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Unsteady Casson Fluid Flow over Stretching Sheet through Porous Medium with Heat Generation and Viscous Dissipation

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Abstract

In this paper, an unsteady magnetohydrodynamic (MHD) natural convection heat transfer flow of electrically conductive non-Newtonian Casson fluid over a stretching sheet through vertical porous plate with an influence of heat generation and viscous dissipation is investigated. Similarity transformations are employed to transform the governing partial differential equations into ordinary differential equations. The transformed equations are then solved numerically by Runge-Kutta and shooting methods. The heat flow characteristics for different values of the parameters namely Casson fluid parameter, unsteadiness parameter, Eckert number, heat generation parameter, permeability porous parameter and Prandtl number are studied and discussed in detail. The increase of permeability porous parameter reduces the velocity field and enhances the fluid temperature. The heat generation source leads to an increase in thermal boundary layer thickness. The higher viscous dissipative heat causes an increase in the fluid temperature.

Key words: Casson fluid, Unsteady MHD, Stretching sheet with porous medium, Heat generation, Viscous dissipation.

1. Introduction

The dynamics of fluid commenced with imaginary “perfect” or “ideal” fluid that is incompressible and with absence of viscosity or elasticity. The analysis of laminar flow and heat transfer occurring over a stretching

sheet in a viscous fluid is extensively used in numerous industrial applications. De Oliveira and Paiva¹ attempted to model different flows through porous media with the consideration of the classical Navier–Stokes equations and used for various applications in the field of fluid dynamics. Many researchers like Magyari and Keller², Elbashbeshy³, Partha *et al.*⁴, Khan⁵ and Sanjayanand and Khan⁶ focused on heat and mass transfer on boundary layer flow in presence of exponentially and continuous stretching sheet under different impacts. A heat transfer flow magnitude past an exponentially stretching sheet has huge applications. Abd El-Aziz⁷ investigated the mixed convection flow of micro polar fluid past an exponentially stretching sheet. Pal⁸ studied mixed convection flow past an exponentially stretching surface with company of a magnetic field. Recently, Nadeem *et al.*⁹ described the flow of Jeffrey fluid and heat transfer past an exponentially stretching sheet. Muthu and Varunkumar¹⁰ analyzed the steady laminar flow of viscous incompressible fluid through a 2-dimensional non-uniform channel with permeable wall. Ishak¹¹ investigated the combined effects of magnetic field and thermal radiation on heat transfer flow over an exponentially stretching sheet. Sahoo and Poncet¹² examined the effects of slip on third grade fluid past an exponentially stretching sheet.

The simplest non-Newtonian fluid is Maxwell model which can be used to predict the stress relaxation. There is another type of non-Newtonian fluid known as Casson fluid. The rheological model of Casson fluid disbars the tangled effects of shear-dependent viscosity from any boundary layer analysis¹³. The Casson fluid is a shear thinning liquid which is assumed to have an infinitesimal viscosity at zero rate of shear, yield stress below which no flow occurs, and zero viscosity at an infinitesimal rate of shear. If shear stress lesser than yield stress is applied to the fluid, it behaves like a solid, whereas if shear stress more than yield stress is applied, it debuts to move¹⁴. The examples for Casson fluid are jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. The blood of human being can also be treated as Casson fluid. Alimohamadi *et al.*¹⁵ simulated the pulsatile blood flow during carotid bifurcation in presence of external magnetic field. Due to the presence of distinct substances like protein, fibrinogen and globulin in aqueous base plasma, human red blood cells can form a chainlike structure known as aggregates or rouleaux. If the rouleaux behave like a plastic solid, then there exists a yield stress that can be notified with the constant yield stress in Casson fluid¹⁶. Bhattacharya *et al.*¹⁷ reported the slip effects on the boundary layer flow of a Casson fluid over porous stretching sheet. Gin and Daripa¹⁸ premeditated 3-layer immiscible porous media and Hele-Shaw flows.

