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Section A

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Economic Production Quantity Model With Generalized Pareto Rate of Production and Weibull Decay Having Selling Price Dependent Demand

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Abstract

Economic production quantity models usually assume that the production is fixed and finite. However, due to various random factors effecting the production, the production process becomes random. This paper deals with the development and analysis of economic production quantity model in which the production is random and follows a generalized Pareto distribution. The generalized Pareto distribution is capable of including different types of production rates. Here it is further assumed that the lifetime of the commodity is random and follows a two parameter Weibull distribution. The Weibull decay includes constant, increasing and decreasing rates of deterioration. It is also assumed that the demand is dependent on selling price. Assuming that shortages are allowed and fully backlogged the instantaneous state of on hand inventory is derived. With suitable cost considerations the total cost function and profit rate function are obtained and minimized with respect to the production uptime and downtime. The optimal production uptime, downtime, production quantity and selling price are derived. A numerical illustration demonstrating the solution procedure of the model is presented. The sensitivity analysis of the model revealed that the production and deteriorating distributions parameters have significant influence on the optimal production schedule and production quantity. This model is extended to the case of without shortages. This model also includes some of the earlier models as particular cases for specific or limiting values of the parameters.

Key words : EPQ model, Generalized Pareto rate of production, Selling price dependent demand, Random production, Weibull decay.

Subject Classification Code: 90 59- Operations Research

1 Introduction

In classical production inventory models it is customary to assume that the demand is constant. But, in many production systems the demand is a function of selling price. Recently much work has been reported in literature regarding selling price dependent demand. In addition to demand another important factor of production level inventory model or EPQ models for deteriorating items is a common phenomenon for scheduling production systems.

Begum *et al.*[2] and Tripathy *et al.*[15] considered Weibull rate of deterioration. Srinivasa Rao *et al.*[13] developed and analyzed an inventory model for deteriorating items with generalized Pareto decay. Srinivasa Rao *et al.*[14] considered the case of additive exponential lifetime. Roy *et al.*[10] introduced an order level inventory model for a deteriorating item, taking the demand rate to be dependent on the sale price of the item and incorporating the concept of the special sale campaign by way of price of reduction is into the model. Madhavi *et al.*[7] developed an inventory model with the assumption that demand is a function of selling price, lifetime of the item is random and follows two parameter exponential distribution and that the deteriorated items are kept in the inventory for second sale. Inventory models for deteriorating items having multivariate demand functions were studied by some authors. Chen *et al.*[3] studied an inventory model with a multivariate demand function of price and time. Khanra *et al.*[4] studied models for deteriorating items having stock level and selling price dependent demand rate. Urban *et al.*[16] studied an inventory model in which demand is a function of price, time and inventory level. Kousar Jaha Begum *et al.*[5] developed an E.P.Q model with the assumptions that the life time of commodity is random and follow a Generalized Pareto Distribution and is assumed that demand is a function of both the time and selling price. Ajay Kumar Agarwal *et al.*[1] studied an inventory model with variable demand rate for deteriorating Items under Permissible Delay in Payments. Varsha Sharma *et al.*[17] studied an inventory model for deteriorating products having demand which is function of selling price. Maragatham *et al.*[8] and Raman Patel *et al.*[9] developed an inventory model with time and price dependent demand.

In all these models it is assumed that the production is finite and constant rate. But in many practical situations the production process is random due to various random factors such as availability of raw material, manpower, power supply, breakdowns etc. Hence recently Sridevi *et al.*[11], Srinivasa Rao *et al.*[12] and Lakshmana Rao *et al.*[6] have developed production level inventory models with the assumption that the production process is random and follows Weibull distribution. Very little work has been reported regarding EPQ models with selling price dependent demand having Weibull rate of decay and generalized Pareto rate of production. Hence in this paper we fill the gap in this area of research by developing and analyzing an EPQ model with generalized Pareto rate of production and Weibull decay having selling price dependent demand. The generalized Pareto rate of production includes constant and time dependent rates of productions as particular cases.

Using the differential equations the instantaneous state of inventory is derived. The total cost function and profit rate function under suitable cost considerations assuming shortages which are fully backlogged are derived. The optimal production uptime and downtime are obtained by minimizing the profit rate function and optimal production quantity and optimal selling price are derived. The sensitivity analysis of the model is included to study the changes in input variables and costs. This model is extended to the case of without shortages.

2 Assumptions :

For developing the model the following assumptions are made:

i) The demand rate is selling price dependent demand. Say $\lambda(s) = a - bs$ (1)
where 'a' and 'b' are constants and 's' is selling price.

ii) The production is finite and follows a Generalized Pareto distribution. The instantaneous rate of production is

$$K(t) = \frac{1}{\alpha - \gamma t} \quad ; 0 < t < \frac{\alpha}{\gamma} \tag{2}$$

iii) Lead time is zero.
iv) Cycle length is T. It is known and fixed.
v) Shortages are allowed and fully backlogged.
vi) A deteriorated unit is lost.
vii) The lifetime of the item is random and follows a two parameter Weibull distribution with probability density function

$$f(t) = \theta\eta t^{\eta-1} e^{-\theta t^{\eta}} \quad ; \theta, \eta > 0, \quad t > 0$$

Therefore the instantaneous rate of deterioration is

$$h(t) = \frac{f(t)}{1-F(t)} = \theta\eta t^{\eta-1} \quad ; \theta, \eta > 0, \quad t > 0 \tag{3}$$

The following notations are used for developing the model.
Q: Production quantity.
A: Setup cost.
C: Cost per unit.
h: Inventory holding cost per unit per unit time.
π: Shortages cost per unit per unit time.

