dfrac{t_1^3-(t_1+t_2)^3}{9}\right)\\ -\left(\dfrac{(1+m_1)^3-t_1^3}{3}\right)\log\left(\dfrac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\end{array}\right\}\end{array}\right\}$$

$$-x_4\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{t_1^4}{4}(t_1+t_2)^2-\dfrac{t_1^6}{6}-\dfrac{1}{12}(t_1+t_2)^6\right)+u_1\left(\dfrac{t_1^4}{4}(t_1+t_2)-\dfrac{t_1^5}{5}-\dfrac{(t_1+t_2)^5}{20}\right)\\ -u_2\left\{\begin{array}{l}\dfrac{(1+m_1)^3}{4}t_2+\left(\dfrac{t_2^2+2t_1t_2}{8}\right)(1+m_1)^2-(1+m_1)\left(\dfrac{t_1^3-(t_1+t_2)^3}{12}\right)\\ -\left(\dfrac{t_1^4-(t_1+t_2)^4}{16}\right)-\left(\dfrac{(1+m_1)^4-t_1^4}{4}\right)\log\left(\dfrac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\end{array}\right\}\end{array}\right\} \quad (18)$$

The holding cost for OW per cycle is given by

$$HC_{OW}=h_2\left[\int_0^{t_1}I_{o1}(t)dt+\int_{t_1}^{t_1+t_2}I_{o2}(t)dt+\int_{t_1+t_2}^{t_1+t_2+t_3}I_{o3}(t)dt\right]$$

$$=h_2\left[(t_1+t_2)-\frac{t_2^2}{2(1+m_2-t_1)}\right.$$

$$+y_1\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left((t_1+t_2)(t_1+t_2+t_3)^2-\dfrac{(t_1+t_2)^3}{3}-\dfrac{2}{3}(t_1+t_2+t_3)^3\right)\\ +v_1\left((t_1+t_2)(t_1+t_2+t_3)-\dfrac{(t_1+t_2)^2}{2}-\dfrac{(t_1+t_2+t_3)^2}{2}\right)\\ -v_2\left\{t_3-(1+m_2-t_1-t_2)\log\left(\dfrac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\right\}\end{array}\right\}$$

$$-y_2\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{(t_1+t_2)^2(t_1+t_2+t_3)^2}{2}-\dfrac{(t_1+t_2)^4}{4}-\dfrac{(t_1+t_2+t_3)^4}{4}\right)\\ +v_1\left(\dfrac{(t_1+t_2)^2(t_1+t_2+t_3)}{2}-\dfrac{(t_1+t_2)^3}{3}-\dfrac{(t_1+t_2+t_3)^3}{6}\right)\\ -v_2\left\{\dfrac{(1+m_2)}{2}t_3-\left(\dfrac{(t_1+t_2)^2-(t_1+t_2+t_3)^2}{4}\right)-\left(\dfrac{(1+m_2)^2-(t_1+t_2)^2}{2}\right)\log\left(\dfrac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\right\}\end{array}\right\}$$

$$+ y_3\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{(t_1+t_2)^3(t_1+t_2+t_3)^2}{3}-\dfrac{(t_1+t_2)^5}{5}-\dfrac{2(t_1+t_2+t_3)^5}{15}\right)\\ +v_1\left(\dfrac{(t_1+t_2)^3(t_1+t_2+t_3)}{3}-\dfrac{(t_1+t_2)^4}{4}-\dfrac{(t_1+t_2+t_3)^4}{12}\right)\\ -v_2\left\{\begin{array}{l}\left(\dfrac{(1+m_2)^2}{3}t_3-(1+m_2)\left(\dfrac{(t_1+t_2)^2-(t_1+t_2+t_3)^2}{6}\right)-\left(\dfrac{(t_1+t_2)^3-(t_1+t_2+t_3)^3}{9}\right)\right)\\ -\left(\dfrac{(1+m_2)^3-(t_1+t_2)^3}{3}\right)\log\left(\dfrac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\end{array}\right.\end{array}\right\}$$

$$-y_4\left\{\begin{array}{l}\dfrac{d_2^2}{4}\left(\dfrac{(t_1+t_2)^4(t_1+t_2+t_3)^2}{4}-\dfrac{(t_1+t_2)^6}{6}-\dfrac{(t_1+t_2+t_3)^6}{12}\right)\\ +v_1\left(\dfrac{(t_1+t_2)^4(t_1+t_2+t_3)}{4}-\dfrac{(t_1+t_2)^5}{5}-\dfrac{(t_1+t_2+t_3)^5}{20}\right)\\ -v_2\left\{\begin{array}{l}\left(\dfrac{(1+m_2)^3}{4}t_3-(1+m_2)^2\left(\dfrac{(t_1+t_2)^2-(t_1+t_2+t_3)^2}{8}\right)-(1+m_2)\left(\dfrac{(t_1+t_2)^3-(t_1+t_2+t_3)^3}{12}\right)\right)\\ -\left(\dfrac{(t_1+t_2)^4-(t_1+t_2+t_3)^4}{16}\right)-\left(\dfrac{(1+m_2)^4-(t_1+t_2)^4}{4}\right)log\left(\dfrac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\end{array}\right.\end{array}\right\} \qquad (19)$$

