Section A

MHD boundary layer stagnation point flow of non-Newtonian Micropolar Nanofluid flow over a permeable vertical plate with chemical reaction effects

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Abstract

The paper presents an investigation of the influence of chemical reaction on steady mixed convection MHD flow of Non-Newtonian micropolar nanofluid past a permeable vertical plate. The effect of Thermophoresis parameter and Brownian motion parameter are incorporated in the energy equation and concentration equation. The governing partial differential equations are converted into a system of ordinary differential equations using similarity transformations. The sysyem of linear equations are solved by using the implicit finite difference method. The effect of different parameters on the dimensionless velocity, temperature and concentration are shown graphically and analyzed. The effect of physical parameters such as material parameter K, suction parameter A, Prandtl number Pr, radiation parameter R, thermophoresis parameter Nt, Brownian motion parameter Nb, chemical reaction parameter δ and magnetic parameter Ha are also being numerically investigated and analyzed.

Key words: MHD, Micropolar nanofluid, Thermophoresis parameter, Brownian motion parameter, radiation and finite difference scheme.

AMS Mathematics Subject Classification: 76A05, 76M20

Introduction

Modeling and analysis of the dynamics of micropolar fluids has been the field of very active research for the last few decades as this class of fluids represents, mathematically, many industrially important fluids such as paints, body fluids, polymers, colloidal fluids, suspension fluids etc. These fluids are defined as fluids

consisting of randomly oriented molecules whose fluid elements undergo translational as well as rotational motions. The theory of a micropolar fluid derives from the need to model the flow of fluids that contain rotating micro-constituents. A micropolar fluid is the fluid with internal structures which coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for granular systems which consist of particles with macroscopic size. Eringen[1] was the first pioneer of formulating the theory of micropolar fluids. His theory introduced new material parameters, an additional independent vector field, the microrotation and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian fluid flow. Although the field of micropolar fluids is rich in literature, some gaps can be observed and needs more study in this field. For instance, Gorla[2], Rees and Bassom[3] investigated the flow of a micropolar fluid over a flat plate. Also, Kelson and Desseaux[4] studied the flow of micropolar fluids on stretching surfaces.

The study of boundary layer flow and heat transfer over a stretching surface particularly in the field of nanofluid has achieved a lot of success in the past years because of its high thermal conductivity and large number of applications in industry and technology. After the pioneering work by Sakiadis, a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces. The problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid was investigated numerically by Khan and Pop. Hassani investigated the boundary layer flow problem of a nanofluid past a stretching sheet analytically. Both the effect of Brownian motion and thermophoresis were considered simultaneously in this case. A numerical investigation on boundary layer flow induced in a nanofluid due to a linearly stretching sheet in the presence of thermal radiation and induced magnetic field was conducted by Gbadeyan *et al.* Srinivas Maripala and Kishan Naikoti[5] investigated the effects of heat source/sink on MHD convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet. Mostafa A.A. Mahmoud[6] studied the thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity on the boundary layer flow and heat transfer of an electrically conducting micropolar nanofluid over a semi infinite continuously stretching sheet with power-law variable variation in the surface temperature in the presence of radiation. Heat and mass transfer have important role in many industrial and technological processes such as manufacturing and metallurgical processes which heat and mass transfer occur simultaneously. The influence of a chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium with heat generation was studied by Mohamed and Abo-Dahab[7]. Recently effect of using micropolar fluid, nanofluid, etc. on flow and heat transfer has been studied by several authors[8-15].

Nanofluid is prepared by mixing nano sized particles in a fluid. The Most commonly used nanoparticles are made of metals, oxides or carbides having sizes up to 100 nm. They are mixed with fluid known as base fluids like water, ethylene glycol or oil. Thermo physical properties of nanofluid are different than the base fluid. Commonly thermal conductivity and viscosity of nanofluids are higher than the base fluid. Hence, the research is carried out to evaluate the effect of nanofluid in enhancement of heat transfer rate in different heat transfer devices. The increase in heat transfer rate mainly depends on type of nanoparticles, size of nanoparticles, shape of nanoparticles, type of base fluid and concentration of nanoparticles in base fluid. Along with the increase in thermal conductivity of the nanofluid, viscosity of the nanofluid also increases, which affects the pumping characteristics of the centrifugal pump used in the heat transfer devices. Many researchers have studied numerically and experimentally the phenomenon of effect of nanofluid on pumping power in different applications.

Nanofluids have become increasingly closer to an engineering reality starting from their initial vision originated more than decades ago[16]. In the last ten years, there has been more attention paid to enhance the convective heat transfer performance of nanofluids[17], due to the recognition in practical applications of nanofluids.

Heat exchangers are widely used in many engineering applications, for example, applications in power production industry, chemical industry, food industry, environmental engineering, waste heat recovery, air conditioning, and refrigeration. For decades, efforts have been made to enhance heat transfer of heat exchangers, reduce the heat transfer time and finally improve energy utilization efficiency. These efforts commonly include passive and active methods such as creating turbulence, extending the exchange surface or the use of a fluid with higher thermophysical properties[18]. The enhancement of thermal conductivity in nanofluids has attracted the interest of many researchers. The Boungiorno[19] model, Kuznetseov and Nield[20] studied the influence of nanoparticles on a natural convection boundary layer flow passing a vertical plate. They considered the temperature and nanoparticle fraction both to be constant along the wall and concluded that the reduced Nusselt number is a decreasing function of the nanofluid numbers . The mixed convection boundary layer flow passing a vertical flat plate embedded in a porous medium filled with a nanofluid was studied by Ahmad and Pop.[21] Furthermore, Eastman *et al.*[22] used pure copper nanoparticles of size less than 10nm and achieved an increase of 40 in thermal conductivity for only 0.3 volume fraction of the solid dispersed in ethyleneglycal. Hwang *et al.*[23] studied a detailed discussion about the effects of thermal conductivities under static and dynamic conditions, energy transfer by nanoparticles dispersion, particles migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on the enhancement of the convective heat transfer coefficient, which are discussed to understand convective heat transfer characteristics of water based nanofluids flowing through a circular tube.