3 EPQ Model with Shortages:

Consider a production system in which the stock level is zero at time t = 0. The stock level increases during the period $(0, t_1)$, due to production after fulfilling the demand and deterioration. The production stops at time t_1 when stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3 the replenishment again starts and fulfils the backlog after satisfying the demand. During (t_3, T) the backorders are fulfilled and inventory level reaches zero at the end of the cycle T. The schematic diagram representing the instantaneous state of inventory is given in Figure 1.

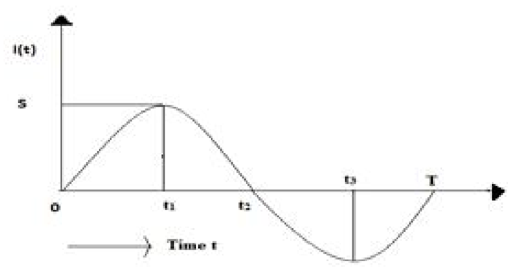


Fig 1: Schematic diagram representing the inventory level

Let I(t) be the inventory level of the system at time 't' ($0 \le t \le T$). The differential equations governing the instantaneous state of I(t) over the cycle of length T are

$$\frac{d}{dt} I(t) + h(t) I(t) = \frac{1}{\alpha - \gamma t} - (a - bs) \quad ; \quad 0 \le t \le t_1 \tag{4}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a-bs) \qquad ; \qquad t_1 \le t \le t_2 \tag{5}$$

$$\frac{d}{dt}I(t) = -(a-bs) \qquad ; \qquad t_2 \le t \le t_3 \tag{6}$$

$$\frac{d}{dt}I(t) = \frac{1}{\alpha-\gamma t} - (a-bs) \qquad ; \qquad t_3 \le t \le T \tag{7}$$

where, h(t) is as given in equation (3), with the initial conditions I(0) = 0, $I(t_1) = S, I(t_2) = 0$ and I(T) = 0.

Substituting h(t) in equations (4) and (5) and solving the differential equations, the on hand inventory at time ' t ' is obtained as

$$I(t) = Se^{\theta(t_1^\eta - t^\eta)} - e^{-\theta t^\eta}\int_t^{t_1}\left(\frac{1}{\alpha-\gamma u} - (a-bs)\right)e^{\theta u^\eta}\,du \;; \quad 0 \le t \le t_1 \tag{8}$$

$$I(t) = Se^{\theta(t_1^\eta - t^\eta)} - (a-bs)e^{-\theta t^\eta}\int_{t_1}^{t} e^{\theta u^\eta}\,du \qquad ; \quad t_1 \le t \le t_2 \tag{9}$$

$$I(t) = (a-bs)(t_2 - t) \qquad ; \quad t_2 \le t \le t_3 \tag{10}$$

$$I(t) = \frac{1}{\gamma}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t}\right) + (a-bs)(T-t) \qquad ; \quad t_3 \le t \le T \tag{11}$$

Production quantity Q in the cycle of length T is

$$Q = \frac{1}{\gamma}\log\left(\frac{\alpha\,(\alpha-\gamma t_3)}{(\alpha-\gamma t_1)(\alpha-\gamma T)}\right) \tag{12}$$

From equation (8) and using the initial condition I(0) = 0, we obtain the value of 'S' as

$$S = e^{-\theta t_1^\eta}\int_0^{t_1}\left(\frac{1}{\alpha-\gamma u} - (a-bs)\right)e^{\theta u^\eta}\,du \tag{13}$$

When t = t_3 , then equations (10) and (11) become

$$I(t_3) = (a-bs)(t_2 - t_3) \quad \text{and} \tag{14}$$

$$(t_3) = \frac{1}{\gamma}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right) + (a-bs)(T-t_3) \qquad \text{respectively.} \tag{15}$$

Equating the equations (14) and (15) and on simplification, one can get

$$t_2 = T + \frac{1}{\gamma(a-bs)}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right) = x(t_3, s) \quad (say) \tag{16}$$

Let K(t_1,t_2,t_3,s) be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Hence the total production cost per unit time becomes

$$K(t_1,t_2,t_3,s) = \frac{A}{T} + \frac{C}{\gamma T}\log\left(\frac{\alpha\,(\alpha-\gamma t_3)}{(\alpha-\gamma t_1)(\alpha-\gamma T)}\right)$$
$$+ \frac{h}{T}\left[\int_0^{t_1}\left[Se^{\theta(t_1^\eta - t^\eta)} - e^{-\theta t^\eta}\int_t^{t_1}\left(\frac{1}{\alpha-\gamma u} - (a-bs)\right)e^{\theta u^\eta}\,du\right]dt\right.$$
$$\left. + \int_{t_1}^{t_2}\left[Se^{\theta(t_1^\eta - t^\eta)} - (a-bs)e^{-\theta t^\eta}\int_{t_1}^{t} e^{\theta u^\eta}\,du\right]dt\right]$$

$$+\frac{\pi}{T}\left[(a-bs)\int_{t_2}^{t_3}(t-t_2)\,dt+(a-bs)\int_{t_3}^{T}(t-T)dt+\frac{1}{\gamma}\int_{t_3}^{T}\log\left(\frac{\alpha-\gamma t}{\alpha-\gamma T}\right)dt\right] \tag{17}$$