The deterioration cost for RW per cycle is given by

$$DC_{RW} = k\int_{t_1}^{t_1+t_2}\theta(t)I_{r2}(t)dt$$

$$= kd_1\left[\begin{array}{l}\dfrac{d_2^2}{4}\left\{\begin{array}{l}\left(t_1(t_1+t_2)^2-\dfrac{t_1^3}{3}-\dfrac{2}{3}(t_1+t_2)^3\right)-d_2\left(\dfrac{t_1^2}{2}(t_1+t_2)^2-\dfrac{t_1^4}{4}-\dfrac{(t_1+t_2)^4}{4}\right)\\ +\dfrac{d_2^2}{2}\left(\dfrac{t_1^3}{3}(t_1+t_2)^2-\dfrac{t_1^5}{5}-\dfrac{2}{15}(t_1+t_2)^5\right)\end{array}\right.\\ +u_1\left\{\begin{array}{l}{}_1\left(t_1(t_1+t_2)-\dfrac{t_1^2}{2}-\dfrac{(t_1+t_2)^2}{2}\right)-d_2\left(\dfrac{t_1^2}{2}(t_1+t_2)-\dfrac{t_1^3}{3}-\dfrac{(t_1+t_2)^3}{6}\right)\\ +\dfrac{d_2^2}{2}\left(\dfrac{t_1^3}{3}(t_1+t_2)-\dfrac{t_1^4}{4}-\dfrac{(t_1+t_2)^4}{12}\right)\end{array}\right.\end{array}\right.$$

$$+u_2\left\{\begin{array}{l}\left(t_2-(1+m_1-t_1)\log\left(\frac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\right)\\ -d_2\left(\left(\frac{1+m_1}{2}\right)t_2+\left(\frac{t_2^2+2t_1t_2}{4}\right)-\left(\frac{(1+m_1)^2-t_1^2}{2}\right)\log\left(\frac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\right)\\ +\frac{d_2^2}{2}\left(\begin{array}{l}\frac{(1+m_1)^2}{3}t_2+\left(\frac{t_2^2+2t_1t_2}{6}\right)(1+m_1)-\left(\frac{t_1^3-(t_1+t_2)^3}{9}\right)\\ -\left(\frac{(1+m_1)^3-t_1^3}{3}\right)\log\left(\frac{1+m_1-t_1}{1+m_1-t_1-t_2}\right)\end{array}\right)\end{array}\right\} \quad (20)$$

The deterioration cost for OW per cycle is given by

$$DC_{OW}=k\left[\int_{t_1}^{t_1+t_2}\theta_2(t)\,I_{o2}(t)dt+\int_{t_1+t_2}^{t_1+t_2+t_3}\theta_2(t)\,I_{o3}(t)dt\ \right]$$

$$=kd_1\left[\begin{array}{l}\frac{d_2^2}{4}\left\{\begin{array}{l}\left((t_1+t_2)(t_1+t_2+t_3)^2-\frac{(t_1+t_2)^3}{3}-\frac{2}{3}(t_1+t_2+t_3)^3\right)\\ -d_2\left(\frac{(t_1+t_2)^2(t_1+t_2+t_3)^2}{2}-\frac{(t_1+t_2)^4}{4}-\frac{(t_1+t_2+t_3)^4}{4}\right)\end{array}\right\}\\ +\frac{d_2^2}{2}\left\{\begin{array}{l}\left(\frac{(t_1+t_2)^3(t_1+t_2+t_3)^2}{3}-\frac{(t_1+t_2)^5}{5}-\frac{2(t_1+t_2+t_3)^5}{15}\right)\end{array}\right\}\end{array}\right]$$

$$+v_1\left\{\begin{array}{l}\left((t_1+t_2)(t_1+t_2+t_3)-\frac{(t_1+t_2)^2}{2}-\frac{(t_1+t_2+t_3)^2}{2}\right)\\ -d_2\left(\frac{(t_1+t_2)^2(t_1+t_2+t_3)}{2}-\frac{(t_1+t_2)^3}{3}-\frac{(t_1+t_2+t_3)^3}{6}\right)\\ +\frac{d_2^2}{2}\left(\frac{(t_1+t_2)^3(t_1+t_2+t_3)}{3}-\frac{(t_1+t_2)^4}{4}-\frac{(t_1+t_2+t_3)^4}{12}\right)\end{array}\right\}$$

