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**Optimal Ordering and Replenishment Policies for Deteriorating Items  
 Having a Fixed Expiry Date with Price and Credit Period Sensitive  
 Demand under Trade Credit**

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**Abstract**

In this paper, an inventory model for deteriorating items with selling price and credit period sensitive demand is developed. The inventory system deals with products which have a fixed expiry date after which the product cannot be sold. Here, a permissible delay period is allowed by the supplier to the retailer to pay all his dues, but if the retailer doesn't pay the entire amount at the end of the delay period, an interest will be charged on the remaining dues. The shortages are also allowed and partially backlogged. This paper provides a procedure to develop the total retailer's profit function per unit time of the system and optimal ordering quantity per cycle for the retailer. Finally, the model is illustrated with numerical examples.

**Key words :** Inventory, deterioration, price and credit period sensitive demand, permissible delay period, and expiry date.

**1. Introduction**

There are some items such as cosmetic products, antidandruff shampoos, toothpastes, sunscreens, and medicine etc. which has a certain expiry date. The time from the date of manufacturing of the products for the time till which the products have sufficient potency to bring about the desired action can be termed as shelf life or expiration date. However, the quality of a product may decline before the expiration date if the product has not been properly stored. For instance, cosmetics exposed to high temperatures or sunlight, or opened and examined by consumers prior to purchase may substantially deteriorate before the expiration date. So, the deterioration before the expiry date of a product plays an important rule in the inventory modelling.

Most of the traditional inventory models under trade credit financing are assuming that buyer pays instantly purchasing cost of the items as soon as the items are received. However, in practice, the supplier offers a permissible delay period known as the trade credit period to retailer to pay all his dues, but if the retailer doesn't pay the entire amount at the end of the delay period, an interest will be charged on the remaining dues. So, in today's competitive business scenario, trade credit is now an invaluable promotional tool for the suppliers to increase profitability through stimulating more sales and a unique opportunity for the retailers to reduce demand uncertainty.

Trade credit was first presented by Haley and Higgins<sup>10</sup> who examined the effect of trade credit on the optimal inventory policy. Later, Chapman *et al.*<sup>4</sup> have developed the optimal ordering policies under different considerations. Goyal<sup>9</sup> established an inventory model for a single item under permissible delay in payments when selling price equal to the purchase cost. Aggarwal and Jaggi<sup>3</sup> extended Goyal<sup>9</sup> model for the deteriorating items. Teng<sup>20</sup> extended Goyal<sup>9</sup> model by considering the difference between the selling price and purchasing cost. Abad<sup>1</sup> developed an optimal pricing and lot sizing inventory model for a reseller considering selling price dependent demand. Abad<sup>2</sup> formulated optimal lot sizing policies for perishable goods in a finite production-inventory model with partial backlogging and lost sales. Dye *et al.*<sup>7</sup> determined optimal selling price and lot size with a variable rate of deterioration and exponential partial backlogging.

Hsu *et al.*<sup>11</sup> developed an optimal ordering decision for deteriorating items with expiration date and uncertain lead time. Kumari *et al.*<sup>12</sup> presented two warehouse inventory model for deteriorating items with partial backlogging under the conditions of permissible delay in payments. Chang *et al.*<sup>5</sup> investigated a partial backlogging inventory model for non instantaneous deteriorating items with stock dependent demand. Liao *et al.*<sup>13</sup> investigated a distribution free newsvendor model with lost sales penalty. Teng and Lou<sup>21</sup> proposed the demand rate is an increasing function of the trade credit period. Lou and Wang<sup>15</sup> studied optimal trade credit and order quantity by considering trade credit with a positive correlation of market sales, but are negatively correlated with credit risk. Liu and Cruz<sup>14</sup> discussed the supply chain networks with corporate financial risks and trade credits under economic uncertainty. Singh and Vishnoi<sup>17</sup> introduced a supply chain inventory model for deteriorating and ameliorating items with price dependent consumption rate under two levels of storage. Wu *et al.*<sup>22</sup> explored optimal credit period and lot size by considering delayed payment time dependent demand under default risk for deteriorating items with expiration dates. Tayal *et al.*<sup>19</sup> presented a multi item inventory model to optimize the unit time profit of inventory management for the products having an expiration date after which the product cannot be sold.

