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Effects of heat and mass flux on MHD free convection flow through a porous medium in presence of radiation and chemical reaction

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Abstract

The present study deals with thermal radiation effect on two dimensional magnetohydrodynamics (MHD) natural convection boundary layer flow of viscous fluid surrounded by porous vertical surface with heat source, chemical reaction and radiation effect. The governing partial differential equations have been converted into a set of ordinary differential equations using non dimensional quantities. Perturbation technique has been employed to solve the system of partial differential equations. Most of the studies so far have presented the numerical and semi analytical solution for flow velocity and temperature in the form of a series solution. The effects of various parameters on flow variables are illustrated graphically, and the physical aspects of the problem are discussed. It has been observed that the velocity increase with increasement in permeability and radiation parameters and reverse trend has been found with respect to magnetic parameter.

Key words : Chemical reaction, porous medium, viscous dissipation, boundary layer, MHD.

MSC Code: 80A20, 76DXX, 76W05, 76S05

B_0 Magnetic flux density

C Non dimensional Concentration parameter

C^* Fluid concentration

C_p Specific molecular diffusivity

C_∞ Species concentration at infinity

C_∞ Species concentration at infinity

D	Coefficient of mass diffusivity
E	Eckert number
F	Radiation Parameter
g	Acceleration due to gravity
Gm	Grashof number for mass transfer
Gr	Grashof number for heat transfer
k_c	Chemical reaction parameter
Pr	Prandtl number
q_r	Constant heat flux
S_c	Schmidt number
Sh	Sherwood number
T	The fluid temperature
T_∞	The fluid temperature at infinity
v_0	Scale of suction velocity

Greek symbols

α	Permeability of porous medium
β_1	Thermal expansion coefficient
β_2	Concentration expansion coefficient
u, v	velocity component along x- and y-direction
η	Distance
ρ	Density of the fluid
τ	Skin fraction
σ	Electrical conductivity
ν	Kinematic viscosity
θ	Dimensionless temperature
λ	Thermal conductivity
μ	Dynamic viscosity

1. Introduction

In many engineering and production areas, flows are stirred not only by temperature differences but also by concentration differences. In many manufacturing industries, transport processes present in which mass and heat transfer phenomenon exist. The process of heat and mass transfer often exists in much chemical production industry, like food, fiber & polymer manufacturing.

In present era MHD is attract the attention of industries due to of its scope in geophysics, agriculture, petroleum industries, astrophysics, geological formations, in exploration and thermal recovery of oil, geothermal reservoirs, underground nuclear waste storage sites, metrology, solar physics and in motion of earth's core, in the field of stellar and planetary magnetospheres etc. In power generation, MHD is attracting remarkable interest because of it offers for much higher thermal efficiencies in electric generation plants. According to Chamka^{2,3} analyzed the unsteady MHD convective heat and mass transfer over a semi-infinite vertical permeable moving plate in presence of heat incorporation. Chaudhary *et. al.*^{4,5} studied the effect of ohmic heating and viscous dissipation

on heat transfer in MHD mixed convection flow and mass transfer past an infinite vertical plate in presence of radiation. They produced solution for the velocity, temperature and concentration profiles. They also investigated an unsteady hydro magnetic free convection flow of elastico-viscous fluid past an infinite vertical plate when the temperature and concentration are assumed to be oscillating with time in presence of Hall effects.

Viscous dissipation has vital role in free convection flow in various processes which operate at high rotational speeds and also in presence of strong gravitational field. The viscous dissipation have an important role in in different industrial processes and generally characterized by the Eckert number. Soundalgekar⁶ and Cortel⁷ descried viscous dissipative heat on the 2 – dimensional unsteady free convective flows past an infinite vertical porous plate when the temperature oscillates in time and with constant suction at the plate. They also analyzed the effect of viscous dissipation on the MHD flow and heat transfer of a visco-elastic fluid over a stretching sheet. The effects of viscous dissipation in presence of radiation over a stretching surface subjected to variable heat flux discussed by Kumar⁸. Kishan and Amrutha⁹ discussed 2-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, electrically conductive and Bossiness fluid flowing over a vertical stretching surface in the presence of uniform magnetic field by taking into account the viscous dissipation with heat, mass transfer chemical reaction and thermal stratification effects. The joint effects of heat generation & viscous dissipation on MHD natural convection flow past a vertical wavy surface analyzed by Parveen & Sujon (2014).

Effect of radiation on MHD flow has become very powerful in many industrial applications. Large number of engineering processes takes place at high temperatures and so the knowledge of radiation and heat transfer is essential for designing various types of industrial equipments. Nuclear power plants, gas turbines and various propulsion devices for different type of aircrafts, missiles and satellites are some important examples of such processes. Subsequently, the impact of radiation on natural convection through a porous vertical plate studied by Hossain *et al.* (1999) analyzed the effect of radiation on free convection from a porous vertical plate. He described the effects of the various parameters on the velocity and temperature. Makinde (2005) analyzed boundary layer free convection flow with thermal radiation & mass transfer past over a moving vertical porous plate. Kulkarni & Patil (2008) analyzed the effect of radiation on unsteady flow of a viscous incompressible fluid past over vertical plate with uniform mass diffusion in the presence of heat source & magnetic field.

