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A note on absolute indexed Riesz Summability factor

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Abstract

In this paper we have established $\left|\overline{N}, p_n, \alpha_n(\delta), \mu\right|_k$, $k \geq 1, \delta \geq 0$, summability faotor using f - power increasing sequence. by extending the result of Paikray *et al.*, who have proved $\left|\overline{N}, p_n; \delta\right|_k, k \geq 1, \delta \geq 0$, summability factor of an infinite series using f - power increasing sequence.

Key words : Quasi-increasing; quasi - f - power increasing; indexed absolute Summability;summability factor.

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1. Introduction

A sequence (a_n) of positive numbers is said to be almost increasing if there exists a positive sequence (b_n) and two positive constants A and B such that

$$Ab_n \leq a_n \leq Bb_n, \text{ for all } n \in N. \tag{1.1}$$

For $0 < \beta < 1$, it is said to be quasi-β-power increasing, if there exists a constant K depending upon β with $K \geq 1$ such that

$$K n^{\beta} a_n \geq m^{\beta} a_m \text{ , for all } n \geq m \text{ .} \tag{1.2}$$

In particular if $\beta = 0$, then (a_n) is a quasi-increasing sequence. It is clear that for any non-negative β, every almost increasing sequence is a quasi-β-power increasing sequence. But the converse is not true in general, as is $(n^{-\beta})$ quasi-β-power increasing but not almost increasing.

Let $f = (f_n)$ be a positive sequence of numbers. Then the positive sequence (a_n) is said to be quasi-f-power increasing, if there exists a constant K depending upon f with $K \geq 1$ such that

$$K f_n a_n \geq f_m a_m \tag{1.3}$$

for $n \geq m \geq 1$. Clearly, if (a_n) is a quasi-f-power increasing sequence, then $(a_n f_n)$ is also a quasi- increasing sequence.

Let $\sum a_n$ be an infinite series with sequence of partial sums $\{s_n\}$. Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty, \text{ as } n \to \infty .$$

Then the sequence-to-sequence transformation

$$T_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \text{ , } P_n \neq 0, \tag{1.4}$$

defines the $(\overline{N}, p_n)$- mean of the sequence (s_n) generated by the sequence of coefficients (p_n). The series $\sum a_n$ is said to be summable $\left|\overline{N}, p_n\right|_k$, $k \geq 1$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{k-1} \left| T_n - T_{n-1} \right|^k < \infty \text{ .} \tag{1.5}$$

The $\sum a_n$ series is said to be summable $\left|\overline{N}, p_n; \delta\right|_k$, $k \geq 1, \delta \geq 0,$ if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{\delta k + k - 1} \left| T_n - T_{n-1} \right|^k < \infty \text{ .} \tag{1.6}$$

Putting $\delta = 1$, the summability method $\left|\overline{N}, p_n; \delta\right|_k$, $k \geq 1, \delta \geq 0$ reduces to the summabillity method $\left|\overline{N}, p_n; \delta\right|_k$, $k \geq 1$.

The series $\sum a_n$ is said to be summable $\left|\overline{N}, p_n, \alpha_n(\delta)\right|_k$, $k \geq 1, \delta \geq 0,$ if

$$\sum_{n=1}^{\infty} (\alpha_n)^k \left(\frac{P_n}{p_n} \right)^{k-1} \left| T_n - T_{n-1} \right|^k < \infty \text{ .} \tag{1.7}$$

Putting $\alpha_n = \left(\frac{P_n}{p_n}\right)^{\delta}$, the summability method $\left|\bar{N}, p_n, \alpha_n(\delta)\right|_k, k \geq 1, \delta \geq 0$ reduces to the summabillity method $\left|\bar{N}, p_n; \delta\right|_k, k \geq 1, \delta \geq 0$

For any real number μ, the series $\sum a_n$ is said to be summable by the summabilty method $\left|\bar{N}, p_n, \alpha_n(\delta), \mu\right|_k$, $k \geq 1, \delta \geq 0$, if

$$\sum_{n=1}^{\infty} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{\mu(k-1)} \left|T_n - T_{n-1}\right|^k < \infty . \tag{1.8}$$

Putting $\mu = 1$, the summabilty method $\left|\bar{N}, p_n, \alpha_n(\delta), \mu\right|_k$ reduces to the summability method $\left|\bar{N}, p_n, \alpha_n(\delta)\right|_k$.