Dye and Yang<sup>8</sup> discussed the sustainable trade credit policy with credit-linked demand and credit risk considering the carbon emission constraints. Chen and Teng<sup>6</sup> extended Teng and Lou's model<sup>21</sup> to consider time varying deteriorating items and default risk rates under two levels of trade credit. Wu and Zhao<sup>23</sup> discussed two retailer-supplier supply chain models with default risk under the trade credit policy. Singh and Singh<sup>18</sup> recently developed an optimal inventory policy for deteriorating items with stock level and selling price dependent demand under the permissible delay in payments. Rastogi *et al.*<sup>16</sup> developed an EOQ model with variable holding cost and partial backlogging under the credit limit policy.

We develop an inventory model for the retailer under the following scenario:

- (1) The supplier provides a permissible delay period to the retailer to pay all his due.
- (2) The trade credit period offered supplier to retailer increases sales of the supplier and revenue of the retailer.
- (3) The demand is a linear function of selling price and the length of credit period.
- (4) Shortages are allowed and partially backlogged.

- (5) The inventory system deals with deteriorating items having a fixed expiry date. Finally, the model is illustrated with numerical examples.

## 2. Assumptions :

To build the mathematical models, the following assumptions are adopted.

- The demand  $D(s, n)$  of the products is a linear function of the selling price and the credit period. For simplicity, the demand rate  $D(s, n)$  may be given by  $D(s, n) = a - bs + cn$ , where  $a, b$ , and  $c$  are non-negative parameters. Also,  $s$  is the selling price of the product and  $n$  is the credit period is offered to the retailer.
- There is a single supplier and single retailer and they deal with a single product.
- Holding cost  $h(t)$  per unit per unit time is assumed to be an increasing function of time and considered as  $h(t) = h_1 + h_2 t$ , where  $h_1, h_2 \geq 0$ .
- Replenishment is instantaneous and lead time is zero.
- Shortages are allowed and partially backlogged with a constant rate  $\eta$  ( $0 \leq \eta < 1$ ).
- A permissible delay period of time  $n$  is allowed by the supplier to the retailer to pay all his dues, but if the retailer doesn't pay all his dues at this time period, an interest will be charged on the remaining dues.
- The inventory system under study deals with deteriorating items having a fixed expiry date and there is no repair or replacement of the deteriorated units. For simplicity, the deterioration rate  $\theta(t)$  is taken as:

$$\theta(t) = \frac{l}{l + v - t}, \text{ where } 0 \leq t \leq v \text{ and the deterioration rate tends to 1 when time tends to expiry date } v \text{ of the product.}$$

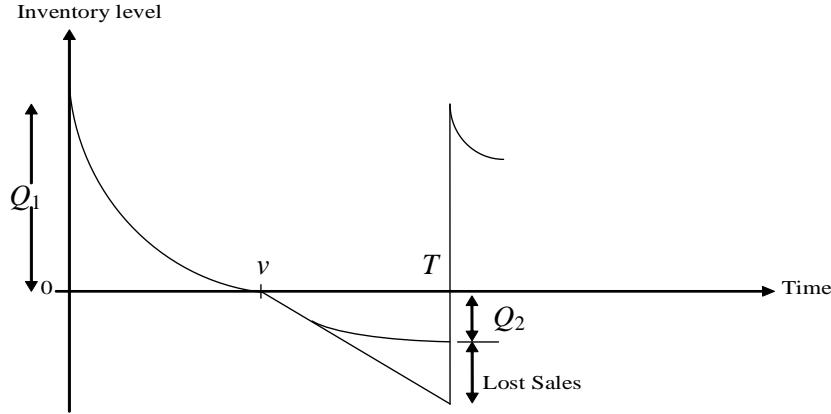
## 3. Notations

To build the mathematical models, the following notations are adopted.