Let $P(t_1, t_2, t_3, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total production cost per unit time. Then the profit rate function is,

$$P(t_1,t_3,s)=s(a-bs)-\frac{A}{T}-\frac{C}{\gamma T}\log\left(\frac{\alpha(\alpha-\gamma t_3)}{(\alpha-\gamma t_1)(\alpha-\gamma T)}\right)$$

$$-\frac{h}{T}\left[\int_{0}^{x(t_3,s)}\left[e^{-\theta t^{\eta}}\int_0^{t_1}\left(\frac{1}{\alpha-\gamma u}-(a-bs)\right)e^{\theta u^{\eta}}du\right]dt\right.$$

$$-\int_0^{t_1}\left[e^{-\theta t^{\eta}}\int_t^{t_1}\left(\frac{1}{\alpha-\gamma u}-(a-bs)\right)e^{\theta u^{\eta}}du\right]dt$$

$$\left.-(a-bs)\int_{t_1}^{x(t_3,s)}\left[e^{-\theta t^{\eta}}\int_{t_1}^{t}e^{\theta u^{\eta}}du\right]dt\right]$$

$$-\frac{\pi}{T\gamma}\left[t_3-T+\left[T-t_3-\frac{1}{\gamma}(\alpha-\gamma t_3)+\frac{1}{2\gamma(a-bs)}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)\right]\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)\right] \tag{18}$$

4 Optimal Ordering Policies of the Model with Shortages:

In this section we obtain the optimal policies of the system under study. To find the optimal values of t_1, t_3 and s, we obtain the first order partial derivatives of $P(t_1, t_2, t_3, s)$ given in equation (18) with respect to t_1, t_3 and s and equate them to zero. The condition for minimization of $P(t_1, t_3, s)$ is

$$D=\begin{bmatrix}\frac{\partial^2 P(t_1,t_3,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1\partial t_3} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1\partial s}\\ \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1\partial t_3} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3^2} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3\partial s}\\ \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1\partial s} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3\partial s} & \frac{\partial^2 P(t_1,t_3,s)}{\partial s^2}\end{bmatrix}<0$$

Where, D is the Hessian matrix.

Differentiating $P(t_1, t_3, s)$ with respect to t_1 and equating to zero, we get

$$\frac{C}{\alpha-\gamma t_1}+he^{\theta t_1^{\eta}}\left[\left(\frac{1}{\alpha-\gamma t_1}-(a-bs)\right)\left[\int_0^{x(t_3,s)}e^{-\theta t^{\eta}}dt-\int_0^{t_1}e^{-\theta t^{\eta}}dt\right]\right.$$

$$\left.+(a-bs)\int_{t_1}^{x(t_3,s)}e^{-\theta t^{\eta}}dt\right]=0 \tag{19}$$

Differentiating $P\,(t_1, t_3, s)$ with respect to t_3 and equating to zero, we get

$$C(\alpha-\gamma T)-he^{-\theta(x(t_3,s))^{\eta}}\left[\frac{1}{(a-bs)}\int_0^{t_1}\left(\frac{1}{\alpha-\gamma u}-(a-bs)\right)e^{\theta u^{\eta}}du-\right.$$

$$\left.\int_{t_1}^{x(t_3,s)}e^{\theta u^{\eta}}du\right]+\frac{\pi}{\gamma}\left[(\alpha-\gamma t_3)(1-\alpha+\gamma T)-\gamma(T-t_3)-\frac{1}{a-bs}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)\right]=0 \tag{20}$$

Differentiating $P\,(t_1, t_3, s)$ with respect to 's' and equating to zero, we get

$$a-2bs-\frac{h}{T}\left[y(t_3,s)e^{-\theta(x(t_3,s))^{\eta}}\int_0^{t_1}\left(\frac{1}{\alpha-\gamma u}-(a-bs)\right)e^{\theta u^{\eta}}du+\right.$$

$$b\int_0^{x(t_3,s)}\left[e^{-\theta t^{\eta}}\int_0^{t_1}e^{\theta u^{\eta}}du\right]dt-b\int_0^{t_1}\left[e^{-\theta t^{\eta}}\int_t^{t_1}e^{\theta u^{\eta}}du\right]dt+$$

$$\left.b\int_{t_1}^{x(t_3,s)}\left[e^{-\theta t^{\eta}}\int_{t_1}^{t}e^{\theta u^{\eta}}du\right]dt-(a-bs)y(t_3,s)e^{-\theta(x(t_3,s))^{\eta}}\int_{t_1}^{x(t_3,s)}e^{\theta u^{\eta}}du\right]$$

$$-\frac{\pi b}{2T\left(\gamma(a-bs)\right)^2}\left(\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)\right)^2=0$$

where, $x(t_3,s)=t_2=T+\frac{1}{\gamma(a-bs)}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)$ and

$$y(t_3,s)=\frac{\partial}{\partial s}x(t_3,s)=\frac{b}{\gamma(a-bs)^2}\log\left(\frac{\alpha-\gamma T}{\alpha-\gamma t_3}\right)$$

Solving the equations (19), (20) and (21) simultaneously, we obtain the optimal time at which production is stopped t_1^*, the optimal time t_3^* at which the production is restarted after accumulation of backorders and the optimal selling price s* .

The optimum production quantity Q* in the cycle of length T is obtained by substituting the optimal values of t_1^* and t_3^* in equation (3.9) as

$$Q^*=\frac{1}{\gamma}\log\left(\frac{\alpha\,(\alpha-\gamma t_3{}^*)}{(\alpha-\gamma t_1{}^*)(\alpha-\gamma T)}\right) \tag{22}$$

5 Results and Discussion on EPQ Model with Shortages:

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 1(a) and Table 1(b). The relationship between the parameters and the optimal values of the replenishment schedule is shown in Figure 2.

