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Some Curvature Properties of LP-Sasakian Manifold with Respect to Quarter Symmetric Non Metric Connection

N.V.C. SHUKLA[1] and RAM ASRAY VERMA[2]

Department of Mathematics & Astronomy, University of Lucknow, Lucknow, 226007, UP (India)
Department of Mathematics & Astronomy, University of Lucknow, Lucknow, 226007, UP (India)
Corresponding Author E-mail address: ramasray1986@gmail.com_

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Abstract

The object of the present paper is to study of various curvature tensor on an Lorentzian para-Sasakian manifold with respect to quarter-symmetric non-metri connection.

Key words : Lorentzian para-Sasakian manifold, Weyl Conformal curvature tensor, Conharmonic curvature tenso, m-Projective curvature tensor, Quasi-Conformal curvature tensor Pseudo W2- curvature tensor.

Subject Classification Code:- 2010 Mathematics 53C07, 53C25.

1 Introduction

In 1975, Golab[27] introduced the notion of quarter – symmetric connection in a Riemannian manifold with affine connection. This was further developed by Yano and Imai[16], Mishra and Pandey[25], Mukhopadhyay, Roy and Barua[28], Biswas and De[26], Sengupta and Biswas[15], Singh and Pandey[24] and many other geometers. In this paper we define and study a quarter – symmetric non – metric connection on an Lorentzian para – Sasakian manifold. Section 1, is introductory. In section 2, we give the definition of Lorentzian para – Sasakian . In section 3, we define a quarter – symmetric non – metric connection. In section 4, Quasi-Conformal curvature tensor of the quarter – symmetric non – metric connection is Lorentzian para – Sasakian manifold is studied. In section 5, m-Projective curvature tensor of the quarter – symmetric non – metric connection is Lorentzian para – Sasakian manifold is studied. In section 6, Weyl-Conformal curvature tensor of the quarter – symmetric non – metric connection is Lorentzian para – Sasakian manifold is studied . In section 7, Psuedo $W_2 -$ curvature tensor of the quarter – symmetric non – metric connection is Lorentzian para – Sasakian manifold is studied and finally In section 8,Conhormonic curvature tensor of the quarter – symmetric non – metric connection is Lorentzian para – Sasakian manifold is studied.

2 Preliminaries :

An $n-$ dimensional differential manifold M^n is a Lorentzian para – Sasakian (LP – Sasakian) manifold if it admits a $(1,1)-$ tensor field ϕ, contravariant vector field ξ, a covariant vector field η, and a Lorentzian metric g, which satisfy

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = -1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\xi) = \eta(X), \tag{2.4}$$

$$(\nabla_X \phi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

$$\nabla_X \xi = \phi X, \tag{2.6}$$

for any vector fields X and Y, where ∇ denotes covariant differentiation with respect to g [17,4] and [18]

In an LP – Sasakian manifold M^n with structure (ϕ,ξ,η,g), it is easily seen that

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \ \ rank(\phi) = (n-1), \tag{2.7}$$

$$F(X,Y) = g(\phi X, Y), \tag{2.8}$$

Then the tensor field F is symmetric $(0,2)$ tensor field

$$F(X,Y) = F(Y,X), \tag{2.9}$$

$$F(X,Y) = (\nabla_X \eta)(Y). \tag{2.10}$$

Definition 1: The quasi – conformal curvature tensor is defined as [22],

$$\begin{aligned} C(X,Y,Z) = {} & aR(X,Y,Z) \\ & + b[S(Y,Z)X - S(X,Z)Y] \\ & + b[g(Y,Z)QX - g(X,Z)QY] \\ & - \frac{r}{(2n+1)}\left[\frac{a}{2n} + b\right][g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{2.11}$$