- $abc$  : Demand parameters
- $h_1, h_2$  : Holding cost parameters
- $\eta$  : Backlogging parameter
- $p$  : Purchasing price of the product per unit
- $s$  : Selling price of the product per unit
- $k$  : Shortage cost per unit per unit time
- $l$  : Lost sales cost per unit per unit time
- $O$  : Ordering cost per order
- $I(t)$  : Inventory level at any time  $t$
- $Q_1$  : Initial stock level
- $Q_2$  : Maximum backordered quantity
- $Q$  : Order quantity per cycle
- $v$  : The expiry date of the product
- $T$  : Cycle time
- $n$  : Permissible delay trade credit is offered to the retailer
- $I_e$  : Rate of interest earned

- $I_c$  : Rate of interest charged
- $U$  : Unpaid amount at the time of payment
- $Z$  : The total retailer's profit per unit time for a complete cycle

#### 4. Mathematical Modelling :



**Fig.1: The graphical representation of behavior of the inventory level over time**

Here, the retailer receives the stocks  $Q_1$  at initially. During the time interval  $[0, v]$ , the inventory level is depleted as a result of cumulative effects of demand and deterioration of the products. Since  $v$  is the expiry of the products, then due to expiration of the products, the saleable stock becomes zero at  $t = v$  and thereafter shortages occur. During the time interval  $[v, T]$ , shortages are accumulated due to demand, until it reaches to a maximum allowable shortage level  $Q_2$  which is partially backlogged. At  $t = T$ , the retailer receives the new stock to clear the previous backlog and to continue the process. The behavior of the inventory level over time during a given cycle is shown in figure1. Therefore, the inventory level at any instant of time  $t$  is described by the following differential equations:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(s, n), \quad 0 \leq t \leq v \quad (1)$$

$$\frac{dI(t)}{dt} = -\eta D(s, n), \quad v \leq t \leq T \quad (2)$$

With the boundary condition  $I(v) = 0$

The solutions of the above differential equations are given by

$$I(t) = (a - bs + cn)\{(I + v - t)\log(I + v - t)\}, \quad 0 \leq t \leq v \quad (3)$$

$$I(t) = -\eta(a - bs + cn)(t - v), \quad v \leq t \leq T \quad (4)$$

With the help of equations (3) and (4), one can get the initial inventory level

$$Q_1 = I(0) = (a - bs + cn)\{(I + v)\log(I + v)\} \quad (5)$$

And the maximum backordered quantity

$$Q_2 = -I(T) = \eta(a - bs + cn)(T - v) \quad (6)$$

Hence, the order quantity per cycle is given by

$$Q = Q_1 + Q_2 = (a - bs + cn)\{(1 + v)\log(1 + v) + \eta(T - v)\} \quad (7)$$

The total retailer's profit per unit time of the system comprises the following components:

a) **Sales Revenue:** The retailer's sales revenue per cycle is given by

$$SR = s \left\{ \int_0^v D(s, n) dt + Q_2 \right\} = s(a - bs + cn)\{v + \eta(T - v)\} \quad (8)$$

b) **Ordering Cost:** The ordering cost per cycle is given by  $OC = O$  (9)

c) **Purchasing Cost:** Since  $Q$  is the total order quantity per cycle, the purchasing cost per cycle is

$$PC = pQ = p(a - bs + cn)\{(1 + v)\log(1 + v) + \eta(T - v)\} \quad (10)$$

d) **Holding Cost:** The cost associated with the holding of the stock is calculated as

$$HC = \int_0^v (h_1 + h_2 t) I(t) dt = (a - bs + cn) \left\{ h_1 \left\{ -\frac{3}{2}v - \frac{5}{4}v^2 + \frac{3}{2}(1 + v)^2 \log(1 + v) \right\} + h_2 \left\{ \frac{1}{6}v + \frac{5}{12}v^2 + \frac{7}{36}v^3 - \frac{1}{6}(1 + v)^3 \log(1 + v) \right\} \right\} \quad (11)$$

e) **Deterioration Cost:** The cost associated with the deteriorated units is calculated as