2. Known Theorems :

Dealing with quasi-β-power increasing sequence Bor and Debnath[2] have established the following theorem:

Theorem 2.1:

Let (X_n) be a quasi-β-power increasing sequence for $0 < \beta < 1$ and (λ_n) be a real sequence.If the conditions

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m), \tag{2.1.1}$$

$$\lambda_n X_n = O(1) \tag{2.1.2}$$

$$\sum_{n=1}^{m} \frac{|t_n|^k}{n} = O(X_m) \tag{2.1.3}$$

$$\sum_{n=1}^{m} \frac{p_n |t_n|^k}{P_n} = O(X_m) \tag{2.1.4}$$

and

$$\sum_{n=1}^{m} n X_n \left|\Delta^2 \lambda_n\right| < \infty \tag{2.1.5}$$

are satisfied, where t_n is the $(C, 1)$ mean of the sequence (na_n).then the series $\sum a_n \lambda_n$ is summable

$\left|\overline{N}, p_n\right|_k, k \geq 1$., where t_n is the $(C, 1)$ mean of the sequence (na_n).
Subsequently Leinder[3] established a similar result reducing certain conditions of Bor and Debnath[2]. He established:

Theorem 2.2:

Let the sequence (X_n) be a quasi-β-power increasing sequence for $0 < \beta < 1$, and (λ_n) be a real sequence satisfying the conditions

$$\sum_{n=1}^{m} \lambda_n = O(m) \tag{2.2.1}$$

and

$$\sum_{n=1}^{m} \left|\Delta\lambda_m\right| = O(m). \tag{2.2.2}$$

Further, suppose the conditions (2.1.3) , (2.1.4) and

$$\sum_{n=1}^{m} nX_n(\beta)\left|\Delta\left|\Delta\lambda_n\right|\right| < \infty, \tag{2.2.3}$$

hold, where $X_n(\beta) = \max\left(n^{\beta} X_n, \log n\right)$. Then the series $\sum a_n\lambda_n$ is summable $\left|\overline{N}, p_n\right|_k, k \geq 1$.

Extending the above results to quasi-f-power increasing sequence, Sulaiman[6] has established the following theorem:

Theorem 2.3:

Let $f = (f_n) = \left(n^{\beta} \log^{\gamma} n\right),\ 0 \leq \beta < 1, \gamma \geq 0$ be a sequence. Let (X_n) be a quasi -f- power sequence and (λ_n) a sequence of constants satisfying the conditions

$$\lambda_n \to 0 \ as\, n \to \infty, \tag{2.3.1}$$

$$\sum_{n=1}^{\infty} nX_n\left|\Delta\left|\Delta\lambda_n\right|\right| < \infty, \tag{2.3.2}$$

$$\left|\lambda_n\right| X_n = O(1), \tag{2.3.3}$$

$$\sum_{n=1}^{\infty} \frac{1}{nX_n^{k-1}}\left|t_n\right|^k = O\left(X_m\right), \tag{2.3.4}$$

and

$$\sum_{n=1}^{\infty} \frac{p_n}{P_n}\frac{1}{X_n^{k-1}}\left|t_n\right|^k = O\left(X_m\right), \tag{2.3.5}$$

where t_n is the $(C, 1)$ mean of the sequence (na_n). Then the series $\sum a_n\lambda_n$ is summable $\left|\overline{N}, p_n\right|_k, k \geq 1$

Recently Paikray *et al.*[4] established the following theorem:

Theorem 2.4:

Let $f = (f_n) = \left(n^{\beta} \log^{\gamma} n\right)$ be a sequence and (X_n) be a quasi-f-power sequence. Let (λ_n) a sequence of constants such that

$$\lambda_n \to 0, \ as \ \ n \to \infty, \tag{2.4.1}$$

$$\sum_{n=1}^{\infty} nX_n \left|\Delta\left|\Delta\lambda_n\right|\right| < \infty, \tag{2.4.2}$$

$$\left|\lambda_n\right| X_n = O(1), \tag{2.4.3}$$

$$\sum_{n=v+1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k-1} \frac{1}{P_{n-1}} = O\left(\frac{P_m}{p_m}\right)^{\delta k-1}, \tag{2.4.4}$$

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k-1} \frac{\left|t_n\right|^k}{X_n^{k-1}} = O\left(X_m\right), \tag{2.4.5}$$

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{\left|t_n\right|^k}{nX_n^{k-1}} = O\left(X_m\right). \tag{2.4.6}$$

Then the series $\sum a_n\lambda_n$ is summable $\left|\overline{N}, p_n; \delta\right|_k, k \geq 1, \delta \geq 0.$

In the mean time Bor[1] has established the following theorem:

Theorem 2.5:

Let $\lambda_n \in BV$. Let (X_n) be quasi -(σ, γ) – power increasing sequence, $\gamma \geq 0, 0 < \sigma < 1$ and there be sequences (β_n) and (λ_n) such that

$$\left|\Delta\lambda_n\right| \leq \beta_n \tag{2.5.1}$$

$$\beta_n \to 0, \ as \ \ n \to \infty, \tag{2.5.2}$$

$$\sum_{n=1}^{\infty} nX_n \left|\Delta\beta_n\right| < \infty \tag{2.5.3}$$

$$\left|\lambda_n\right| X_n = O(1) \tag{2.5.4}$$

If (p_n) is a sequence such that

$$P_n = O(np_n) \tag{2.5.5}$$

$$P_n \Delta p_n = O(P_n P_{n-1}) \tag{2.5.6}$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{|t_n|^k}{n} = O(X_m), \ as \ m \to \infty, \tag{2.5.7}$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k-1} \frac{1}{P_{n-1}} = O\left(\left(\frac{P_\nu}{p_\nu}\right)^{\delta k-1} \frac{1}{P_\nu}\right), \ as \ m \to \infty,. \tag{2.5.8}$$

Then the series $\sum a_n \frac{P_n}{p_n} \lambda_n$ is summable $\left|\overline{N}, p_n; \delta\right|_k, k \geq 1, \frac{1}{k} > \delta \geq 0.$

In what follows in this paper, using quasi -f- power increasing sequence and the reference cited in[5], we prove the following new result on $\left|\overline{N}, p_n, \alpha_n(\delta), \mu\right|_k, k \geq 1, \delta \geq 0$ summability:

3. Main theorem:

Theorem 3.1:

Let $f = (f_n) = \left(n^{\beta} (\log n)^{\gamma}\right)$ be a sequence and (X_n) be a quasi-f-power increasing sequence. Let (λ_n) be a sequence of constants such that

$$\lambda_n \to 0, \ as \ \ n \to \infty, \tag{3.1.1}$$

$$\sum_{n=1}^{\infty} nX_n |\Delta|\Delta\lambda_n|| < \infty \tag{3.1.2}$$

$$|\lambda_n| X_n = O(1), \tag{3.1.3}$$

$$\sum_{n=\nu+1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{1}{P_{n-1}^k} = O\left((\alpha_m)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\right), \tag{3.1.4}$$

$$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m), \tag{3.1.5}$$

$$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{p_n}{P_n}\right)^{(\mu-1)(k-1)} \frac{|t_n|^k}{nX_n^{k-1}} = O(X_m), \tag{3.1.6}$$

where (t_n) is the n th $(C, 1)$ mean of the sequence $(n a_n)$.

Then the series $\sum a_n \lambda_n$ is summable $\left|\overline{N}, p_n, \alpha_n(\delta), \mu\right|_k, k \geq 1, \delta \geq 0$.

In order to prove the theorem, we require the following lemma.