Definition 2: The M-projective curvature tensor (w^*) is defined as [22],

$$\begin{aligned} w^*(X,Y,Z) = {} & R(X,Y,Z) \\ & - \frac{1}{2(n-1)}[S(Y,Z)X - S(X,Z)Y] \end{aligned} \tag{2.12}$$

$$-\frac{1}{2(n-1)}[g(Y,Z)QX-g(X,Z)QY].$$

***Definition 3:** The conhormonic curvature tensor* (C^*) *is defined as* [23],

$$C^*(X,Y,Z)=R(X,Y,Z) \tag{2.13}$$

$$-\frac{1}{(n-2)}[S(Y,Z)X-S(X,Z)Y]$$

$$-\frac{1}{(n-2)}[g(Y,Z)QX-g(X,Z)QY].$$

Definition 4 : The Weyl conformal curvature tensor κ *is defined as* [22],

$$\kappa(X,Y,Z)=R(X,Y,Z) \tag{2.14}$$

$$-\frac{1}{n-2}[g(Y,Z)QX-g(X,Z)QY$$

$$+S(Y,Z)X-S(X,Z)Y]$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y].$$

Definition 5 : Pseudo W_2 *curvature tensor* W_2 *is defined as* [22],

$$W_2(X,Y,Z)=a_2R(X,Y,Z) \tag{2.15}$$

$$+b_2[g(Y,Z)QX-g(X,Z)QY]$$

$$-\frac{r}{n}\left[\frac{a_2}{n-1}+b_2\right][g(Y,Z)X-g(X,Z)Y].$$

3 Quarter – symmetric non – metric Connection in an LP – Sasakian manifold :

Let (M^n,g) be an LP – Sasakian manifold with Levi – Civita connection ∇ we define linear connection $\overline{\nabla}$ on M^n by

$$\overline{\nabla}_XY=\nabla_XY-\eta(X)\phi Y-\eta(X)Y-\eta(Y)X+g(X,Y)\xi, \tag{3.1}$$

for all vector fields X and Y

Using (3.1) the torsion tensor $\overline{T}$ of M^n with respect to the connection $\overline{\nabla}$ is given by

$$T(X,Y)=\eta(Y)\phi X-\eta(X)\phi Y. \tag{3.2}$$

A linear connection satisfying (3.2) is called quarter – symmetric connections[1,2,3,6,9]

further Using (3.1) we have

$$(\overline{\nabla}_Xg)(Y,Z)=2\eta(X)g(Y,Z)+2\eta(X)g(\phi Y,Z) \tag{3.3}$$

A linear connection ∇ defined by (3.2) satisfies (3.2) and (3.3) is called quarter – symmetric non – metric connection.

Let $\overline{\nabla}$ be a linear connection in M^n given by

$$\overline{\nabla}_X Y = \nabla_X Y + H(X,Y) \tag{3.4}$$

Now we shall determine the tensor field H such that $\overline{\nabla}$ satisfies (3.2) and (3.3)

From (3.4), we have

$$T(X,Y) = H(X,Y) - H(Y,X) \tag{3.5}$$

Denote

$$G(X,Y,Z) = (\overline{\nabla}_X g)(Y,Z) \tag{3.6}$$

From (3.4) and (3.6) we have

$$g(H(X,Y),Z) + g(H(X,Z),Y) = -G(X,Y,Z) \tag{3.7}$$

From (3.4), (3.6), (3.7) and (3.3) we have

$$\begin{aligned} g(\overline{T}(X,Y),Z) + g(\overline{T}(Z,X),Y) + g(\overline{T}(Z,Y),X) &= g(H(X,Y),Z) - g(H(Y,X),Z) \\ &\quad + g(H(Z,X),Y) - g(H(X,Z),Y) \\ &\quad + g(H(Z,Y),X) - g(H(Y,Z),X) \\ &= g(H(X,Y),Z) + G(X,Y,Z) \\ &\quad + G(Y,X,Z) - G(Z,X,Y) \\ &= 2g(H(X,Y),Z) \\ &\quad + 2\eta(X)g(Y,Z) + 2\eta(X)g(\phi Y,Z) \\ &\quad + 2\eta(Y)g(X,Z) + 2\eta(Y)g(\phi X,Z) \\ &\quad + 2\eta(Z)g(X,Y) + 2\eta(Z)g(\phi X,Y) \end{aligned}$$