The purpose of study is to investigate the effects of MHD on non-Newtonian micro polar nanofluid flow with radiation, thermophoresis parameter, Brownian motion parameter and chemical reaction. The non-linear boundary value problem arising from the non dimensionalization and local non similarity method is solved by using an implicit finite difference scheme along with Gauss-Seidal method. The C-programming code is run for the solution of the system equations.

Mathematical formulation

Consider the two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar nanofluid implinging perpendicular on a permeable wall and flowing away along the *x*-axis. A uniform magnetic field β_o is applied normal to the walls. And using the boundary layer approximation and neglecting the dissipation, the equation of energy for temperature T with thermophoresis parameter Nt and thermal radiation, the equation of mass for concentration C with Brownian motion parameter Nb and chemical reaction. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar nanofluid are governed by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = (\mu + k)\left(\frac{\partial^2 u}{\partial y^2}\right) + k\frac{\partial N}{\partial y} - \frac{\sigma\beta_0^2}{\rho}u \tag{2}$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2} - \frac{k_f}{j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{\partial q_r}{\partial y} + \iota\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - R^*\left(C - C_\infty\right) \qquad (5)$$

Where N is the microrotation or angular velocity whose direction of rotation is in the *x-y*-plane, μ Is the viscosity of the fluid, ρ is the density, c_p is the specific heat capacity at constant presuure of the fluid, k_f is the thermal conductivity of the fluid, j, γ and k
are the microinertia per unit mass, spin gradient viscosity and vortex viscosity respectively which are assumed to be constant.

The appropriate physical boundary conditions of Eqs. (1)-(5) are

$$u(x,0) = 0; \qquad v(x,0) = -v_0 \qquad N(x,0) = -\eta\frac{\partial u}{\partial y}$$

$$y = \infty: \quad u(x, y) = U(x) = \alpha x, \qquad v(x, y) = 0, \qquad N(x, y) = 0$$

$$y = 0; \qquad T = T_w, \qquad\qquad C = C_w \qquad\qquad (6)$$

$$y = \infty; \qquad T = T_\infty, \qquad\qquad C = C_\infty$$

where η is a constant and $0 \le \eta \le 1$. The case $\eta = 1/2$ indicates the vanishing of the antisymmetric part of the stress tensor and denotes weak concentration of microelements, which will be considered here. Using the transform we have

$$\eta = \sqrt{\frac{\alpha}{v}}\, y \qquad\qquad u = \alpha x f'(\eta), \qquad\qquad v = -\sqrt{\alpha v}\, f(\eta)$$

$$N = \alpha x\sqrt{\frac{\alpha}{v}}\, g(\eta), \qquad g(\eta) = -\frac{1}{2}f''(\eta), \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \qquad\qquad (7)$$

After using the transformation (7), for micropolar fluid, there are two equations in which one is for angular velocity or microrotation and physically it is important in micropolar fluid. In this study, we have two equations

$f(\eta)$ and g(η) which g(η) equal to $-\dfrac{1}{2}f''(\eta)$

Eqns (2) and (3) reduce to the single equation as Eq. (8a)

$$\left(1 + \frac{K}{2}\right)f_4(\eta) + f(\eta)f_3(\eta) - f_2(\eta)^2 - Ha^2 f_2 + 1 = 0 \qquad (8a)$$

$$\theta''\left(1 + \frac{4}{3}R\right) + Prf\theta' + PrN_b\theta'\phi' + PrN_t\theta'^2 = 0 \qquad (8b)$$

$$\phi'' + Scf\phi' + \frac{N_t}{N_b}\theta'' - Sc\delta\phi = 0 \qquad (8c)$$

Subject to the boundary conditions

$$f(0) = A \qquad f'(0) = 0 \qquad f'(\infty) = 1$$

$$\theta(0) = 1 \qquad \theta(\infty) = 0 \qquad (8d)$$

$$\phi(0) = 1 \qquad \phi(\infty) = 0$$

Where prime denotes differentiation with respect to η

$$K = \frac{k}{\mu} \qquad\qquad \text{material parameter}$$

$$\Pr = \frac{\mu c_p}{k} \qquad\qquad \text{Prandtl number}$$

$$Sc = \frac{\mu}{\rho D} \qquad\qquad \text{Schmidt number}$$

$$\delta = \frac{xR^*}{U} \qquad\qquad \text{Chemical reaction parameter}$$

$$R = \frac{4\sigma^* T_\infty^3}{k^* k} \qquad\qquad \text{Radiation parameter}$$

$$Ha = \frac{\sigma \beta_0^2}{\rho \alpha} \qquad\qquad \text{Magnetic parameter}$$

$$N_t = \frac{\tau D_T (T_w - T_\infty)}{v T_\infty} \qquad\qquad \text{Thermophoresis Parameter}$$

$$N_b = \frac{\tau D_B (C_w - C_\infty)}{v} \qquad\qquad \text{Brownian motion Parameter}$$

$$T_w = \left[(\mu + k) \frac{\partial u}{\partial y} + kN \right]_{y=0} \qquad \text{Skin friction} \qquad\qquad (9)$$

$$C_f = \frac{T_w}{\rho U^2} \qquad\qquad \text{Skin friction coefficient} \qquad\qquad (10)$$

By using the above we have

$$\operatorname{Re}_x^{1,2} = \left(1 + \frac{K}{2} \right) f''(0) \qquad \text{where} \qquad\qquad (11)$$

$$\operatorname{Re}_x^{1/2} = \frac{xU}{v} \qquad\qquad \text{Reynolds number}$$

The heat transfer from the surface to the fluid is computed by application of Fourier's law

$$q = \left[\left(\frac{16\sigma^* T_\infty^3}{3K^*}\right)\frac{\partial T}{\partial y}\right]_{y=0}$$ (12)