$$DC = d \left\{ Q_1 - \int_0^v D(s, n) dt \right\} = d(a - bs + cn)\{(1 + v)\log(1 + v) - v\} \quad (12)$$

f) **Shortage Cost:** The shortage cost per cycle is

$$SC = k \int_v^T \{-I(t)\} dt = \frac{k}{2} \eta(a - bs)(T - v)^2 \quad (13)$$

g) **Lost Sales Cost:** During the shortage, all the customers do not wait up for the next arrival. They make their purchases from any other place. So, the cost associated with the lost sales during shortages can be calculated as

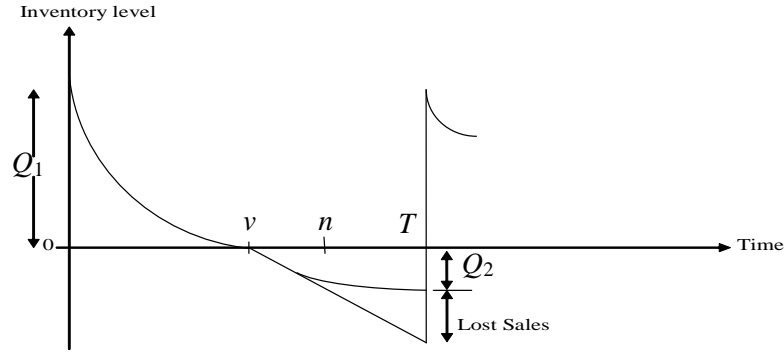
$$LSC = l \int_v^T (1 - \eta) D(s, n) dt = l(a - bs)(1 - \eta)(T - v) \quad (14)$$

According to the trade credit periods offered by the supplier to the retailer and the expiration period of the product, the following cases may arise:-

**Case 1:** When the credit period is greater than or equal to the expiry date of the product i.e.  $n \geq v$

**Case 2:** When the credit period is less than the expiry date of the product i.e.  $n < v$

**4.1 Case 1: When  $n \geq v$**



**Fig.2: Case 1: When**

In this case, retailer settles the account at time  $t = n$  which is greater than or equal to the expiry date of the products. The retailer sells all the stock at time  $t = v$  and deposits the sales revenue in an interest bearing account. Therefore, the interest earned in this case is given by

$$IE_I = sI_e \left[ \int_0^v D(s, n) t dt + (n - v) \int_0^v D(s, n) dt \right] = sI_e (a - bs + cn) \left\{ nv - \frac{1}{2} v^2 \right\} \quad (15)$$

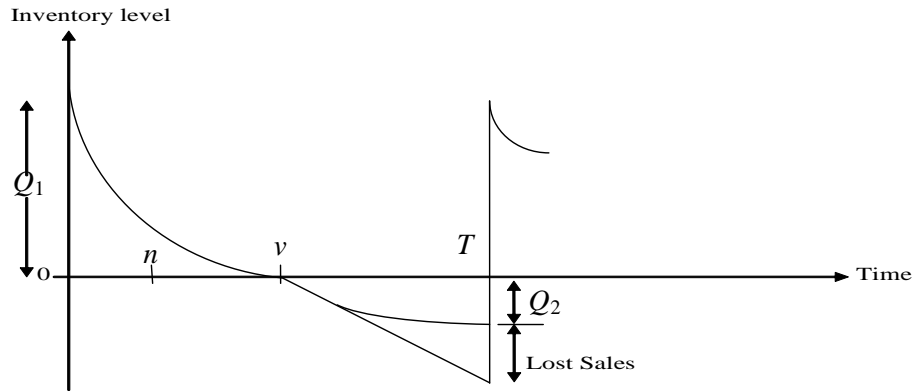
At time  $t = v$ , the retailer has sold all the stock and has enough money to pay all his dues. So, the interest charged by the supplier to the retailer will be zero i.e.

$$IC_I = 0 \quad (16)$$

Hence, in this case, the total retailer's profit per unit time of the system is given by

$$Z_I = \frac{1}{T} \{ SR - OC - PC - HC - DC - SC - LSC - IC_I + IE_I \} \quad (17)$$