The increasing demand for chemical reactions in many chemical production industries are attracting scientist and researchers. In many chemical processes the chemical reaction occur between a foreign mass and the fluid in which the plate is moving. Such type of processes occur in many industrial applications like polymer production, food processing & glassware. Chambre and Young (1958) analyzed the effect of first order chemical reaction in the neighborhood of a horizontal plate. Chen and Yuh¹⁴ dealt with steady of combined heat and mass transfer effects for both conditions of uniform wall temperature/concentration and uniform heat/mass flux. The scope of first order homogeneous chemical reaction on the flow past an infinite vertical plate in presence of uniform mass & heat fluxes analyzed by Das *et. al.*¹⁵. Muthucumarswamy (2002) analyzed the thermal stratification in presence of chemical reaction over a vertical stretching surface. The effects of chemical reaction, thermophoresis and variable viscosity on a study of hydro magnetic flow with heat and mass transfer over a flat plate in the presence of absorption& generation of heat was examined by Seddeek (2005). Raptis and Perdakis¹⁸ studied the effect of magnetic field and chemical reaction on a viscous flow over a non linear stretching sheet. Kabeir and Abdou (2007) analyzed the effect of chemical reaction to a micro polar flow over an isothermal vertical cone. The effect of chemical reaction & radiation on free convection flow through porous medium with variable suction in the presence of uniform magnetic field was examined by Sudheer Babu & Satyanarayana²⁰. Mahapatra *et. al.*²¹ have studied the effect of chemical reaction on a free convection flow through a porous medium bounded by a vertical infinite surface. They also found that the velocity & concentration increase during a generative reaction and decrease in a destructive reaction. Chandra Shekar & Kishan²² discussed the

unsteady convective flow of heat and mass transfer of a viscous, incompressible, electrically conducting Newtonian fluid past a vertical permeable plate in the presence of a homogeneous first order chemical reaction in presence of thermal radiation. The unsteady free convection flow through a non-homogeneous porous medium having variable permeability bounded by an infinite porous vertical plate in slip flow regime in presence of chemical reaction & thermal radiation have studied by Ibrahim and Suneetha²³.

When In chemical reaction, a bulk amount of heat is generated .Due to this reason, the temperature of the body increased. The insulation is required to maintain the body temperature. Porous media play in important role to insulate a heated body for maintaining temperature during process. So, it is necessary to study a flow through a porous medium and to estimate the effect of chemical reaction on mass & heat transfers. Yamamoto *et al.*²⁴ discussed acceleration of convection in a porous permeable medium along an arbitrary smooth surface. Raptis and Singh²⁵ discussed flow past an impulsively vertical plate in a porous medium by a finite difference method. Sattar *et al.*²⁶ studied unsteady free convection flow along a vertical porous plate embedded in a porous medium. Thermal radiation effect on unsteady MHD flow past a vertical porous plate immersed in a porous medium was analyzed by Samad and Rahman²⁷. Sharma and Singh²⁸ discussed the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Sharma *et al.*²⁹ have analyzed the heat and mass transfer effects on unsteady MHD free convective flow along a vertical porous plate with internal heat generation and variable suction. The influence of thermal radiation on magnetohydrodynamic (MHD) flow of Cu-water nanofluid past a wedge in the occurrence of viscous-Ohmic dissipation and chemical reaction has been analyzed by Pandey & Kumar³⁰. An unsteady two-dimensional free convection flow of a viscous incompressible fluid past an impulsively started semi-infinite vertical cylinder adjacent to a non Darcian porous media in the presence of chemical reaction of first order is investigated by Vasu *et al.*³¹

The present research work is concerned with the effect of thermal radiation on magnetohydrodynamic free convection flow on steady viscous incompressible fluid in presence of chemical reaction over a vertical surface in presence of uniform heat and mass flux. The classical model for radiation effect introduced by Cogley *et al.*¹ is used. The Perturbation technique has been applied to convert the governing non-linear partial differential equations into a system of ordinary differential equations. Equations are solved analytically.

2. Mathematical formulation of the problem :

In present work, we study two dimensional flow of a viscous incompressible, electrically conducting fluid through a porous medium occupying semi infinite region of space bounded by a vertical infinite surface. The uniform magnetic field has been applied normal to the direction of flow. Here we consider that a chemically reactive species is emitted from the surface and diffuses into the fluid. The x^* axis taken along the plate and the y^* axis is perpendicular to the plate. The induced magnetic field is neglected under the consideration that the magnetic Reynolds number is small. It is considered that the reaction take place in the whole stream.