OR

$$\begin{aligned} H(X,Y) = \frac{1}{2}\left\{\overline{T}(X,Y) + \overline{T}^{\circ}(X,Y) + \overline{T}^{\circ}(Y,X)\right\} \\ -\eta(X)Y - \eta(Y)X + g(X,Y)\xi \\ -\eta(X)\phi Y - \eta(Y)\phi X + g(\phi X,Y)\xi \end{aligned} \tag{3.8}$$

Where $\overline{T}$ be a tensor field of type $(1,2)$ defined by

$$g(\overline{T}^{\circ}(X,Y),Z) = g(\overline{T}(Z,X),Y) \tag{3.9}$$

From (3.2) and (3.9) we have

$$\overline{T}^{\circ}(X,Y) = \eta(X)\phi Y - g(\phi X,Y)\xi \tag{3.10}$$

Using (3.2) and (3.10) in (3.8) we get

$$H(X,Y) = -\eta(X)\phi Y - \eta(X)Y - \eta(Y)X + g(X,Y)\xi \tag{3.11}$$

This implies

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y - \eta(X)Y - \eta(Y)X + g(X,Y)\xi \tag{3.12}$$

Theorem 1 : Let (M^n, g) be an LP – Sasakian manifold with almost Lorentzian – Para contact metric structure (ϕ, ξ, η, g) admitting a quarter – symmetric non – metric connection $\overline{\nabla}$ which satisfies (3.2) and (3.3) then the quarter–symmetric non – metric connection is given by (3.12)

Curvature tensor of an LP – Sasakian manifold with respect to the Quarter – symmetric non – metric Connection $\overline{\nabla}$

Let $\overline{R}$ and R be the curvature tensor of the connection $\overline{\nabla}$ and ∇ respectively

$$\overline{R}(X,Y)Z = \overline{\nabla}_X \overline{\nabla}_Y Z - \overline{\nabla}_Y \overline{\nabla}_X Z - \overline{\nabla}_{[X,Y]} Z \tag{3.13}$$

:

From (3.13) and (3.1), we get

$$\begin{aligned} \overline{R}(X,Y)Z = {} & \overline{\nabla}_X [\nabla_Y Z - \eta(Y)\phi Z - \eta(Y)Z \\ & -\eta(Z)Y + g(Y,Z)\xi] \\ & -\overline{\nabla}_Y [\nabla_X Z - \eta(X)\phi Z - \eta(X)Z \\ & -\eta(Z)X + g(X,Z)\xi] \\ & -\nabla_{[X,Y]} Z - \eta([X,Y])\phi Z \\ & -\eta([X,Y])Z - \eta(Z)[X,Y] \\ & + g([X,Y],Z)\xi \end{aligned} \tag{3.14}$$

Using (2:5) (2:6) and (3:1) in (3:14)

$$\begin{aligned} \overline{R}(X,Y)Z = {} & R(X,Y)Z + F(Y,Z)X - F(X,Z)Y \\ & + g(Y,Z)\phi X - g(X,Z)\phi Y \\ & + \eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X \\ & + g(Y,Z)X - g(X,Z)Y \\ & + \eta(X)F(Y,Z)\xi - \eta(Y)F(X,Z)\xi \\ & + 2\eta(X)g(Y,Z)\xi - 2\eta(Y)g(X,Z)\xi \end{aligned} \tag{3.15}$$

Where

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z,$$

is the curvature tensor of D with respect to the Riemannian Connection.

Contracting (3.15) we get

$$\overline{S}(Y,Z) = S(Y,Z) + (n-3)g(\phi Y,Z) + (\psi + n - 3)g(Y,Z) - (\psi + 2)\eta(Y)\eta(Z), \tag{3.16}$$

From (3.16)

$$g(\overline{Q}Y,Z) = g(QY,Z) + (n-3)g(\phi Y,Z) + (\psi + n - 3)g(Y,Z) - (\psi + 2)\eta(Y)\eta(Z), \tag{3.17}$$