**4.2 Case 2: When  $n < v$**



**Fig.3: Case 2: When**

In this case, the retailer sells the items and deposits the sales revenue into an interest bearing account and earns

interest on it during the time interval  $[0, n]$ . The total interest earned by the retailer on the sales revenue during the time interval  $[0, n]$  is calculated as

$$IE_2[0, n] = sI_e \int_0^n D(s, n) t dt = \frac{sI_e(a - bs + cn)n^2}{2} \quad (18)$$

And the total sales revenue generated by the retailer during the time interval  $[0, n]$  is given by

$$SR[0, n] = s \int_0^n D(s, n) dt = s n (a - bs + cn) \quad (19)$$

Based on the difference in the total amount of money in the account and the purchasing cost at the time of payment, two different subcase may arise:-

**Subcase 2.1:** When  $SR[0, n] + IE_2[0, n] \geq pQ$

**Subcase 2.2:** When  $SR[0, n] + IE_2[0, n] < pQ$

#### 4.2.1 Subcase 2.1: When $SR[0, n] + IE_2[0, n] \geq pQ$

In this subcase, the retailer has enough money in his account to settle the account at time  $t = n$ . Therefore, the interest charged in this subcase will be zero i.e.

$$IC_{2.1} = 0 \quad (20)$$

And interest earned in this subcase is given by

$$IE_{2.1} = sI_e \left\{ \int_0^n D(s, n) t dt + \int_n^v D(s, n) t dt \right\} = sI_e \int_0^v D(s, n) t dt = sI_e (a - bs + cn) \frac{v^2}{2} \quad (21)$$

Hence, the total retailer's profit per unit time of this inventory system is given by

$$Z_{2.1} = \frac{I}{T} \{SR - OC - PC - HC - DC - SC - LSC - IC_{2.1} + IE_{2.1}\} \quad (22)$$

#### 4.2.2 Subcase 2.2: When $SR[0, n] + IE_2[0, n] < pQ$

In this subcase, the retailer does not have enough money in his account to settle the account at time  $t = n$ .

Therefore, interest will be charged on the unpaid amount during the time interval  $[n, v]$ . The unpaid amount is

$$U = pQ - \{SR[0, n] + IE_2[0, n]\} = (a - bs + cn) \left\{ p(1 + v) \log(1 + v) + \eta(T - v) - sn - \frac{1}{2} sI_e n^2 \right\} \quad (23)$$

So, the interest charged on this unpaid amount will be

$$IC_{2.2} = I_c(v-n)U = I_c(v-n)(a-bs+cn)\left\{p(1+v)\log(1+v) + \eta(T-v) - sn - \frac{I}{2} s I_e n^2\right\} \quad (24)$$

And interest earned will be

$$IE_{2.2} = IE_{2.1} = sI_e \left\{ (a-bs+cn) \frac{v^2}{2} \right\} \quad (25)$$

Hence, the total retailer's profit per unit time of this inventory system is given by

$$Z_{2.2} = \frac{1}{T} \{SR - OC - PC - HC - DC - SC - LSC - IC_{2.2} + IE_{2.2}\} \quad (26)$$

### 5. Solution Procedure

To maximize the total retailer profit per unit time with respect to expiry date and cycle time, the necessary conditions are

$$\frac{\partial Z}{\partial v} = 0 \quad \text{and} \quad \frac{\partial Z}{\partial T} = 0 \quad (27)$$

The non-linearity of the equation (27) will not permit us to obtain the closed form solution. So, we recommend following solution procedure:

**Step 1:** Assign numerical values to all inventory parameters.

**Step 2:** Solve equations in (27) simultaneously with the help of Mathematical software for each case. Here, we have used Mathematica.