4. Lemma:

Lemma 4.1[3]:

Let $f = (f_n) = \left(n^{\beta} (\log n)^{\gamma}\right), 0 \leq \beta < 1, \gamma \geq 0$, be a sequence and (X_n) be a quasi -*f*-power increasing sequence, then

$$n X_n \Delta\lambda_n = O(1) \tag{4.1.1}$$

and

$$\sum_{n=1}^{m} X_n \Delta\lambda_n < \infty, \text{ as } m \to \infty. \tag{4.1.2}$$

5. Proof of the Theorem 3.1:

Let (T_n) be the sequence of $(\overline{N}, p_n)$ mean of the series $\sum_{n=1}^{\infty} a_n \lambda_n$. Then

$$T_n = \frac{1}{P_n} \sum_{\upsilon=0}^{n} p_{\upsilon} \sum_{r=0}^{\upsilon} a_r \lambda_r$$

$$= \frac{1}{P_n} \sum_{\upsilon=0}^{n} (P_n - P_{\upsilon-1}) a_{\nu} \lambda_{\nu}$$

Hence for $n \geq 1$

$$T_n - T_{n-1} = \frac{p_n}{P_n P_{n-1}} \sum_{\upsilon=1}^{n} P_{\upsilon-1} a_{\nu} \lambda_{\nu}$$

$$= \frac{p_n}{P_n P_{n-1}} \sum_{\upsilon=1}^{n} \nu a_{\nu} \left(\frac{1}{\nu} P_{\nu-1} \lambda_{\nu}\right)$$

$$= \frac{(n+1)}{n} \frac{p_n}{P_n} t_n \lambda_n + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} p_{\nu-1} t_{\nu} \lambda_{\nu} \frac{\nu+1}{\nu} + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_{\nu} t_{\nu} \frac{\nu+1}{\nu} \Delta\lambda_{\nu}$$

$$+ \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_{\nu} t_{\nu} \frac{\lambda_{\nu+1}}{\nu}$$

$$= T_{n1} + T_{n2} + T_{n3} + T_{n4} \text{ (say).}$$

In order to prove the theorem, using Minkowski's inequality it is enough to show that

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,j}\right| < \infty,\ j = 1,2,3,4.$$

Applying Hölder's inequality, we have

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,1}\right|^k$$

$$= \sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{(n+1)P_n}{np_n}t_n\lambda_n\right|^k$$

$$= O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{\left|t_n\right|^k}{X_n^{k-1}}\left(X_n\left|\lambda_n\right|\right)^{k-1}\left|\lambda_n\right|$$

$$= O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{\left|t_n\right|^k}{X_n^{k-1}}\left|\lambda_n\right|$$

$$= O(1)\sum_{n=1}^{m-1}\left(\sum_{\nu=1}^{n}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1}\frac{\left|t_\nu\right|^k}{X_\nu^{k-1}}\right)\Delta\left|\lambda_n\right| + O(1)\left(\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1}\frac{\left|t_\nu\right|^k}{X_\nu^{k-1}}\right)\left|\lambda_m\right|$$

$$= O(1)\sum_{n=1}^{m-1}X_n\,\Delta\left|\lambda_n\right| + O(1)X_m\left|\lambda_m\right|$$

$$= O(1)$$

Next,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,2}\right|^k$$

$$= \sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{p_n}{P_nP_{n-1}}\sum_{\nu=1}^{n-1}p_{\nu-1}t_\nu\lambda_\nu\frac{(\nu+1)}{\nu}\right|^k$$

$$= O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}}\sum_{\nu=1}^{n-1}p_{\nu-1}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k\left(\frac{1}{P_{n-1}}\sum_{\nu=1}^{n-1}p_\nu\right)^{k-1}$$

$$= O(1)\sum_{\nu=1}^{m} p_{\nu-1}|t_\nu|^k|\lambda_\nu|^k \sum_{n=\nu+1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}}$$

$$= O(1)\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1}|t_\nu|^k|\lambda_\nu|^k$$

$$= O(1).$$

Again,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}|T_{n,3}|^k$$

$$= \sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{p_n}{P_n P_{n-1}}\sum_{\nu=1}^{n-1}p_\nu t_\nu\frac{(\nu+1)}{\nu}\Delta\lambda_\nu\right|^k$$