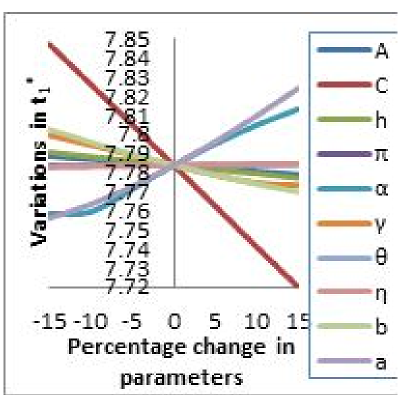
It is observed that the costs are having a significant influence on the optimal production quantity and replenishment schedules. As the setup cost 'A' decreases, the optimal production downtime t_1^*, the optimal production quantity Q^* and the profit rate P^* are increasing and the

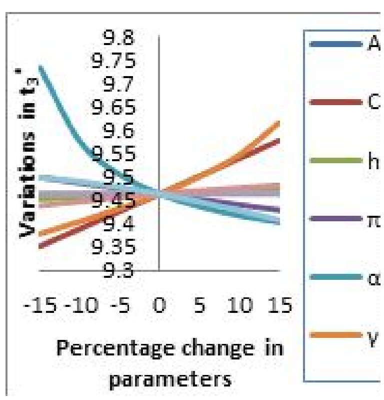
Table 1(a) Sensitivity Analysis of the Model – With Shortages

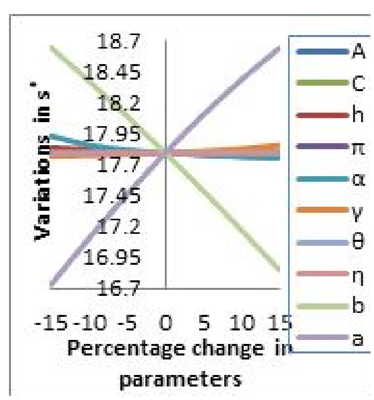
Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
θ	t_1^*	7.7887	7.7871	7.7855	7.7839	7.7822	7.7806	7.779
	t_3^*	9.4573	9.4596	9.4619	9.4642	9.4665	9.4688	9.4711
	s^*	17.7875	17.7906	17.7936	17.7966	17.7997	17.8027	17.8057
	Q^*	29.2339	29.22	29.2061	29.1922	29.1783	29.1644	29.1505
	P^*	74.1892	72.9544	71.7197	70.4849	69.2501	68.0154	66.7806
c	t_1^*	7.8472	7.8262	7.8051	7.7839	7.7625	7.7411	7.7201
	t_3^*	9.3525	9.3895	9.4267	9.4642	9.502	9.54	9.5782
	s^*	17.7874	17.7905	17.7936	17.7966	17.7996	17.8025	17.805
	Q^*	29.8175	29.6113	29.4028	29.1922	28.9794	28.7645	28.5475
	P^*	74.3954	73.0689	71.7653	70.4849	69.2281	67.9952	66.7842
h	t_1^*	7.7907	7.7884	7.7861	7.7839	7.7816	7.7794	7.7771
	t_3^*	9.4531	9.4568	9.4605	9.4642	9.4679	9.4716	9.4753
	s^*	17.8367	17.8233	17.81	17.7966	17.7833	17.77	17.7566
	Q^*	29.2563	29.2349	29.2135	29.1922	29.1709	29.1496	29.1283
	P^*	70.0055	70.1646	70.3244	70.4849	70.6461	70.8081	70.9707
π	t_1^*	7.7836	7.7837	7.7838	7.7839	7.784	7.7841	7.7842
	t_3^*	9.4992	9.4875	9.4759	9.4642	9.4526	9.441	9.4293
	s^*	17.7994	17.7985	17.7976	17.7966	17.7957	17.7948	17.7938
	Q^*	29.0537	29.1	29.1461	29.1922	29.2381	29.284	29.3297
	P^*	70.1435	70.2557	70.3695	70.4849	70.6019	70.7205	70.8407
α	t_1^*	7.7584	7.7594	7.7713	7.7839	7.7951	7.8049	7.8132
	t_3^*	9.7335	9.5758	9.5052	9.4642	9.4372	9.418	9.4036
	s^*	17.9335	17.8586	17.8205	17.7966	17.78	17.7677	17.7581
	Q^*	50.5859	39.452	33.3184	29.1922	26.1418	23.7575	21.8239
	P^*	45.0981	58.2576	65.5364	70.4849	74.1849	77.1077	79.5008
γ	t_1^*	7.7995	7.7945	7.7892	7.7839	7.7787	7.7746	7.7734
	t_3^*	9.3791	9.4054	9.4333	9.4642	9.5008	9.5477	9.6162
	s^*	17.7706	17.7777	17.7862	17.7966	17.8102	17.8288	17.8569
	Q^*	24.6423	25.8791	27.3622	29.1922	31.5418	34.7442	39.567
	P^*	76.4693	74.7751	72.8208	70.4849	67.5602	63.6472	57.8171