In the field of fluid mechanics, destruction of undulating velocity gradient due to viscous stress is known as viscous dissipation. This partial irreversible process is often referred to as conversion of kinetic energy into internal energy of the fluid (heating up the fluid due to viscosity since dissipation is greater in the regions with large gradients). Pop¹⁹ introduced the concept of energy dissipation and transport in nano scale structures in the energy efficient circuits and energy conversion systems. Motsumi and Makinde²⁰ examined the effects of viscous dissipation parameter (Eckert number) and diffusion effects on boundary layer flow of Nano-fluids over a moving flat plate. Depending on the admissible grouping of variables (parameterization), Eckert number and Brinkmann number ($P_r \times E_c$) can be used to quantify viscous dissipation. Abd El-Aziz²¹ analyzed the unsteady mixed convection in the flow of air over a semi-infinite stretching sheet taking into account the effect of viscous dissipation. Yasin *et al.*²² reported the effects of viscous dissipation and Joule heating on 2-dimensional stagnation point flow. Animasaun and Aluko²³ noted negligible effect of Eckert number on velocity profiles when dynamic viscosity of air is assumed to vary linearly with temperature. Recently, Haspot²⁴ investigated the mathematical properties of solutions of the Navier-Stokes equations for viscous fluids. Raju and Sandeep²⁵ focused on the motion of Casson fluid over a moving wedge with slip and observed a decrement in the temperature field with rising values of Eckert number. Sueur²⁶ investigated the issue of the

inviscid limit for compressible Navier-Stokes equations in an impermeable domain.

Dissipation of compact spirals owing to molecular viscosity close to the wall is coherent in the non-Newtonian Casson fluid flow over greater flat thermally stratified dissolving plane of a uprising paraboloid. Numerical investigations of micro-polar fluid flow over a nonlinear stretching sheet have amplified the body of cognizance on fluid flow, boundary layer analysis, and heat and mass transfer taking into account the effects of a temperature dependent viscosity²⁷, MHD in the presence of Cattaneo-Christov heat flux²⁸, thermo electromagnetic flow of a viscous fluid in 3D domain²⁹, flow within boundary layer over an exponentially stretching surface embedded in a thermally stratified medium³⁰, electrically conducting laminar fluid flow under MHD phenomenon³¹ and steady natural convection flow in vertical annular micro-channel having viscosity in presence of velocity slip and temperature jump at the annular micro-channel surfaces³². In the recent research, Alam *et al.*³³ concluded that thermal boundary layer would decrease with an increasing temperature dependent viscosity. Jha and Aina³⁴ obtained analytical solution for fully developed mixed convection flow of viscous, incompressible fluid in the of presence transverse magnetic field.

The purpose of this study is to widen the flow and heat transfer analysis in boundary layer flow of a Casson fluid over an exponentially stretching sheet. The converged effects of heat generation and viscous dissipation in presence of porous medium are investigated. These terms are incorporated in temperature and motion equations. Using similarity transformations, a third order ordinary differential equation related to the equation of motion and a second order differential equation related to the energy equation are derived. Numerical calculations of these equations are computed by shooting method approach. Finally, estimation of skin friction coefficient and Nusselt number which are very important in industrial applications are also discussed.

2. Problem Formulation :

A two dimensional and incompressible heat transfer viscous flow of a Casson fluid past a stretching sheet which coincides with the plane $y = 0$ is considered in this study. The fluid flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis, so that the wall is stretched keeping the origin fixed. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as follows;

$$\tau_{i,j} = \begin{cases} e_{i,j} \left(2\mu_\beta + p_y \sqrt{\frac{2}{\pi}} \right), & \pi > \pi_c \\ e_{i,j} \left(2\mu_\beta + p_y \sqrt{\frac{2}{\pi_c}} \right), & \pi < \pi_c \end{cases}$$