Contracting (3.17) with respect to Z we get (3.18)

$$\overline{Q}Y = QY + (n-3)\phi Y + (\psi + n - 3)Y - (\psi + 2)\eta(Y)\xi, \tag{3.18}$$

$$\overline{r} = r + 2(n-1)\psi + (n-1)(n-2.) \tag{3.19}$$

where $\overline{S}$ and $\overline{r}$ are the Ricci tensor and scalar curvature with respect to $\overline{\nabla}$ and $\psi = trace\ \phi$

4 Quasi – conformal Curvature tensor on an LP – Sasakian manifold with respect to the Quarter – symmetric non – metric Connection $\overline{\nabla}$:

Let $\overline{C^*}$ and C^* denote the quasi – conformal curvature tensor with respect to connection $\overline{\nabla}$ and ∇ respectively[22],

$$\overline{C^*}(X,Y,Z) = a\overline{R}(X,Y,Z) + b\left[\overline{S}(Y,Z)X - \overline{S}(X,Z)Y\right] + b\left[g(Y,Z)\overline{Q}X - g(X,Z)\overline{Q}Y\right] - \frac{r}{(2n+1)}\left[\frac{a}{2n} + b\right]\left[g(Y,Z)X - g(X,Z)Y\right], \tag{4.1}$$

and

$$C^*(X,Y,Z) = aR(X,Y,Z) + b\left[S(Y,Z)X - S(X,Z)Y\right] + b\left[g(Y,Z)QX - g(X,Z)QY\right] - \frac{r}{(2n+1)}\left[\frac{a}{2n} + b\right]\left[g(Y,Z)X - g(X,Z)Y\right]. \tag{4.2}$$

Using (:15) ; (3:16) & (3:18) in (4:1) , we find

$$\overline{C^*}(X,Y,Z) = C^*(X,Y,Z) + \left[F(Y,Z)X - F(X,Z)Y\right]\left[a + b(n-3)\right] \tag{4.3}$$

$$
\begin{aligned}
&+[g(Y,Z)\phi X - g(X,Z)\phi Y][a+b(n-3)]\\
&+a[\eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X]\\
&+[g(Y,Z)X - g(X,Z)Y][a+2(\psi+n-3)\\
&-\left\{\frac{2(n-1)\psi+(n-1)(n-2)}{2n+1}\right\}\left(\frac{a}{2n}+b\right)]\\
&+a[\eta(X)F(Y,Z)\xi - \eta(Y)F(X,Z)\xi]\\
&+[\eta(X)g(Y,Z)\xi - 2\eta(Y)g(X,Z)\xi][2a-b(\psi+2)]\\
&-b(\psi+2)\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}.
\end{aligned}
$$

If

$$
\begin{aligned}
&[F(Y,Z)X - F(X,Z)Y][a+b(n-3)]\\
&+[g(Y,Z)\phi X - g(X,Z)\phi Y][a+b(n-3)]\\
&+a[\eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X]\\
&+[g(Y,Z)X - g(X,Z)Y][a+2(\psi+n-3)\\
&-\left\{\frac{2(n-1)\psi+(n-1)(n-2)}{2n+1}\right\}\left(\frac{a}{2n}+b\right)]\\
&= -a[\eta(X)F(Y,Z)\xi - \eta(Y)F(X,Z)\xi]\\
&-[\eta(X)g(Y,Z)\xi - 2\eta(Y)g(X,Z)\xi][2a-b(\psi+2)]\\
&+b(\psi+2)\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}
\end{aligned}
\tag{4.4}
$$

then from equation (4.3) we get

$$\overline{C^*}(X,Y,Z) = C^*(X,Y,Z),$$

Hence, we can state the following theorem;

Theorem 2 : In an LP – Sasakian manifold Quasi – conformal Curvature tensor with respect to Riemannian connection is equal to the Quasi – conformal Curvature tensor with respect to Quarter – symmetric non – metric Connection $\overline{\nabla}$ if equation (4.4) holds.