Table 1(b) Sensitivity Analysis of the Model – With Shortages

Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
θ	t_1^*	7.783	7.7833	7.7836	7.7839	7.7842	7.7844	7.7847
	t_3^*	9.4645	9.4644	9.4643	9.4642	9.4641	9.464	9.4639
	s^*	17.7955	17.7958	17.7962	17.7966	17.797	17.7974	17.7978
	Q^*	29.191	29.1914	29.1918	29.1922	29.1926	29.193	29.1934
	P^*	70.5001	70.495	70.49	70.4849	70.4799	70.4748	70.4698
η	t_1^*	7.783	7.7833	7.7836	7.7839	7.7842	7.7844	7.7847
	t_3^*	9.4645	9.4644	9.4643	9.4642	9.4641	9.464	9.4639
	s^*	17.7954	17.7958	17.7962	17.7966	17.797	17.7974	17.7978
	Q^*	29.191	29.1914	29.1918	29.1922	29.1926	29.193	29.1934
	P^*	70.5002	70.4951	70.49	70.4849	70.4798	70.4747	70.4697
b	t_1^*	7.8026	7.7959	7.7896	7.7839	7.7787	7.774	7.7699
	t_3^*	9.4387	9.4479	9.4565	9.4642	9.4712	9.4774	9.4827
	s^*	18.647	18.3754	18.0917	17.7966	17.4912	17.1762	16.8526
	Q^*	29.3485	29.2921	29.2399	29.1922	29.1491	29.1108	29.0776
	P^*	87.7922	81.7651	75.9933	70.4849	65.2453	60.2771	55.5807
a	t_1^*	7.7561	7.7634	7.7728	7.7839	7.7962	7.8097	7.8241
	t_3^*	9.4989	9.4907	9.4789	9.4642	9.4474	9.4288	9.4087
	s^*	16.7352	17.1094	17.4642	17.7966	18.1048	18.3871	18.6424
	Q^*	28.9724	29.0268	29.1015	29.1922	29.2954	29.4088	29.5303
	P^*	36.6672	47.5851	58.8665	70.4849	82.4099	94.6082	107.044







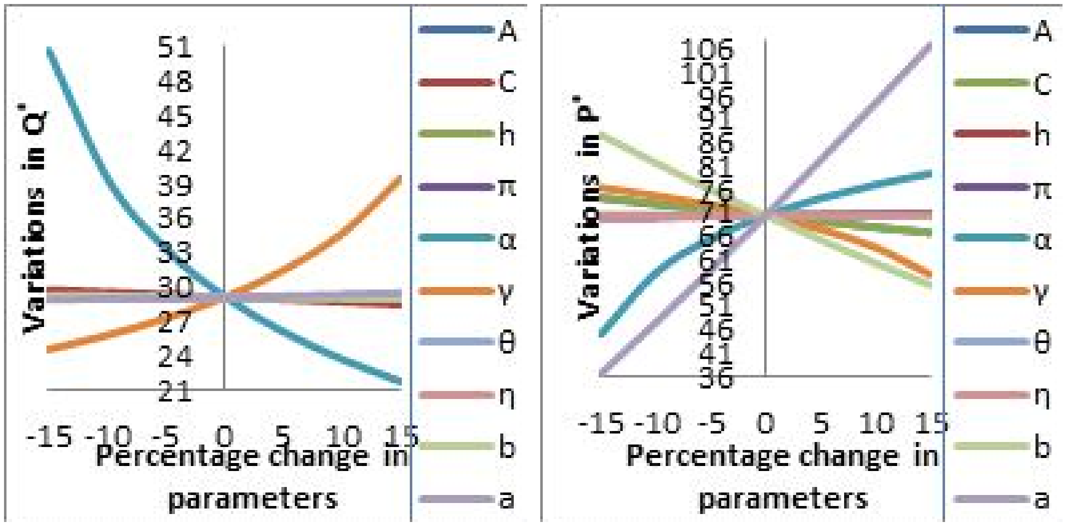


Fig 2: Relationship between parameters and optimal values with shortages

optimal production uptime t_3^* and the optimal selling price s^* are decreasing. As the cost per unit 'C' decreases, the optimal production downtime $\boldsymbol{t_1^*}$, the optimal production quantity Q^* and the profit rate P^* are increasing and the optimal production uptime t_3^* and the optimal selling price s^* are decreasing. As the holding cost 'h' decreases the optimal production downtime $\boldsymbol{t_1^*}$, the optimal selling price s^* and the optimal production quantity Q^* are increasing and the optimal production uptime t_3^* and the profit rate P^* are decreasing. As the shortage cost 'π' decreases, the optimal production downtime t_3^* and the optimal selling price s^* are increasing and the optimal production downtime $\boldsymbol{t_1^*}$, the optimal production quantity Q^* and the profit rate function P^* are decreasing.

As the production rate parameter 'γ' decreases, the optimal production downtime $\boldsymbol{t_1^*}$ and the profit rate function P^* are increasing and the optimal production downtime t_3^*, the optimal selling price s^* and the optimal ordering quantity Q^* are decreasing. As production parameter 'α' decreases, the optimal production downtime t_3^*, the optimal selling price s^* and the optimal production quantity Q^* are increasing and the optimal production downtime $\boldsymbol{t_1^*}$ and the profit rate P^* are decreasing. As deteriorating parameter θ decreases, the optimal production uptime t_3^*, the profit rate P^* are increasing and the optimal production downtime $\boldsymbol{t_1^*}$, the optimal selling price s^* and the optimal production quantity Q^* are decreasing. Another deteriorating parameter η decreases, the optimal production uptime t_3^* and the profit rate P^* are increasing and the optimal production downtime $\boldsymbol{t_1^*}$, the optimal selling price s^* and the optimal production quantity Q^* are decreasing. As demand parameter *a* decreases, the optimal production up time t_3^* increases and the optimal production downtime $\boldsymbol{t_1^*}$, the optimal selling price s^*, the optimal production quantity Q^* and the profit rate P^* are decreasing. Another demand parameter *b* decreases the optimal production downtime $\boldsymbol{t_1^*}$, the optimal selling price s^* the optimal production quantity Q^* and the profit rate P^* are increasing and the optimal production up time t_3^* decreases.