Where $\pi = e_{i,j} e_{i,j}$ and $\beta = \frac{\mu_\beta \sqrt{2\pi_c}}{p_y}$. The unsteady fluid and heat flows start at $t = 0$. Sheet emerges out

of a slit at origin (0, 0) and moves with non-uniform velocity $U = \frac{xd}{(1-\alpha t)}$ if $t < \alpha^{-1}$ where $d > 0$ and

$\alpha \geq 0$ is stable with dimension (time)⁻¹. The governing equations such kind of flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\gamma(1+\beta)}{\beta} \frac{\partial^2 u}{\partial y^2} - \frac{\gamma}{k'} u \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k_1 \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) + \frac{\gamma}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

With the boundary conditions,

$$\begin{aligned} u = U(x, t), \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (4)$$

And $T_w = T_\infty + \frac{T_0 dx^2}{2\gamma(1-\alpha t)^{\frac{3}{2}}}$ if $t < \alpha^{-1}$, Introduce $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ & $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ and dimensionless

similarity transformations.

$$\eta = y \sqrt{\frac{d}{\gamma(1-\alpha t)}}, \quad \psi = \sqrt{\frac{\gamma d}{(1-\alpha t)}} x f(\eta), \quad T = \left[T_\infty + \frac{T_0 dx^2}{2\gamma(1-\alpha t)^{\frac{3}{2}}} \right] \theta(\eta) \quad (5)$$

On simplifying the equations (2), (3) and boundary conditions (4) with the help of equation (5) the final nonlinear differential equations with boundary conditions are given by,

$$\left(\frac{1+\beta}{\beta} \right) f''' + \left(f - \frac{E\eta}{2} \right) f'' - (f')^2 - (K + E)f' = 0 \quad (6)$$

$$\frac{1}{P_r} \theta'' + \left(f - \frac{E\eta}{2} \right) \theta' + \left(Q - \frac{3E}{2} - 2f' \right) \theta + E_c (f'')^2 = 0 \quad (7)$$

And the corresponding non-dimensional boundary conditions are

$$\begin{aligned} f' = 1, \quad f = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \\ f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (8)$$

$$\text{Here } E = \frac{\alpha}{d} \quad K = \frac{\gamma(1-\alpha t)}{d k'} \quad P_r = \frac{\gamma}{k_1} \quad Q = \frac{(T_w - T_\infty)(1-\alpha t) Q_0}{T_w \rho d C_p} \quad E_c = \frac{d^2 x^2}{T_w (1-\alpha t)^2 C_p}$$

3. Numerical Solution :

The set of nonlinear ordinary differential equations (6) and (7) with the boundary conditions (8) can be solved by fourth order Runge-Kutta method along with shooting technique for the prescribed parameters like Permeability porous parameter (K), Prandtl number (P_r), Casson fluid parameter (β), unsteadiness parameter (E), heat generation parameter (Q), and Eckert number (E_c). By applying shooting method, the unknown surface conditions $f''(0)$, $f'(0)$, and $\theta'(0)$ can be obtained. A program for the Runge-Kutta technique

can be set up to solve the equations (6) and (7) with the boundary conditions (8). A step size of $\Delta\eta = 0.01$ can be chosen and value of η_∞ can be found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$.

4. Results and Discussion

In order to evaluate the accuracy of this study, a comparison with available results of Sharidan *et al.*³⁵, Chamkha *et al.*³⁶ and Mukhopadhyay *et al.*³⁷ corresponding to the coefficient of skin-friction $f''(0)$ for unsteady viscous incompressible fluid flow is given in Table1.

Table 1: The values of $f''(0)$ for different values of unsteadiness parameter E for unsteady viscous fluid ($Pr = 0.7, Ec = Q = K = 0, \beta \rightarrow \infty$).