$$-v_2\left\{\begin{array}{l}\left(t_3-(1+m_2-t_1-t_2)\log\left(\frac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\right)\\ -d_2\left(\frac{(1+m_2)}{2}t_3-\left(\frac{(t_1+t_2)^2-(t_1+t_2+t_3)^2}{4}\right)-\left(\frac{(1+m_2)^2-(t_1+t_2)^2}{2}\right)\log\left(\frac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\right)\\ +\frac{d_2^2}{2}\left(\begin{array}{l}\frac{(1+m_2)^2}{3}t_3-(1+m_2)\left(\frac{(t_1+t_2)^2-(t_1+t_2+t_3)^2}{6}\right)\\ -\left(\frac{(t_1+t_2)^3-(t_1+t_2+t_3)^3}{9}\right)-\left(\frac{(1+m_2)^3-(t_1+t_2)^3}{3}\right)\log\left(\frac{1+m_2-t_1-t_2}{1+m_2-t_1-t_2-t_3}\right)\end{array}\right)\end{array}\right\} \quad (21)$$

The shortage cost per cycle is given by

$$SC = s_1 \int_{t_1+t_2+t_3}^{T} -I_n(t)\,dt = \frac{s_1 d_1}{\lambda}\left[t_4 - \frac{1}{\lambda}\log(1+\lambda t_4)\right] \tag{22}$$

The lost sales cost per cycle is given by

$$LS = l_1 \int_{t_1+t_2+t_3}^{T} d_1\left(1 - \frac{1}{1+\lambda(T-t)}\right)dt = \frac{l_1 d_1}{\lambda}\left[t_4 - \frac{1}{\lambda}\log(1+\lambda t_4)\right] \tag{23}$$

Thus the optimum inventory cost (TIC) per cycle per unit time is given by

$$TIC = \frac{1}{T}[OC + HC_{RW} + HC_{OW} + DC_{RW} + DC_{OW} + SC + LS] \tag{24}$$

The cost function is highly non-linear. To minimize the total cost per unit time, the optimal values of t_2 and T can be obtained by solving the following system of equations simultaneously

$$\frac{\partial TIC}{\partial t_2} = 0 \tag{25}$$

$$\frac{\partial TIC}{\partial T} = 0 \tag{26}$$

$$\frac{\partial^2 TIC}{\partial t_2^2} > 0,\ \frac{\partial^2 TIC}{\partial T^2} > 0 \text{ and } \left(\frac{\partial^2 TIC}{\partial t_2^2}\right)\left(\frac{\partial^2 TIC}{\partial T^2}\right) - \left(\frac{\partial^2 TIC}{\partial t_2 \partial T}\right)^2 > 0 \tag{27}$$

Solution Procedure :

Step 1. Set the value of the parameters in equation (24).

Step 2. Now using the equations (25) and (26) obtain t_2^* and T^* and from the equation (24) obtain TIC^*.

Step 3. Putting the value of t_2^* and T^* in the system of equation (27) and check the optimality. If satisfied then go to stop otherwise repeat the process from step 1 to 3.

Conclusions

In this paper we developed a two warehouses inventory model with the maximum life time. In actual conditions, the maximum products/items deteriorate due to expiration of their maximum life time. Such type of model with time varying deterioration function of the time with assuming non-instantaneous items has not yet been proposed. This proposed model can be applied to the inventory system for stock-dependent demand. The study of this paper developed for the optimal inventory cost with solution procedure. Further, for research extension in this paper with variable holding cost, ramp type demand, quadratic demand, price and multi valued demand, inflation, reliability and trade credit etc.

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Appendices

$$u_1 = d_2 - \frac{d_2^2}{2}(1+m_1),\ u_2 = 1+u_1(1+m_1),\ v_1 = d_2 - \frac{d_2^2}{2}(1+m_2),\ v_2 = 1+v_1(1+m_2),$$

$$x_1 = (1+m_1)d_1,\ x_2 = d_1 + d_2 x_1,\ x_3 = d_1 d_2 + \frac{x_1 d_2^2}{2},\ x_4 = \frac{d_1 d_2^2}{2}.$$

$$y_1 = (1+m_2)d_1,\ \ y_2 = d_1 + d_2 y_1,\ \ y_3 = d_1 d_2 + \frac{y_1 d_2^2}{2},\ \ y_4 = \frac{d_1 d_2^2}{2} = x_4.$$