6 EPQ Model without Shortages:

In this section the inventory model for deteriorating items without shortages is developed and analyzed. Here, it is assumed that shortages are not allowed and the stock level is zero at time $t = 0$. The stock level increases during the period $(0, t_1)$ due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T). At time T the inventory reaches zero. The schematic diagram representing the instantaneous state of inventory is given in Figure 3.

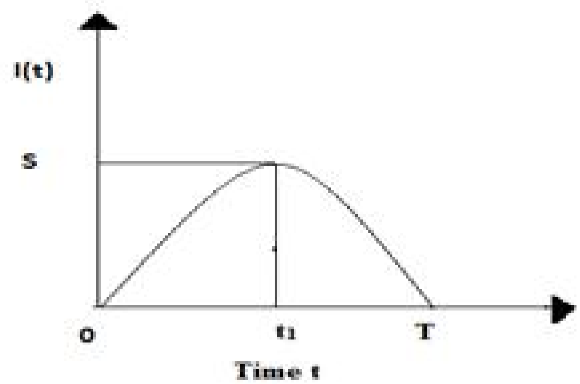


Fig 3: Schematic diagram representing the inventory level

Let I(t) be the inventory level of the system at time 't' ($0 \le t \le T$). Then the differential equations governing the instantaneous state of I(t) over the cycle of length T are

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{1}{\alpha - \gamma t} - (a - bs)\ ; \qquad 0 \le t \le t_1 \tag{23}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a - bs) \quad ; \qquad t_1 \le t \le T \tag{24}$$

where, h(t) is as given in equation (3), with the initial conditions I(0) = 0, $I(t_1) = S$ and I(T) = 0.

Substituting h(t) in equations (23) and (24) and solving the differential equations, the on hand inventory at time ' t ' is obtained as

$$I(t) = Se^{\theta(t_1^\eta - t^\eta)} - e^{-\theta t^\eta}\int_t^{t_1}\left(\frac{1}{\alpha-\gamma u} - (a - bs)\right)e^{\theta u^\eta}du\ ; \quad 0 \le t \le t_1 \tag{25}$$

$$I(t) = Se^{\theta(t_1^\eta - t^\eta)} - (a - bs)e^{-\theta t^\eta}\int_{t_1}^{t} e^{\theta u^\eta}du \quad ; \quad t_1 \le t \le T \tag{26}$$

Production quantity Q in the cycle of length T is

$$Q = \frac{1}{\gamma}\log\left(\frac{\alpha}{(\alpha-\gamma t_1)}\right)$$

From equation (25) and using the initial condition I(0) = 0, we obtain the value of 'S' as

$$S = e^{-\theta t_1^\eta} \int_0^{t_1} \left(\frac{1}{\alpha-\gamma u} - (a - bs)\right) e^{\theta u^\eta} du \tag{28}$$

Let $K(t_1, s)$ be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore, the total production cost per unit time becomes

$$\begin{aligned} K(t_1, s) = {} & \frac{A}{T} + \frac{C}{\gamma T} \log\left(\frac{\alpha}{\alpha-\gamma t_1}\right) \\ & + \frac{h}{T}\left[\int_0^{t_1} \left[S e^{\theta(t_1^\eta - t^\eta)} - e^{-\theta t^\eta} \int_t^{t_1} \left(\frac{1}{\alpha-\gamma u} - (a-bs)\right) e^{\theta u^\eta} du\right] dt\right] \\ & + \int_{t_1}^T \left[S e^{\theta(t_1^\eta - t^\eta)} - (a-bs) e^{-\theta t^\eta} \int_{t_1}^t e^{\theta u^\eta} du\right] dt \end{aligned} \tag{29}$$

Let $P(t_1, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total production cost per unit time. We have,

$$\begin{aligned} P(t_1, s) = {} & s(a-bs) - \frac{A}{T} - \frac{C}{\gamma T} \log\left(\frac{\alpha}{\alpha-\gamma t_1}\right) \\ & - \frac{h}{T}\left[\int_0^T \left[e^{-\theta t^\eta} \int_0^{t_1} \left(\frac{1}{\alpha-\gamma u} - (a-bs)\right) e^{\theta u^\eta} du\right] dt\right. \\ & \left. - \int_0^{t_1} \left[e^{-\theta t^\eta} \int_t^{t_1} \left(\frac{1}{\alpha-\gamma u} - (a-bs)\right) e^{\theta u^\eta} du\right] dt - (a-bs) \int_{t_1}^T \left[e^{-\theta t^\eta} \int_{t_1}^t e^{\theta u^\eta} du\right] dt\right] \end{aligned} \tag{30}$$

7 Optimal ordering policies of the model without shortages :

In this section we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1 and s, we equate the first order partial derivatives of $P(t_1,s)$ with respect to t_1 and andequate them to zero. The condition for minimum of $P(t_1,s)$ is

$$D = \begin{vmatrix} \frac{\partial^2 P(t_1,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,s)}{\partial t_1 \partial s} \\ \frac{\partial^2 P(t_1,s)}{\partial t_1 \partial s} & \frac{\partial^2 P(t_1,s)}{\partial s^2} \end{vmatrix} < 0$$

Differentiating $P(t_1,s)$ with respect to t_1 and equating to zero, we get

$$\frac{C}{\alpha-\gamma t_1} + h e^{\theta t_1^\eta} \left[\left(\frac{1}{\alpha-\gamma t_1} - (a-bs)\right)\left[\int_0^T e^{-\theta t^\eta} dt - \int_0^{t_1} e^{-\theta t^\eta} dt\right] + (a-bs) \int_{t_1}^T e^{-\theta t^\eta} dt\right] = 0 \tag{31}$$

Solving the equations (31), we obtain the optimal time t_1^* of t_1 at which the production is to be stopped.