Introducing the transformed variables, the expression for q becomes and the heat transfer coefficient, in terms of the Nusselt number Nu can be expressed as

$$Nu = \frac{q}{k(T_w - T_\infty)\sqrt{\dfrac{\alpha}{v}}} \qquad \text{Nusselt number}$$ (13)

Then we have $Nu = -\theta'(0)$

$$j_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$Sh_x = \frac{j_w}{D(C_w - C_\infty)\sqrt{\dfrac{\alpha}{v}}} \qquad \text{Sherwood number}$$ (14)

Numerical solutions :

Applying Quasi-linearization technique[24] to the non-linear equation (8a) we obtain as

$$(1 + K/2)f''' + (F)f'' + (-2F' - Ha^2)f' + F''f = FF'' - F'F' - 1$$ (15)

Where assumed F is the value of f at n^{th} iteration and f is at $(n+1)^{th}$ iteration. The convergence criterion is fixed as $|F - f| < 10^{-5}$

Using an implicit finite difference scheme for the equation (16), (8b) and (8c) we obtain

$$a[i]f[i+2] + b[i]f[i+1] + c[i]f[i] + d[i]f[i-1] = e[i]$$ (16)

$$a_1\theta[i+1\} + b_1\theta[i] + c_1\theta[i-1] = d[i]$$ (17)

$$a_2\phi[i+1] + b_2[i]\phi[i] + c_2[i]\phi[i-1] = d_2[i]$$ (18)

$a[i] = A[i] \qquad b[i] = -3A[i] + h*B[i] + 0.5*h^2*C[i]$

$c[i] = 3A[i] - 2hB[i] + h^3 D[i]$

$d[i] = -A[i] + hB[i] - 0.5*h^2 C[i] \qquad\qquad e[i] = h^3 E[i]$

$a_1[i] = A_i[i] + 0.5*h*B_1[i] \qquad\qquad b_1[i] = -2A_1[i]$

$c_1[i] = A_1[i] - 0.5*h*B_1[i] \qquad\qquad d_1[i] = h^2*D_1[i]$

$a_2[i] = A_2[i] + 0.5*h*B_2[i] \qquad\qquad b_2[i] = -2A_2[i] + h^2 C_2[i]$

$c_2[i] = A_2[i] - 0.5*hB_2[i] \qquad\qquad d_2[i] = h^2 D_2[i]$

$A[i] = 1 + \dfrac{K}{2} \qquad\qquad B[i] = F \qquad\qquad C[i] = -2F' - Ha^2$

$$D[i] = F''$$ $$E[i] = FF'' - F'F' - 1$$
$$A_1[i] = 1 + (4|3)R$$ $$B_1[i] = Prf + PrN_b\phi' + 2PrN_t\theta'$$
$$D_1[i] = PrN_t\theta'\theta'$$
$$A_2[i] = 1 \quad B_2[i] = Scf[i] \quad C_2[i] = -Sc\delta \quad D_2[i] = -N_t/N_b$$

Results and Discussion

Numerical calculations are carried out for different values of dimensionless parameters such as material parameter K, suction parameter A, Prandtl number Pr, thermophoresis parameter Nt, Brownian motion parameter Nb, radiation parameter R, chemical reaction parameter δ, and magnetic number Ha and a representative set of results is reported in graphs from figures 1 to 8. These results are obtained to show that the flow field is influenced appreciably by A, K, Pr, R, δ, Ha, Nt and Nb.

Figures 1(a)- 1(c) display the effect of magnetic parameter Ha for velocity, temperature and concentration profiles respectively. The velocity profile f' decrease with the increase of magnetic parameter Ha, whereas the temperature and concentration profiles increase with the increase of magnetic parameter Ha. It is because that applications of transverse magnetic field will result a resistive type of force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing the velocity.

Figures 2(a)-2(b) show the effect of radiation parameter R on temperature and concentration profiles respectively. It is noticed that the temperature distribution increase with the increase value of radiation parameter while the concentration profile decrease with the increase of radiation parameter *R*.

The effect of material parameter K on velocity, temperature and concentration profiles is displayed in figs. 3(a)-3(c). It is seen that the velocity profile, temperature and concentration profiles decrease with the increase of material parameter K.

Figures 4(a)–4(c) represent the dimensionless velocity, temperature and concentration profiles for the values of A respectively. From these figures it is clear that $f'(\eta)$ increase with the increase of suction parameter A, whereas the temperature and concentration profiles decrease with the increase of suction parameter A.

The effect of thermophoresis parameter Nt is displayed in figs. 5(a)-5(b) for temperature and concentration respectively. Temperature and concentration profiles increase with the increase of thermopheresis parameter. The effect of Brownian motion parameter Nb for temperature and concentration is displayed in figure 6(a) and 6(b) respectively. With the increase of Brownian motion parameter Nb it is noticed that temperature profile increases whereas concentration profile decreases.

The effect of dimensionless Prandtl number Pr on temperature distribution θ is placed in fig.7(a) It can be seen that the temperature profile decrease with the increase of Pr value. The effect of Prandtl number Pr is to decrease the concentration profile is observed from fig. 7(b).