5 m – Projective Curvature tensor on an LP – Sasakian manifold with respect to the Quarter – symmetric non – metric Connection $\overline{\nabla}$:

Let $\overline{w^*}$ and w^* denote the M-projective curvature tensor $(w^*)$with respect to connection $\overline{\nabla}$ and ∇ respectively. thenM-projective curvature tensor (w^*) is defined as[22],

$$\overline{w^*}(X,Y,Z) = \overline{R}(X,Y,Z), \tag{5.1}$$

$$-\frac{1}{2(n-1)}\left[\bar{S}(Y,Z)X-\bar{S}(X,Z)Y\right]$$

$$-\frac{1}{2(n-1)}\left[g(Y,Z)\bar{Q}X-g(X,Z)\bar{Q}Y\right],$$

And

$$w^{*}(X,Y,Z)=R(X,Y,Z)\tag{5.2}$$

$$-\frac{1}{2(n-1)}\left[S(Y,Z)X-S(X,Z)Y\right]$$

$$-\frac{1}{2(n-1)}\left[g(Y,Z)QX-g(X,Z)QY\right],$$

Using $(3.15),(3.153.16)\&\,3.153.163.18$ in (5.1), we find

$$\overline{W^{*}}(X,Y)Z=W^{*}(X,Y)Z+\left\{\frac{n+1}{2(n-1)}\right\}\left[F(Y,Z)X-F(X,Z)Y\right]\tag{5.3}$$

$$+\left\{\frac{n+1}{2(n-1)}\right\}\left[g(Y,Z)\phi X-g(X,Z)\phi Y\right]$$

$$+\left[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X\right]$$

$$+\left(1-\frac{(\psi+n-3)}{2(n-1)}\right)\left[g(Y,Z)X-g(X,Z)Y\right]$$

$$+\left[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi\right]$$

$$+\left(2+\frac{\psi+2}{2(n-1)}\right)\left[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi\right]$$

$$+\frac{(\psi+2)}{2(n-1)}\left[\eta(Y)\eta(Z)X-\eta(X)\eta(Z)Y\right],$$

If

$$\left\{\frac{n+1}{2(n-1)}\right\}\left[F(Y,Z)X-F(X,Z)Y\right]\tag{5.4}$$

$$+\left\{\frac{n+1}{2(n-1)}\right\}\left[g(Y,Z)\phi X-g(X,Z)\phi Y\right]$$

$$+\left[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X\right]$$

$$=-\left(1-\frac{(\psi+n-3)}{2(n-1)}\right)\left[g(Y,Z)X-g(X,Z)Y\right]$$

$$-\left[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi\right]$$

$$-\left(2+\frac{\psi+2}{2(n-1)}\right)\left[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi\right]$$

$$-\frac{(\psi+2)}{2(n-1)}[\eta(Y)\eta(Z)X-\eta(X)\eta(Z)Y].$$

then from equation (5.3) we get

$$\overline{W^*}(X,Y)Z=W^*(X,Y)Z,$$

Hence, we can state the following theorem;

Theorem 3: In an LP — *Sasakian manifold m-Projective Curvature tensor with respect to Riemannian connection is equal to the m-Projective Curvature tensor with respect to Quarter* — *symmetric non* — *metric Connection* $\overline{\nabla}$ *if equation* (??) *holds.*

6 Weyl conformal Curvature tensor on an LP — *Sasakian manifold with respect to the Quarter* — *symmetric non* — *metric Connection* $\overline{\nabla}$ *:*

Let $\overline{\kappa}$ and κ denote the Weyl conformal curvature tensor with respect to connection $\overline{\nabla}$ and ∇ respectively . then

$$\overline{\kappa}(X,Y,Z)=\overline{R}(X,Y,Z)-\frac{1}{n-2}\left[g(Y,Z)\overline{Q}X-g(X,Z)\overline{Q}Y+\overline{S}(Y,Z)X-\overline{S}(X,Z)Y\right]+\frac{\overline{r}}{(n-1)(n-2)}\left[g(Y,Z)X-g(X,Z)Y\right], \tag{6.1}$$

And

$$\kappa(X,Y,Z)=R(X,Y,Z)-\frac{1}{n-2}\left[g(Y,Z)QX-g(X,Z)QY+S(Y,Z)X-S(X,Z)Y\right]+\frac{r}{(n-1)(n-2)}\left[g(Y,Z)X-g(X,Z)Y\right], \tag{6.2}$$