Differentiating $P(t_1,s)$ with respect to s and equating to zero, one can get

$$T(a-2bs)-hb\left[\int_0^T\left[e^{-\theta t^{\eta}}\int_0^{t_1}e^{\theta u^{\eta}}du\right]dt-\int_0^{t_1}\left[e^{-\theta t^{\eta}}\int_0^{t_1}e^{\theta u^{\eta}}du\right]dt\right.$$
$$\left.+\int_{t_1}^{T}\left[e^{-\theta t^{\eta}}\int_{t_1}^{t}e^{\theta u^{\eta}}du\right]dt\right]=0 \qquad (32)$$

Solving the equations (32), we obtain the optimal value of s of t_1 at which the production is to be stopped. The optimal production quantity Q^* in the cycle of length T is obtained by substituting the optimal values of t_1 in equation (27) as

$$Q^*=\frac{1}{\gamma}\log\left(\frac{\alpha}{(\alpha-\gamma t_1{}^*)}\right) \qquad (33)$$

8 Results and Discussion of the model without shortages:

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2(a) and Table 2(b). The relationship between the parameters and the optimal values of the production schedule is shown in Figure 4.

It is observed that the costs are having a significant influence on the optimal production quantity and production schedules. As the setup cost 'A' decreases, the optimal production downtime t_1^*, the optimal selling price s^*, the optimal production quantity Q^* and the profit rate P^* are increasing. As the cost per unit 'C' decreases, the optimal production downtime t_1^*, the optimal selling price s^*, the optimal production quantity Q^* and the profit rate P^* are increasing. As the holding cost 'h' decreases, the optimal production downtime t_1^*, the optimal selling price s^* and the optimal production quantity Q^* are increasing and the profit rate P^* decreases. As the production rate parameter 'γ' decreases, the optimal production downtime t_1^*, the optimal selling price s^* and the profit rate function P^* are increasing and the optimal production quantity Q^* decreases. As production rate parameter 'α' decreases, the optimal production quantity Q^*

Table 2(a) Sensitivity Analysis of the Model – Without Shortages

Variation Parameters	Optimal policies	-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	7.8306	7.8292	7.8277	7.8262	7.8248	7.8233	7.8218
	S^*	16.594	16.5927	16.5914	16.5901	16.5888	16.5875	16.5862
	Q^*	16.0302	16.0257	16.0213	16.0168	16.0123	16.0079	16.0034
	P^*	47.8934	46.6495	45.4056	44.1617	42.9178	41.6739	40.4301
C	t_1^*	7.8873	7.867	7.8466	7.8262	7.8058	7.7852	7.7647
	S^*	16.5924	16.5916	16.5908	16.5901	16.5893	16.5886	16.5879
	Q^*	16.2024	16.1405	16.0787	16.0168	15.9549	15.8931	15.8312
	P^*	46.3622	45.6224	44.889	44.1617	43.4408	42.726	42.0175

h	t_1^*	7.8394	7.835	7.8306	7.8262	7.8218	7.8175	7.8131
	S^*	16.6335	16.619	16.6046	16.5901	16.5756	16.5612	16.5467
	Q^*	16.0567	16.0434	16.0301	16.0168	16.0035	15.9903	15.9771
	P^*	43.82	43.933	44.0469	44.1617	44.2774	44.394	44.5114
α	t_1^*	7.7749	7.7974	7.8138	7.8262	7.8359	7.8435	7.8497
	S^*	16.5838	16.5863	16.5883	16.5901	16.5916	16.5929	16.5941
	Q^*	20.4351	18.7126	17.2589	16.0168	14.9435	14.0068	13.1821
	P^*	38.0591	40.4352	42.4433	44.1617	45.6483	46.9473	48.0924
γ	t_1^*	7.8392	7.8354	7.8311	7.8262	7.8207	7.8145	7.8073
	S^*	16.5914	16.591	16.5906	16.5901	16.5896	16.589	16.5884
	Q^*	15.0054	15.3232	15.6597	16.0168	16.3966	16.8013	17.2335
	P^*	45.5209	45.0933	44.6411	44.1617	43.6523	43.1101	42.5318
θ	t_1^*	7.826	7.8261	7.8261	7.8262	7.8263	7.8264	7.8265
	S^*	16.5863	16.5876	16.5888	16.5901	16.5913	16.5925	16.5937
	Q^*	16.016	16.0163	16.0165	16.0168	16.0171	16.0173	16.0176
	P^*	44.1887	44.1796	44.1706	44.1617	44.1529	44.1442	44.1355
η	t_1^*	7.826	7.826	7.8261	7.8262	7.8263	7.8264	7.8265
	S^*	16.5862	16.5875	16.5888	16.5901	16.5914	16.5926	16.5939
	Q^*	16.016	16.0163	16.0165	16.0168	16.0171	16.0174	16.0177
	P^*	44.1894	44.1801	44.1709	44.1617	44.1527	44.1436	44.1347

Table 2(a) Sensitivity Analysis of the Model – Without Shortages

Variation Parameters	Optimal policies	-15%	-10%	-5%	0%	5%	10%	15%
b	t_1^*	7.8432	7.8372	7.8316	7.8262	7.8212	7.8165	7.812
	S^*	17.5178	17.2164	16.907	16.5901	16.2664	15.9367	15.5967
	Q^*	16.0683	16.0501	16.033	16.0168	16.0016	15.9874	15.9744
	P^*	58.5317	53.4649	48.6749	44.1617	39.9233	35.9559	32.9889
a	t_1^*	7.7958	7.8057	7.8158	7.8262	7.8369	7.8479	7.8592
	S^*	15.6196	15.9553	16.279	16.5901	16.888	17.1722	17.4422
	Q^*	15.925	15.9546	15.9852	16.0168	16.0493	16.0826	16.117
	P^*	18.5274	26.7998	35.3482	44.1617	53.2288	62.5369	72.0719