E	Sharidan <i>et al.</i>	Chamka <i>et al.</i>	Mukhopadhyay <i>et al.</i>	Present study
0.8	-1.261042	-1.261512	-1.261479	-1.261043
1.2	-1.377722	-1.378052	-1.377850	-1.377725

To analyze the behavior of velocity and temperature profiles for Casson fluid, acatalectic numerical computation is carried out for various values of the parameters which describe the flow characteristics and the results are discussed in the following sections.

4.1 Effects of Casson fluid parameter :

The velocity and temperature profiles for different value of Casson fluid parameter (β) with $Pr = 0.7, E = 0.2, Q = 0.3, K = 0.5, Ec = 0.2$ are shown in the Figure 1 and Figure 2 respectively. As β increases, velocity is found to be decreased. The decreasing yielding stress suppresses the velocity profile, which means that the fluid behaves as Newtonian fluid since Casson fluid parameter becomes large. Also, the momentum boundary layer thickness decreases as β increases. From Figure 2, it is noticed that the effect of increasing β leads to enhance the temperature profile. The thickening of the thermal boundary layer occurs due to increase in elasticity stress parameter.

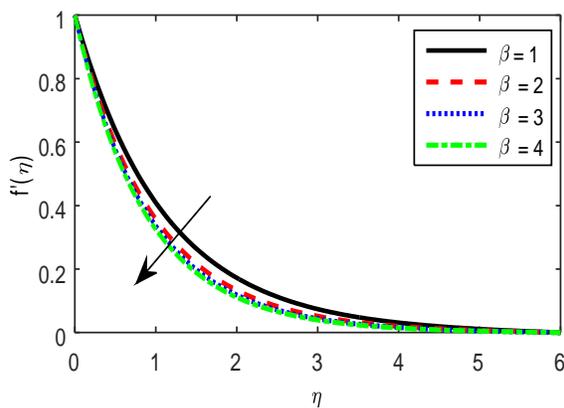


Figure 1: Effects of β on Velocity

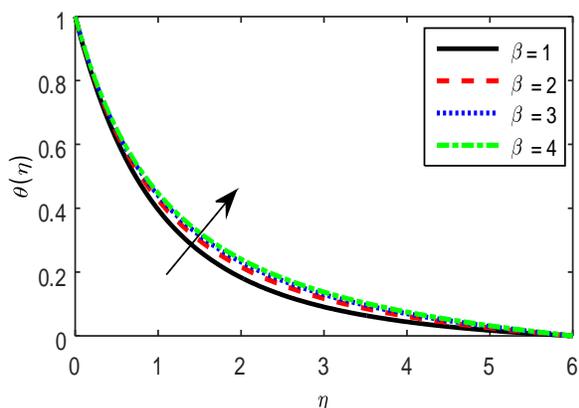


Figure 2: Effects of β on Temperature

4.2 Effects of unsteadiness parameter

The effect of unsteadiness parameter (E) on velocity and temperature profiles is represented in Figure 3 and Figure 4 respectively. As E increases, velocity decreases and this implies that an accompanying reduction of the thickness of the momentum boundary layer near the wall but away from the wall fluid velocity rises with increasing steadiness. From Figure 4, it is noticed that temperature also decreases as E increases, since fluid flow is caused solely by the stretching sheet. The sheet surface temperature is more than free stream temperature. So, it is essential to note that the rate of cooling is much faster for higher values of E whereas it takes more time for cooling in the steady flows.