$$= O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^k}\sum_{\nu=1}^{n-1}p_\nu^k\frac{|t_\nu|^k}{X_\nu^{k-1}}|\Delta\lambda_\nu|\left(\sum_{\nu=1}^{n-1}X_\nu|\Delta\lambda_\nu|\right)^{k-1}$$

$$= O(1)\sum_{\nu=1}^{n-1}p_\nu^k\frac{|t_\nu|^k}{X_\nu^{k-1}}|\Delta\lambda_\nu|\sum_{n=\nu+1}^{m+1}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^k}$$

$$= O(1)\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)}\frac{|t_\nu|^k}{\nu X_\nu^{k-1}}(\nu|\Delta\lambda_\nu|)$$

$$= O(1)\sum_{\nu=1}^{m-1}\left(\sum_{r=1}^{\nu}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}\frac{|t_r|^k}{r X_r^{k-1}}\right)(\nu|\Delta\lambda_\nu|) + O(1)(m|\Delta\lambda_m|)\sum_{r=1}^{m}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}$$

$$= O(1)\sum_{\nu=1}^{m-1}X_\nu\left(-|\Delta\lambda_\nu| + (\nu+1)|\Delta|\Delta\lambda_\nu||\right) + O(1)mX_m|\Delta\lambda_m|$$

$$= O(1).$$

Finally,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}|T_{n,4}|^k$$

$$= \sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{\mu(k-1)} \left| \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} p_\nu t_\nu \frac{\lambda_{\nu+1}}{\nu} \right|^k$$

$$= O(1) \sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{1}{P_{n-1}^k} \sum_{\nu=1}^{n-1} \frac{p_\nu}{\nu} |t_\nu|^k |\lambda_\nu|^k \left(\sum_{\nu=1}^{n-1} \frac{p_\nu}{\nu}\right)^{k-1}$$

$$= O(1) \sum_{\nu=1}^{m} \frac{p_\nu}{\nu} |t_\nu|^k |\lambda_\nu|^k \sum_{n=\nu+1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{1}{P_{n-1}}$$

$$= O(1) \sum_{\nu=1}^{m} (\alpha_\nu)^{k\mu} \left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)} \frac{|t_\nu|^k}{\nu X_\nu^{k-1}} \left(X_\nu |\lambda_\nu|\right)^{k-1} |\lambda_\nu|$$

$$= O(1) \sum_{\nu=1}^{m-1} \left\{ \sum_{r=1}^{\nu} (\alpha_r)^{k\mu} \left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)} \frac{|t_r|^k}{r X_r^{k-1}} \right\} |\Delta\lambda_\nu| + O(1) \sum_{r=1}^{m} (\alpha_r)^{k\mu} \left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)} \frac{|t_r|^k}{r X_r^{k-1}}$$

$$= O(1) \sum_{\nu=1}^{m-1} X_\nu |\Delta\lambda_\nu| + O(1)\, m X_m |\Delta\lambda_m|$$

$$= O(1).$$

This completes the proof of the theorem.

6. Corollaries:

Corollary 6.1:

Putting $\mu = 1$ in the conditions of our theorem, the series $\sum a_n \lambda_n$ is summable $\left|\overline{N}, p_n, \alpha_n(\delta)\right|_k$, $k \geq 1, \delta \geq 0.$

Corollary 6.2:

Putting $\mu = 1$ and $\alpha_n(\delta) = \left(\frac{P_n}{p_n}\right)^\delta$ in the conditions of our theorem, the series $\sum a_n \lambda_n$ is summable $\left|\overline{N}, p_n, \delta\right|_k$, $k \geq 1, \delta \geq 0.$

Corollary 6.3:

Putting $\mu = 1$ and $\alpha_n(\delta) = \left(\frac{P_n}{p_n}\right)^\delta$ and $\delta = 0$ in the conditions of our theorem, the series $\sum a_n \lambda_n$ is summable $\left|\overline{N}, p_n\right|_k, \quad k \geq 1.$

7. Conclusion

The present theorem generalizes the theorem of Sulaiman[6] and Paikray[4].

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