Figure 8 illustrate the variation of concentration profile fpr difference values of δ. The effect of increase of chemical reaction parameter δ is to decrease concentration profile is noticed from the figure. Table (1) shown the value of skin friction coefficient $f''(0)$ and the rate of heat transfer coefficient $-\theta'(0)$ for different values of K and A for fixed values of *Pr.* It is observed from the table the skin friction coefficient $f''(0)$ decrease with the increase of *K* whereas the value of $f''(0)$ increase with the increase of *A*. It can also be seen that the heat transfer coefficient $-\theta'(0)$ decrease with the increase of *K* value. The heat transfer coefficient $-\theta'(0)$ increase with the increase of *A* value from -2 to 2.
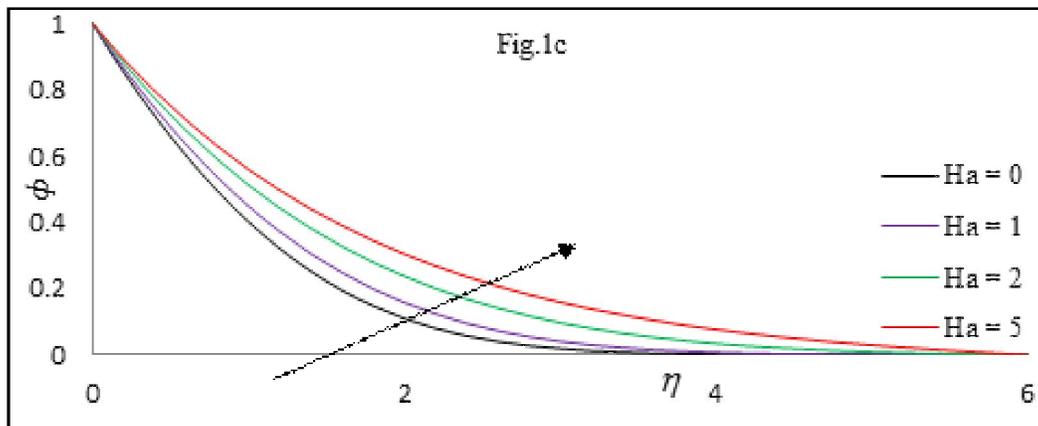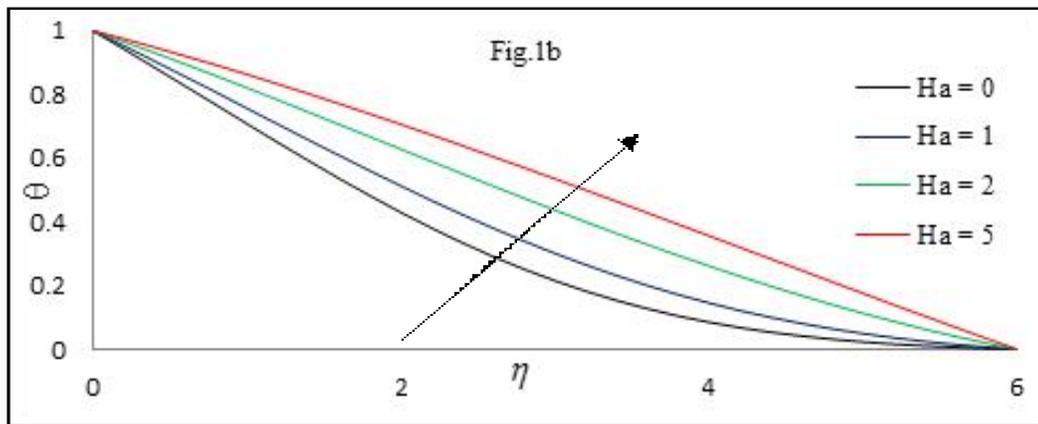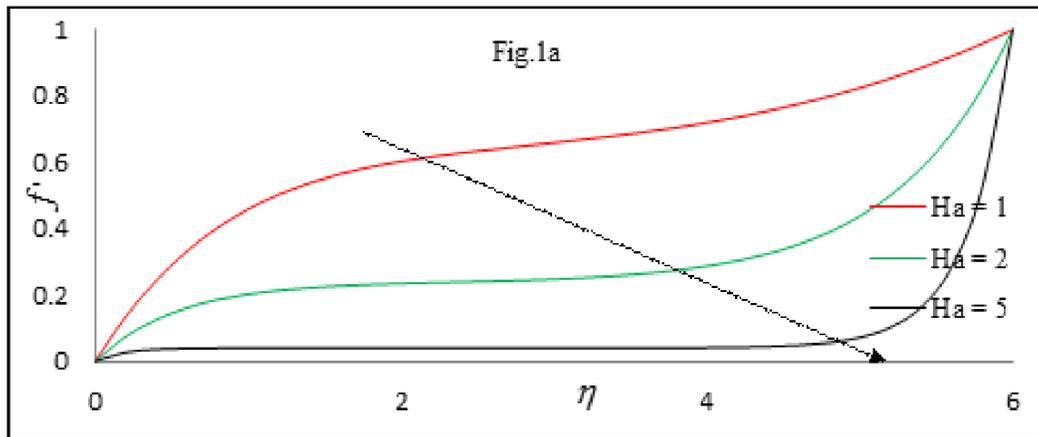
Fig.1 Effects of Magnetic parameter Ha for A=0, K=1.0, Pr=0.72, δ=0.4, R=2,
Nt=0.1 and Nb=0.1 (a) velocity profile (b) temperature profile (c) concentration profile
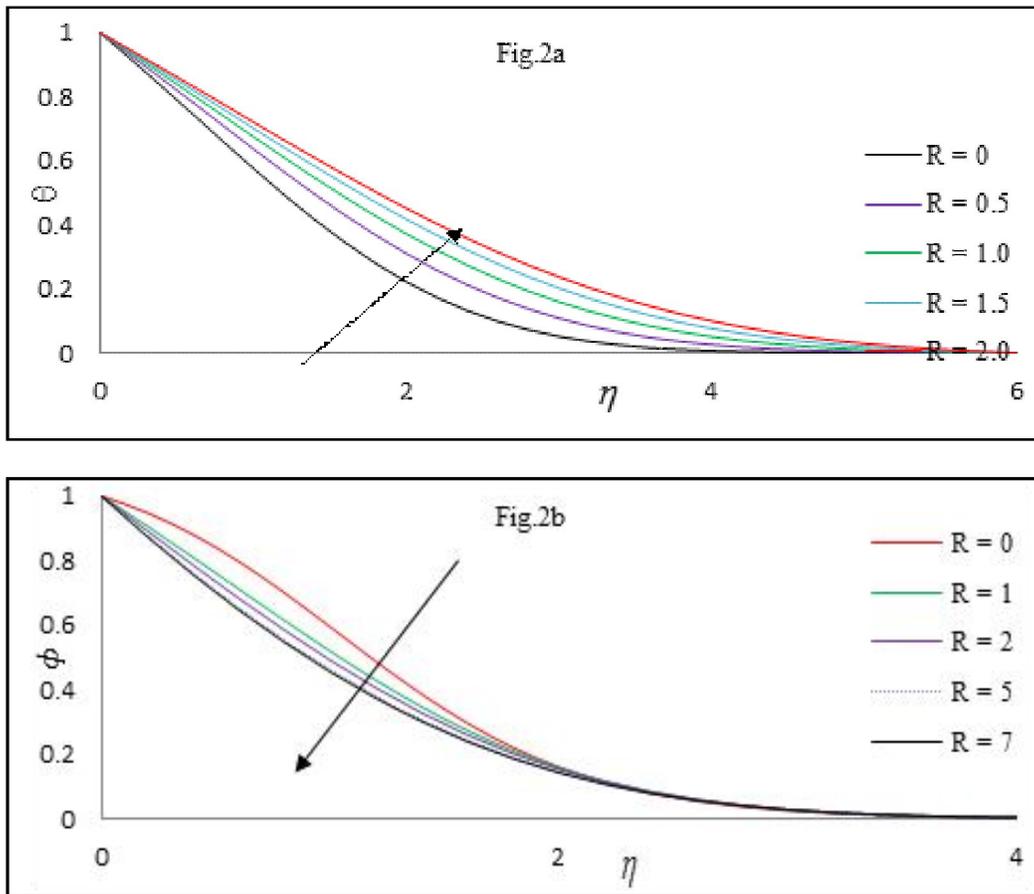
Fig. 2 Effects of Radiation parameter R for A=0, K=1.0, Pr=0.72, δ=0.4, Ha=0.5,
Nt=0.1 and Nb=0.1 (a) temperature profile (b) concentration profile