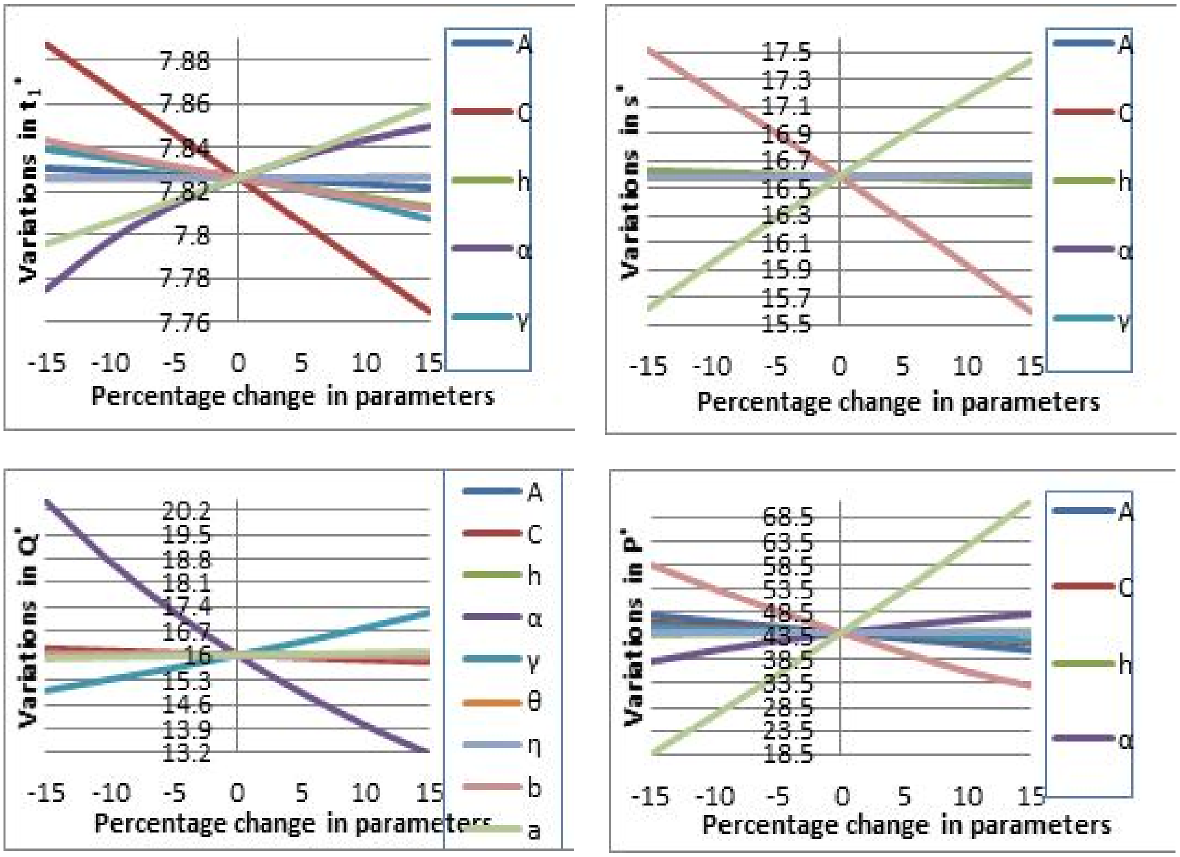


Fig 4: Relationship between parameters and optimal values with shortages

increases and the optimal production downtime t_1^*, the optimal selling price s^* and the profit rate P^* are decreasing. As deteriorating parameter θ decreases, the profit rate increases and the optimal production downtime t_1^*, the optimal production quantity Q^* and the optimal selling price s^* are decreasing. As another deteriorating parameter 'η' decreases, the profit rate P^* increases and the optimal production downtime t_1^*, the optimal production quantity Q^* and the optimal selling price s^* are decreasing. As demand parameter 'a' decreases, the optimal production downtime t_1^*, the optimal selling price s^* the optimal production quantity Q^* and the profit rate P^* are decreasing. As another demand parameter 'b' decreases, the optimal production downtime t_1^*, the optimal selling price s^* the optimal production quantity Q^* and the profit rate P^* are increasing.

Conclusions

This paper addresses the derivation of optimal ordering policies of an EPQ model with the assumption that the production process is random and follows a generalized Pareto distribution. Further it is assumed that the life time of the commodity is random and follows a Weibuull distribution. The generalized Pareto distribution characterizes the production process more close to the reality. The Weibull rate deterioration can include increasing/decreasing/constant rates of deterioration for different values of parameters. The sensitivity analysis of the model reveals that the production distribution parameters have significant influence on the optimal values of the production uptime, production downtime, production quantity and profit. The deterioration distribution parameter also influencing the optimal values of the model. The production and deterioration distribution parameters can be estimated by using historical data. The production manager can obtain the optimal production downtime and uptime, by estimating the parameters and costs. It is also observed that the model with shortages has less production cost per a unit time than that of without shortages. This model also includes some of the earlier models as particular cases for specific or limiting values of the parameters. This model can be extended for the cases of changing money value (inflation) and multicommodity production systems which will be taken elsewhere.

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