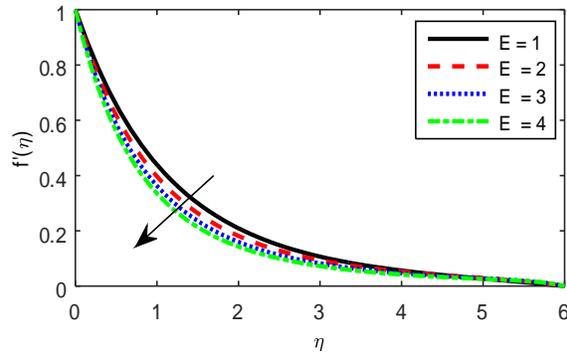


Figure 3: Effects of E on Velocity

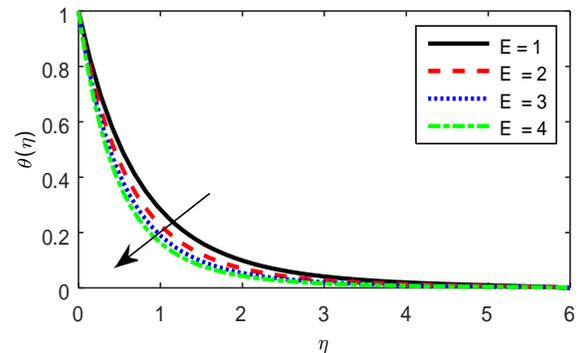


Figure 4: Effects of E on Temperature

4.3 Effects of permeability porous parameter :

The influence of permeability porous parameter (K) on velocity and temperature profiles is represented in Figure 5 and Figure 6 respectively. As K increases, velocity decreases and temperature increases. The effect of increasing values of K opposes the flow in the boundary layer region, which results in more heat transfer from the sheet to the fluid. This is because of presence of the porous medium to increase the resistance to the fluid motion. This causes the fluid velocity to decrease, due to which there is rise in the temperature in the boundary layer.

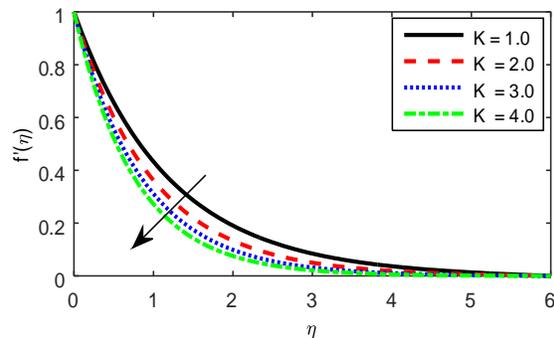


Figure 5: Effects of K on Velocity

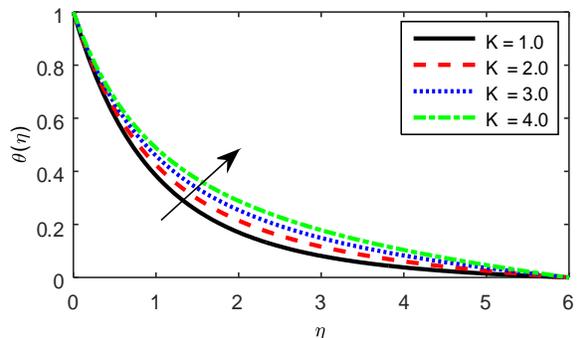


Figure 6: Effects of K on Temperature

4.4 Effects of Prandtl number, heat generation parameter and Eckert number :

It is observed from Figure 7 displays that the temperature decreases as Prandtl number (Pr) increases, which means that an increase in Prandtl number reduces the thermal boundary layer thickness. This is because

of the fact that as Pr decreases, the thickness of the thermal boundary layer becomes higher than the thickness of the velocity boundary layer according to the well-known relation as given in the equation (9).

$$\frac{\delta_v}{\delta_T} \approx Pr \tag{9}$$

where ‘ δ_v ’ is the thickness of the velocity boundary layer and ‘ δ_T ’ is the thickness of the thermal boundary layer. Hence, the thickness of the thermal boundary layer increases as Pr decreases, and therefore temperature profile decreases with increase of Pr .

The temperature profile for different values of heat generation parameter (Q) is portrayed in Figure 8.

It is observed that for increasing value of Q , the temperature exponentially increases. It is noticed that the internal heat generation enhances the heat transport and hence the heat generation source leads to a larger thermal diffusion layer which may increase thermal boundary layer thickness.