Table (1). Results of $f''(0)$ and - θ'(0) for different values of K and A when Pr = 0.7

K	A	f'(0)	-θ'(0)
0	1	1.419836	0.403682
0.5	1	-1.201999	0.398784
1.0	1	-1.055735	0.39472
1.5	1	-0.9498	0.391236
2.0	1	-0.86894	0.388179
1	-2	0.378311	0.030677
1	-1	0.519675	0.108716
1	0	0.737433	0.234122
1	1	1.055735	0.39472
1	2	1.494145	0.573393
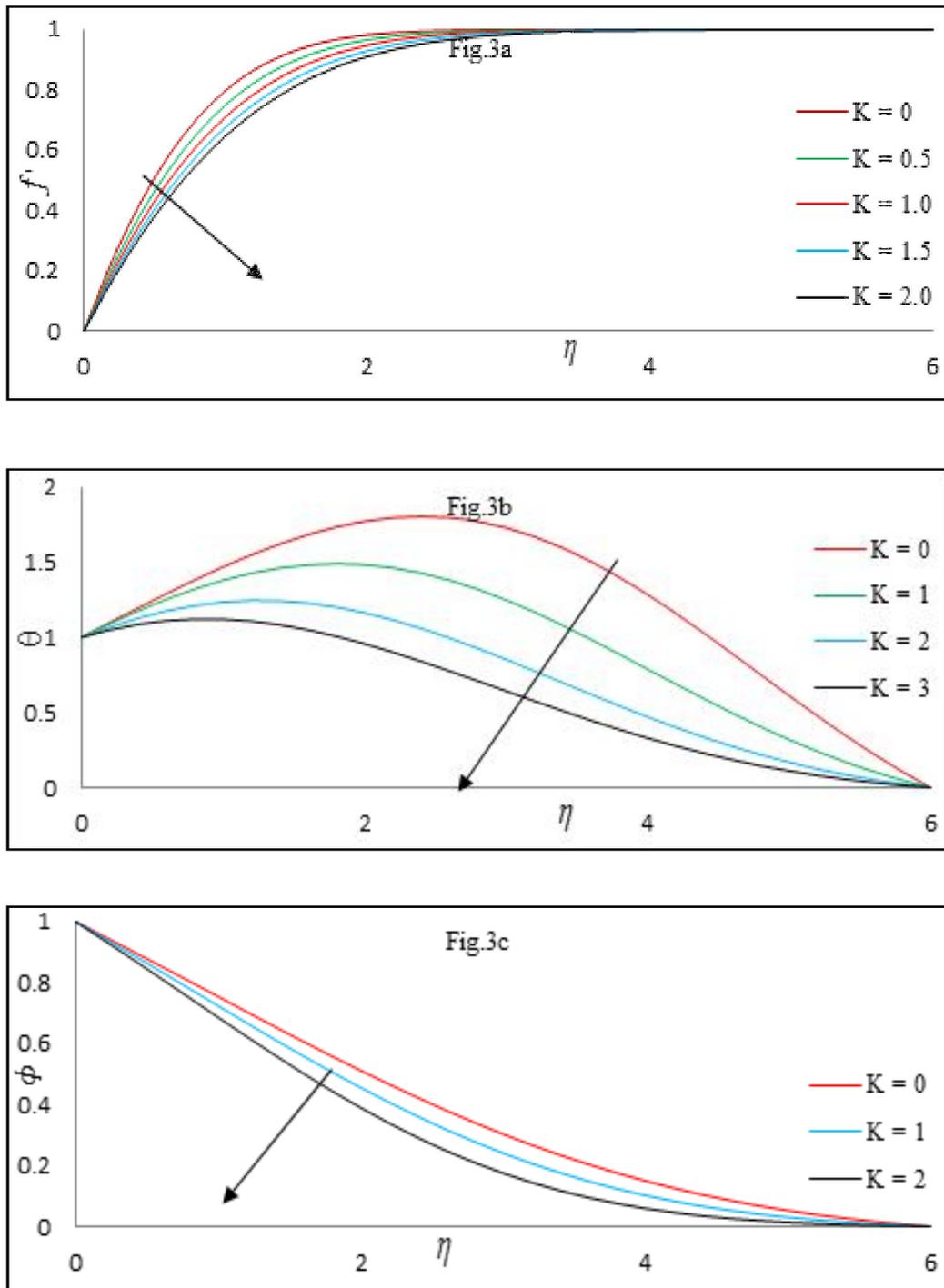
Fig.3 Effects of material parameter K for A = 0, Pr=0.72, R=2.0, δ=0.4, Nt=0.1,
Nb=0.1 and Ha=0.5 (a) velocity profile (b) temperature profile (c) concentration profile
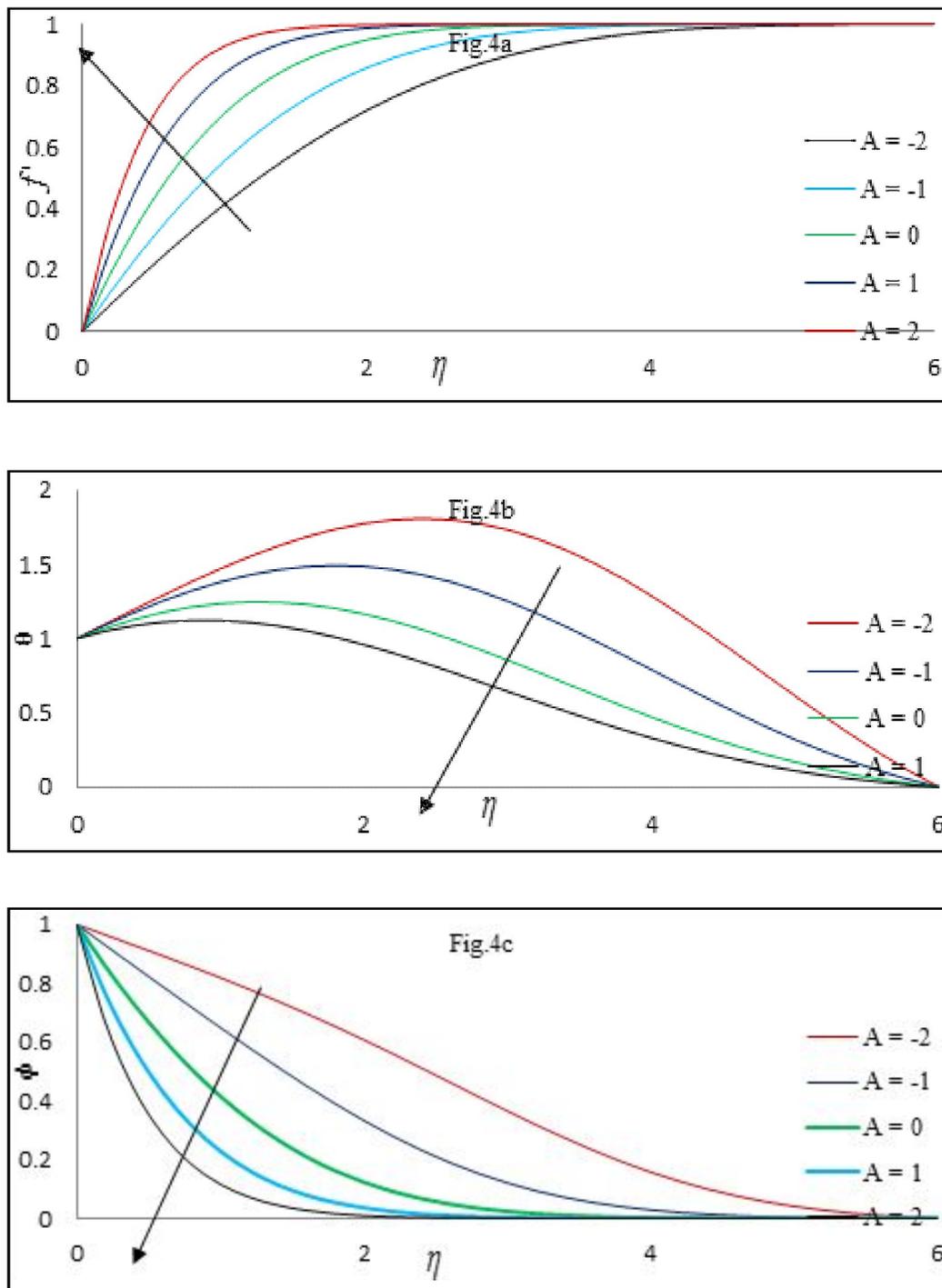
Fig. 4 Effects of Suction parameter A for K=1.0, Pr=0.72, δ=0.4, R = 2, Nt=0.1,