**Step 3:** Test sufficient conditions i.e.

$$\frac{\partial^2 Z}{\partial v^2} < 0, \quad \frac{\partial^2 Z}{\partial T^2} < 0, \quad \text{and} \quad \left( \frac{\partial^2 Z}{\partial v^2} \times \frac{\partial^2 Z}{\partial T^2} \right) - \left( \frac{\partial^2 Z}{\partial v \partial T} \right)^2 > 0.$$

**Step 4:** If sufficient conditions are satisfied, then find out the total retailer profit per unit time for each case using equations (17), (22), and (26) respectively and order quantity  $Q$  using equation (7).

### 6. Numerical Examples

We consider the following examples to illustrate the mathematical formulation.

#### 6.1 Example for Case 1: When $n \geq v$

The following numerical values of the parameter in proper unit are considered as input for numerical and graphical analysis of the model:

$$a = 300, \quad b = 1.5, \quad c = 175, \quad h_1 = 1, \quad h_2 = 0.5, \quad \eta = 0.95, \quad p = 60, \quad s = 150, \quad d = 20, \\ k = 15, \quad l = 12, \quad O = 250, \quad I_e = 0.08, \quad \& \quad n = 2.8.$$

The output of the model is:  $v = 1.7502$ ,  $T = 6.5920$ ,  $Q = 3137.36$ ,  $\& \quad Z_1 = 8985.77$ .



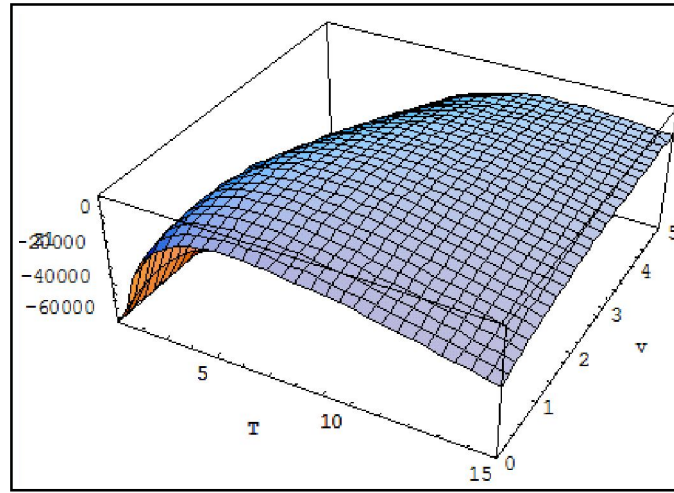


Fig. 4: Concavity of total retailer's profit function w. r. t. expiry date and cycle time

### 6.2 Example for Case 2.1: When $SR[0,n] + IE_2[0,n] \geq pQ$

The following numerical values of the parameter in proper unit are considered as input for numerical and graphical analysis of the model:

$$a = 300, \quad b = 1.5, \quad c = 175, \quad h_1 = 1, \quad h_2 = 0.5, \quad \eta = 0.95, \quad p = 60, \quad s = 150, \quad d = 20, \\ k = 15, \quad l = 12, \quad O = 250, \quad I_e = 0.08, \quad \& \quad n = 2.8.$$

The output of the model is:

$$v = 2.3650, \quad T = 7.4037, \quad Q = 3769.73, \quad \& \quad Z_{2,1} = 7400.91.$$

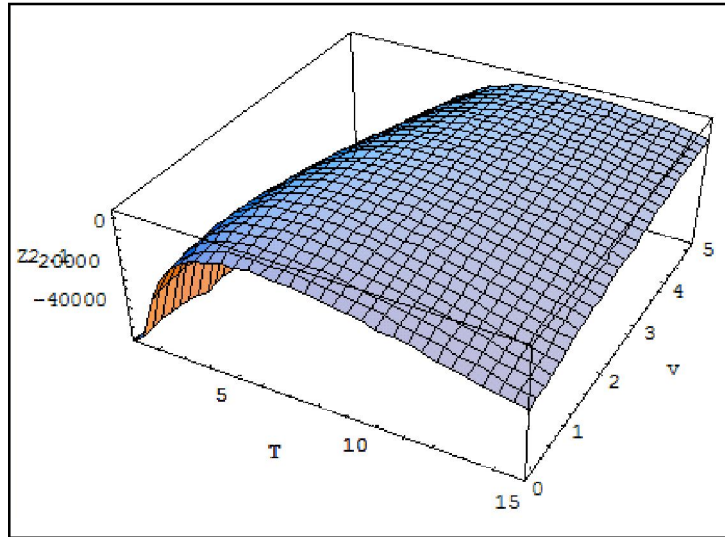


Fig. 5: Concavity of the total retailer's profit function with respect to cycle time