The temperature profile for the various values of Eckert number (Ec) with is shown in Figure 9. The renovation of kinetic energy into internal energy by work done in opposition to the viscous fluid stresses is integrated by the Ec . The positive Ec number implies the cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes an increase in the fluid temperature as well as the fluid velocity.

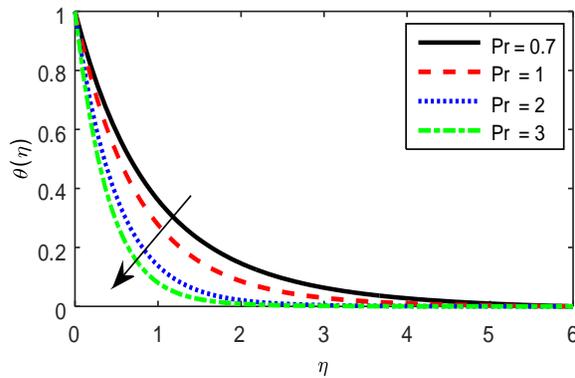


Figure 7: Effects of Pr on Temperature

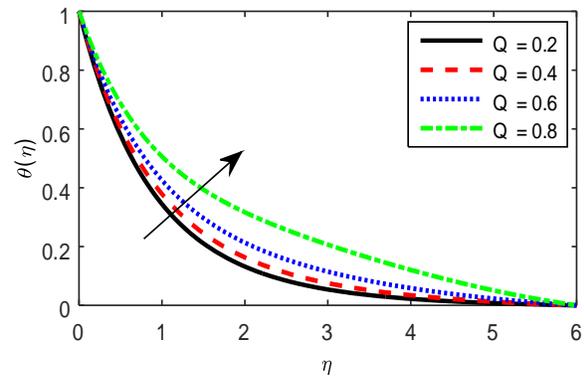


Figure 8: Effects of Q on Temperature

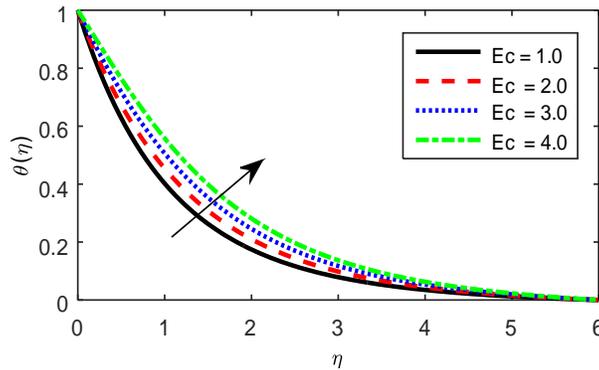


Figure 9: Effects of Ec on Temperature

From the Table 2, it is noted that skin friction coefficient decreases with the increasing values of K , β and E and no effect of Ec , Q and Pr . Nusselt number decreases with an increase in K , Ec , β and Q but the opposite trend is observed with an increase in Pr and E .

Table 2: Numerical values of $-f''(0)$ and $-\theta'(0)$ for various values of the flow parameters

K	Ec	β	Q	Pr	E	$-f''(0)$	$-\theta'(0)$
0.2	0.2	0.5	0.3	0.7	0.2	-0.671148	1.112167
0.4	0.2	0.5	0.3	0.7	0.2	-0.718659	1.095466
0.6	0.2	0.5	0.3	0.7	0.2	-0.763405	1.079508
0.5	0.1	0.5	0.3	0.7	0.2	-0.741351	1.103714
0.5	0.2	0.5	0.3	0.7	0.2	-0.741351	1.087398
0.5	0.3	0.5	0.3	0.7	0.2	-0.741351	1.071082
0.5	0.2	0.3	0.3	0.7	0.2	-0.618316	1.129693
0.5	0.2	0.6	0.3	0.7	0.2	-0.786007	1.071778
0.5	0.2	0.9	0.3	0.7	0.2	-0.882963	1.037507
0.5	0.2	0.5	0.2	0.7	0.2	-0.741351	1.125480
0.5	0.2	0.5	0.4	0.7	0.2	-0.741351	1.046658
0.5	0.2	0.5	0.6	0.7	0.2	-0.741351	0.953010
0.5	0.2	0.5	0.3	1.0	0.2	-0.741351	1.336908
0.5	0.2	0.5	0.3	2.0	0.2	-0.741351	1.966166
0.5	0.2	0.5	0.3	3.0	0.2	-0.741351	2.446008
0.5	0.2	0.5	0.3	0.7	0.2	-0.741351	1.087398
0.5	0.2	0.5	0.3	0.7	0.4	-0.773578	1.167906
0.5	0.2	0.5	0.3	0.7	0.6	-0.804979	1.242390