Nb=0.1 and Ha=0.5 (a) velocity profile (b) temperature profile (c) concentration profile
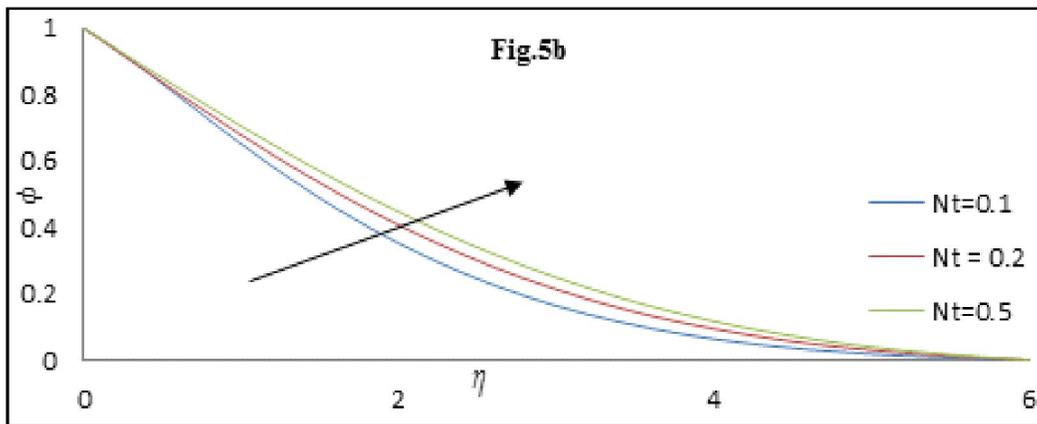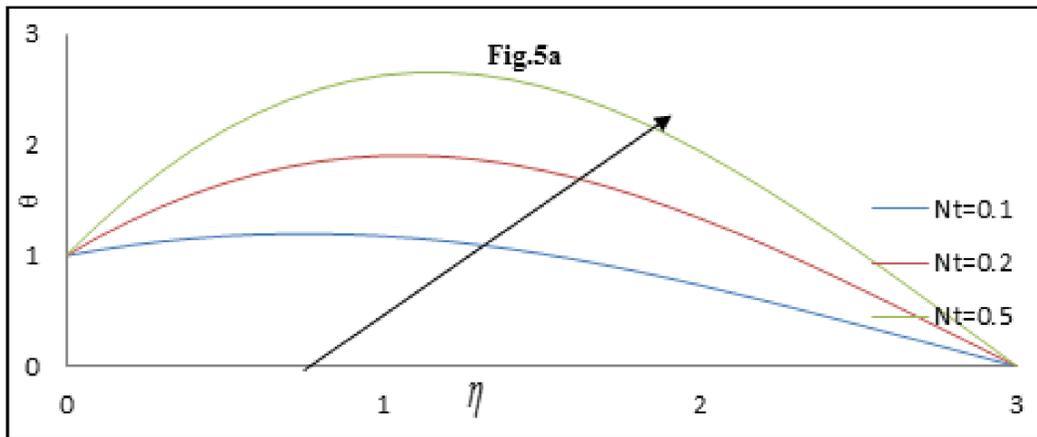
Fig.5 Effects of thermophoresis parameter Nt for K=1.0, Pr=0.72, δ=0.4, R = 2,
A=0, Nb=0.1 and Ha=0.5 (a) temperature profile (b) concentration profile
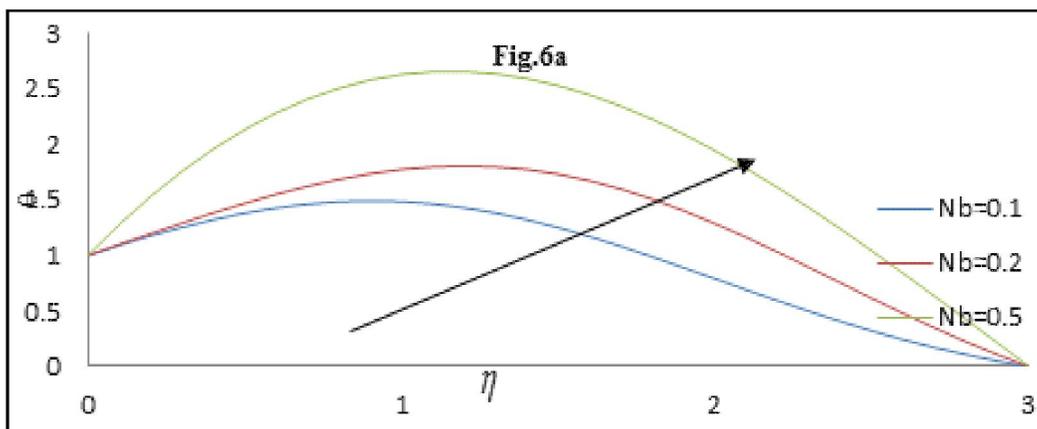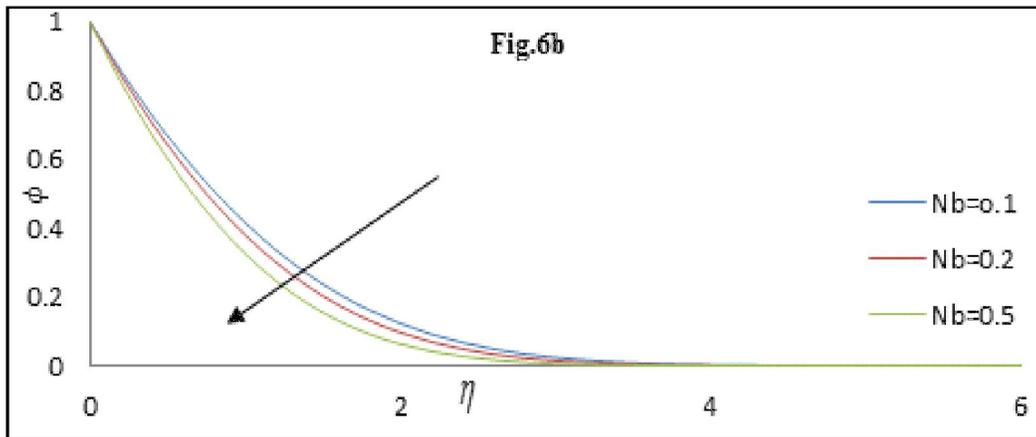
Fig. 6 Effects of Brownian motion parameter Nb for A=0, K=1.0, Pr=0.72, δ=0.4,
R = 2, Nt=0.1 and Ha=0.5 (a) temperature profile (b) concentration profile
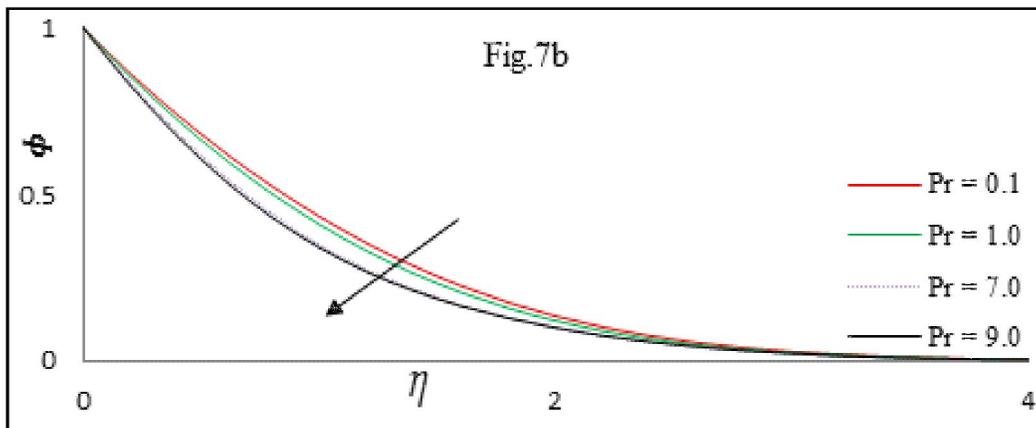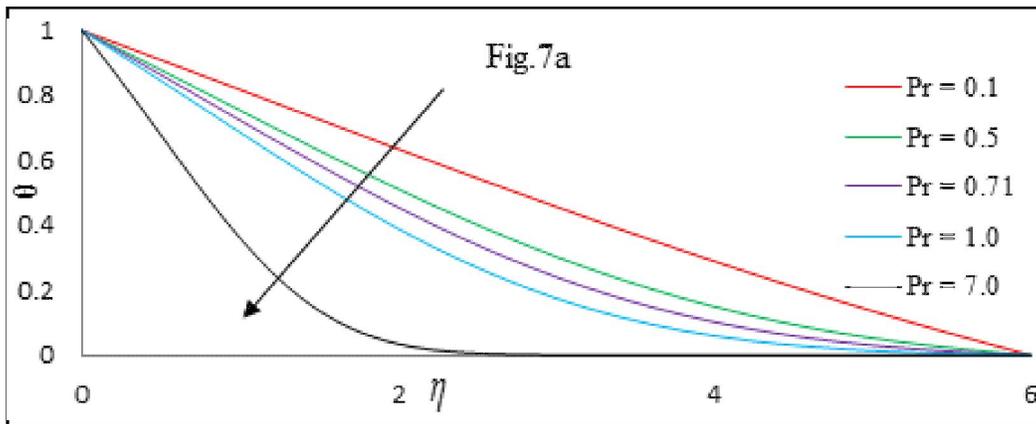
Fig.7 Effects of Prandtl number Pr for A=0, K=1.0, Ha = 0.5, Nt=0.1, Nb=0.1,