### 6.3 Example for Case 2.2: When $SR[0,n] + IE_2[0,n] < pQ$

The following numerical values of the parameter in proper unit are considered as input for numerical and graphical analysis of the model:

$$a = 300, \quad b = 1.5, \quad c = 175, \quad h_1 = 1, \quad h_2 = 0.5, \quad \eta = 0.95, \quad p = 60, \quad s = 150, \quad d = 20, \\ k = 15, \quad l = 12, \quad O = 250, \quad I_e = 0.08, \quad I_c = 0.12, \quad \& \quad n = 2.8.$$

The output of the model is:  $v = 3.0718$ ,  $T = 8.1054$ ,  $Q = 4462.11$ , &  $Z_1 = 7424.24$ .

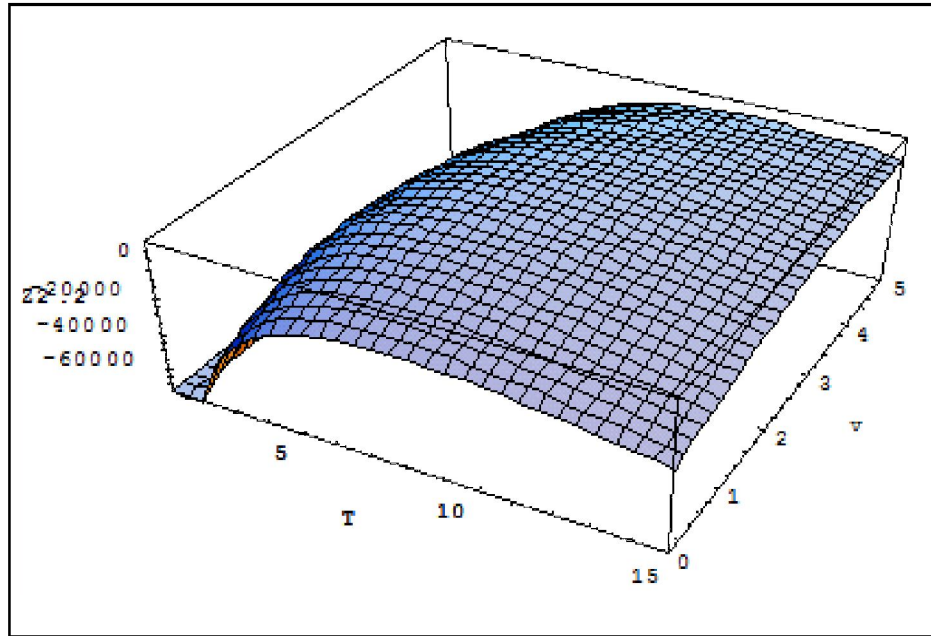


Fig. 6: Concavity of the total retailer's profit function with respect to cycle time

## 7. Conclusion

In this paper, ordering and replenishment strategies have been studied for the retailer when the product has a fixed expiry date and is deteriorating in nature. It is established under a permissible delay period that is allowed by the supplier to the retailer to pay all his dues. The inventory model has been developed with selling price and credit sensitive demand pattern. Here, the shortages are allowed and partially backlogged. The realistic situation that holding cost changes with time is incorporated. The model has been solved analytically by maximizing the total retailer profit per unit time. By the numerical and graphical analysis the proposed model has been verified finally. The obtained results indicate the validity and stability of the model.

The proposed model can further be enriched by taking more realistic assumptions, such as inflation induced demand rate, finite replenishment rate, probabilistic demand rate, and ramp type demand rate etc. It can also be extended in inflationary and fuzzy environments.

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