5. Conclusion

The numerical solutions for unsteady natural convection heat transfer flow of a Casson fluid over a stretching sheet through porous medium in presence of heat generation and viscous dissipation are given in this study. The velocity boundary layer thickness decreases and thermal boundary layer thickness increases with the increase of Casson fluid parameter. The velocity and temperature of Casson fluid decrease significantly with an increase in unsteadiness parameter. The effect of increasing permeability porous parameter suppresses the velocity field whereas enhances the temperature. An increase in Prandtl number decreases the temperature so that this parameter can be used to increase the rate of cooling in conducting flows. The heat generation enhances the heat transport and heat generation source leads to increase thermal boundary layer thickness. The higher viscous dissipative heat causes an increase in the fluid temperature.

Nomenclature

E	- Unsteadiness parameter
K	- Permeability porous parameter
Pr	- Prandtl number
Q	- Heat generation parameter

β	- Casson fluid parameter
Ec	- Eckert number
$-f''(0)$	- Skin friction coefficient
$-\theta'(0)$	- Nusselt number
u, v	- Velocity components in x and y directions
T	- Temperature component
γ	- The fluid's kinematic viscosity
k_1	- The fluid's thermal diffusivity
k'	- Permeability porous parameter in dimension form
C_p	- Specific heat
Q_0	- Heat generation parameter in dimensional form
T_0	- Reference temperature
T_∞	- Free stream temperature (constant)
ψ	- Stream function
$\tau_{i,j}$ and $e_{i,j}$	- (i, j) th components of the stress tensor and deformation rate
π_c	- Critical value of π based on the non-Newtonian model
p_y	- Fluid's yield stress
μ_β	- Plastic dynamic viscosity of the non-Newtonian fluid.

Scope of future work:

We now present some informal observations pertaining to possible extension of this paper to different problems. We briefly outline some interesting problems which can be taken up in the future.

The heat and mass transfer over an irregular surfaces very significant. Because it is found in many practical real time applications. To the best our knowledge and understanding, the studies pertaining to non-Newtonian fluid flow over an unsteady cone have not received much attention. We can formulate many research problems with various flow configurations. For example some of them are listed below.

- Carreau fluid flow over wedge with variable fluid properties.
- Non-Newtonian fluid flow over cone with gyrotactic microorganisms and flux conditions.
- Heat and mass transfer investigations of magnetohydrodynamic Casson fluid flow over a rotating cone in a rotating frame with heat lux and wall temperature conditions.
- Free convection of MHD non-Newtonian over rotating plate with gyrotactic microorganism and flux conditions.
- Heat and mass transfer on magnetohydrodynamic Jeffrey fluid flow over a rotating cone or plate in a

rotating frame with Darcy's law porosity and cross diffusion.

- Free convection of MHD non-Newtonian over rotating plate with partial slip, gyrotactic microorganism and flux conditions.

In our ongoing study and investigations, we have observed that all the problems which we have proposed above may be solved by using Keller box method, Homotopy analysis method, and implicit finite difference scheme of Crank-Nicolson type and Runge-Kutta with shooting technique.

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