Using (3.15),(3.153.16)(3.153.163.18)&(3.153.163.183.19)in (6.1), we find

$$\begin{aligned}\overline{\kappa}(X,Y)Z=\kappa(X,Y)Z&+\left(\frac{1}{n-2}\right)[F(Y,Z)X-F(X,Z)Y]\\&+\left(\frac{1}{n-2}\right)[g(Y,Z)\phi X-g(X,Z)\phi Y]\\&+[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X]\\&+[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi]\\&+\left(2+\frac{\psi+2}{n-2}\right)[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi]\\&-\left[\frac{n-4}{n-2}\right][\{g(Y,Z)X-g(X,Z)Y\}],\end{aligned} \tag{6.3}$$

If

$$\left(\frac{1}{n-2}\right)[F(Y,Z)X-F(X,Z)Y] \tag{6.4}$$

$$+\left(\frac{1}{n-2}\right)[g(Y,Z)\phi X-g(X,Z)\phi Y]$$

$$+[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X]$$

$$=-[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi]$$

$$-\left(2+\frac{\psi+2}{n-2}\right)[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi]$$

$$+\left[\frac{n-4}{n-2}\right][\{g(Y,Z)X-g(X,Z)Y\}],$$

then from equation (6.3) we get

$$\overline{\kappa}(X,Y)Z=\kappa(X,Y)Z,$$

Hence, we can state the following theorem;

Theorem 4 : In an LP — Sasakian manifold Weyl conformal Curvature tensor with respect to Riemannian connection is equal to the Weyl conformal Curvature tensor with respect to Quarter — symmetric non — metric Connection $\overline{\nabla}$ if equation (6.4) holds.

7 Pseudo W_2 – Curvature tensor on an LP — Sasakian manifold with respect to the Quarter — symmetric non — metric Connection $\overline{\nabla}$:

Let $\overline{W_2}$ and W_2 denote the Pseudo W_2 curvature tensor with respect to connection $\overline{\nabla}$ and ∇ respectively.[22],

$$\overline{W_2}(X,Y,Z)=a_2\overline{R}(X,Y,Z) \tag{7.1}$$

$$+b_2\left[g(Y,Z)\overline{Q}X-g(X,Z)\overline{Q}Y\right]$$

$$-\frac{\overline{r}}{n}\left[\frac{a_2}{n-1}+b_2\right][g(Y,Z)X-g(X,Z)Y],$$

and

$$W_2(X,Y,Z)=a_2R(X,Y,Z) \tag{7.2}$$

$$+b_2[g(Y,Z)QX-g(X,Z)QY]$$

$$-\frac{r}{n}\left[\frac{a_2}{n-1}+b_2\right][g(Y,Z)X-g(X,Z)Y],$$

Using (3.15), $(3.153.16)(3.153.163.18)$ & $(3.153.163.183.19)$ in (7.1), we find

$$\overline{W_2}(X,Y)Z=W_2(X,Y)Z+a_2[F(Y,Z)X-F(X,Z)Y] \tag{7.3}$$

$$+a_2[g(Y,Z)\phi X-g(X,Z)\phi Y]$$

$$+a_2[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X]$$

$$+(2a_{2-}b_2(\psi+2))[2\eta(X)g(Y,Z)\xi-2\eta(Y)g(X,Z)\xi]$$

$$+\left[a_2+b_2(\psi+n-3)-\left[\frac{2(n-1)\psi+(n-1)(n-2)}{n}\right]\right.$$

$$\left.\left\{\frac{a_2}{n-1}+b_2\right\}\{g(Y,Z)X-g(X,Z)Y\}\right],$$

If

$$a_2[F(Y,Z)X-F(X,Z)Y] \tag{7.4}$$
$$+a_2[g(Y,Z)\phi X-g(X,Z)\phi Y]$$
$$+a_2[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X]$$
$$=-(2a_{2\_}b_2(\psi+2))[2\eta(X)g(Y,Z)\xi-2\eta(Y)g(X,Z)\xi]$$
$$-\left[a_2+b_2(\psi+n-3)-\left[\frac{2(n-1)\psi+(n-1)(n-2)}{n}\right]\right.$$
$$\left.\left\{\frac{a_2}{n-1}+b_2\right\}\{g(Y,Z)X-g(X,Z)Y\}\right],$$

then from equation (7.3) we get

$$\overline{W_2}(X,Y)Z=W_2(X,Y)Z,$$

Hence, we can state the following theorem;