δ =0.4 and R = 2 (a)temperature profile (b) concentration profile
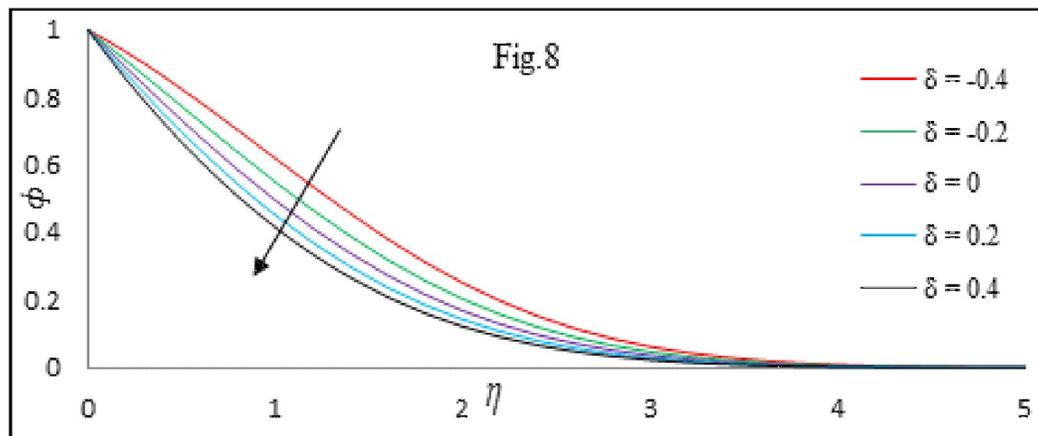
Fig. 8 Effects of chemical reaction parameter δ on concentration profile
A=0, K=1.0, Pr=0.72, Nt=0.1, Nb=0.1, R = 2 and Ha=0.5

Conclusion

The problem of steady mixed convection MHD stagnation point flow of non newtonian micropolar fluid with radiation, thermophoresis parameter, Brownian motion parameter and chemical reaction are investigated.
1.   The velocity profile is decreased with the increase of material parameter K and magnetic parameter Ha. The reverse is in the case of   suction parameter A.
2.   The temperature profile increases with the increase of magnetic parameter Ha, radiation parameter R, thermophoresis parameter Nt and Brownian motion parameter Nb. The temperature profile decreases with the increase of suction parameter A, material parameter K and Prandtl number Pr.
3.   The concentration profile increase with the increase of magnetic parameter Ha and thermophoresis parameter Nt whereas the concentration profiles decreases with increase of material parameter K, suction parameter A, Prandtl number Pr, chemical reaction parameter δ, Brownian motion parameter Nb and radiation parameter R.
The aim and scope of the paper to analyze unsteady MHD fluid flow with above mentioned parameters.

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