Theorem 5: In an LP — Sasakian manifold Pseudo W_2 – Curvature tensor with respect to Riemannian connection is equal to the Pseudo W_2 – Curvature tensor with respect to Quarter — symmetric non — metric Connection $\overline{\nabla}$ if equation (??) holds.

8 Conhormonic Curvature tensor on an LP — Sasakian manifold with respect to the Quarter — symmetric non — metric Connection $\overline{\nabla}$:

Let $\overline{H}$ and H denote the conhormonic curvature tensor with respect to connection $\overline{\nabla}$ and ∇ respectively.[23],

$$\overline{H}(X,Y,Z)=\overline{R}(X,Y,Z) \tag{8.1}$$
$$-\frac{1}{(n-2)}\left[\overline{S}(Y,Z)X-\overline{S}(X,Z)Y\right]$$
$$-\frac{1}{(n-2)}\left[g(Y,Z)\overline{Q}X-g(X,Z)\overline{Q}Y\right],$$

And

$$H(X,Y,Z)=R(X,Y,Z) \tag{8.2}$$
$$-\frac{1}{(n-2)}[S(Y,Z)X-S(X,Z)Y]$$

$$-\frac{1}{(n-2)}\left[g(Y,Z)QX-g(X,Z)QY\right],$$

Using (3.15), (3.153.16)(3.153.163.18) & (3.153.163.183.19) in (8.1), we find

$$\overline{H}(X,Y)Z=H(X,Y)Z+\left\{\frac{1}{n-2}\right\}\left[F(Y,Z)X-F(X,Z)Y\right] \tag{8.3}$$

$$+\left\{\frac{1}{n-2}\right\}\left[g(Y,Z)\phi X-g(X,Z)\phi Y\right]$$

$$+\left[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X\right]$$

$$+\left(1-\frac{(\psi+n-3)}{n-2}\right)\left[g(Y,Z)X-g(X,Z)Y\right]$$

$$+\left[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi\right]$$

$$+\left(2+\frac{\psi+2}{n-2}\right)\left[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi\right]$$

$$+\frac{(\psi+2)}{n-2}\left[\eta(Y)\eta(Z)X-\eta(X)\eta(Z)Y\right],$$

If

$$\left\{\frac{1}{n-2}\right\}\left[F(Y,Z)X-F(X,Z)Y\right] \tag{8.4}$$

$$+\left\{\frac{1}{n-2}\right\}\left[g(Y,Z)\phi X-g(X,Z)\phi Y\right]$$

$$+\left[\eta(X)\eta(Z)\phi Y-\eta(Y)\eta(Z)\phi X\right]$$

$$=-\left(1-\frac{(\psi+n-3)}{n-2}\right)\left[g(Y,Z)X-g(X,Z)Y\right]$$

$$-\left[\eta(X)F(Y,Z)\xi-\eta(Y)F(X,Z)\xi\right]$$

$$-\left(2+\frac{\psi+2}{n-2}\right)\left[\eta(X)g(Y,Z)\xi-\eta(Y)g(X,Z)\xi\right]$$

$$-\frac{(\psi+2)}{n-2}\left[\eta(Y)\eta(Z)X-\eta(X)\eta(Z)Y\right],$$

then from equation (8.3) we get

$$\overline{H}(X,Y)Z=H(X,Y)Z,$$

Hence, we can state the following theorem;

Theorem 6: In an LP – Sasakian manifold conhormonic Curvature tensor with respect to Riemannian connection is equal to the conhormonic Curvature tensor with respect to Quarter – symmetric non – metric

Connection $\overline{\nabla}$ *if equation* (8.4) *holds.*

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