The fluid flow under the above considerations along an extremely porous medium, the governing boundary layer equations for this problem given as

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$v^* \frac{\partial u^*}{\partial y^*} = v \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_1(T^* - T_\infty) + g\beta_2(C^* - C_\infty) - \frac{vu^*}{k} - \frac{\sigma B_0^2 u^*}{\rho} \quad (2)$$

$$v^* \frac{\partial T^*}{\partial y^*} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{v}{C_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{\lambda}{\rho C_p} \frac{\partial q_r}{\partial y^*} \quad (3)$$

$$v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - k_c C^* \quad (4)$$

We neglect the chemical reaction in view of low species concentration case.

The solution of Eq. (1) is

$$v^* = \text{const} \tan t = -v_0 \quad (5)$$

Here v_0 is constant suction velocity normal to the plate.

Cogley *et al.*¹ have shown that the radiative heat flux is given by

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T_\infty)I^* \quad (6)$$

Where $I^* = \int K_\lambda \frac{\partial e_\lambda}{\partial T^*} d\lambda$, K_λ being the absorption coefficient at the plate and e_λ be the Plank's function

In this case the relevant boundary conditions are:

$$\left. \begin{aligned} u^* &= 0, T^* = T_\infty, C^* = C_\infty \text{ for any value of } y^*, t \leq 0 \\ u^* &= 0, \frac{\partial T^*}{\partial y^*} = -\frac{q}{\lambda}, \frac{\partial C^*}{\partial y^*} = -\frac{m}{D}, y^*, t > 0 \\ u^* &\rightarrow 0, T^* \rightarrow T_\infty, y^* \rightarrow \infty, t > 0 \end{aligned} \right\} \quad (7)$$

Using equation (5), equations (1), (2), (3) and (4) reduce to

$$-v^* \frac{\partial u^*}{\partial y^*} = v \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_1(T^* - T_\infty) + g\beta_2(C^* - C_\infty) - \frac{vu^*}{k} - \frac{\sigma B_0^2 u^*}{\rho} \quad (8)$$

$$-v^* \frac{\partial T^*}{\partial y^*} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{v}{C_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y^*} \quad (9)$$

$$-v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - k_c C^* \quad (10)$$

Now we consider the non dimensional parameters are

$$\left. \begin{aligned} f(\eta) &= \frac{u^*}{v_0}, \eta = \frac{v_0 y^*}{v}, P_r = \frac{v \rho C_p}{\lambda}, F = \frac{4v^2 I^*}{\lambda v_0^2}, \alpha = \frac{v_0^2 k}{v^2}, C = \frac{(C^* - C_\infty) D}{m}, \\ G_r &= \frac{g\beta_1 q v}{\lambda v_0^2}, G_m = \frac{g\beta_2 m v}{D v_0^2}, S_c = \frac{v}{D}, \theta = \frac{(T^* - T_\infty) \lambda}{v q}, K_r = \frac{k_c v}{v_0^2}, M = \frac{\sigma v B_0^2}{\rho v_0^2}, E = \frac{\lambda v_0^2}{q C_p} \end{aligned} \right\} \quad (11)$$

The non-dimensional structure of eqs. (8)- (10) reduce to

$$f'' + f' - \left(\frac{1}{\alpha} + M\right)f = -G_r\theta - G_m C \quad (12)$$

$$\theta'' + P_r\theta' - F\theta = -EP_r(f')^2 \quad (13)$$

$$C'' + S_C C' = S_C k_r C \quad (14)$$

Where all primes are function of η .

Corresponding boundary conditions are

$$\left. \begin{aligned} \eta = 0, f = 0, \theta' = -1, C' = -1 \\ \eta \rightarrow \infty, f \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \end{aligned} \right\} \quad (15)$$

To solve the solution of equations (12)-(14) using boundary conditions (15), let us expand f , θ and C in powers of E , which is assumed to be very small.

$$\left. \begin{aligned} f &= f_0 + Ef_1 + o(E^2) \\ \theta &= \theta_0 + E\theta_1 + o(E^2) \\ C &= C_0 + Ef_1 + o(E^2) \end{aligned} \right\} \quad (16)$$

Using equations (16) into eqs. (12)-(14) and equating like powers of E and excluding the higher powers of E , we get

$$f_0'' + f_0' - \delta f_0 = -G_r\theta_0 - G_m C_0 \quad (17)$$

$$f_1'' + f_1' - \delta f_1 = -G_r\theta_1 - G_m C_1 \quad (18)$$

$$\theta_0'' + P_r\theta_0' - F\theta_0 = 0 \quad (19)$$

$$\theta_1'' + P_r\theta_1' - F\theta_1 = -P_r f_0'^2 \quad (20)$$

$$C_0'' + S_C C_0' = S_C k_r C_0 \quad (21)$$

$$C_1'' + S_C C_1' = S_C k_r C_1 \quad (22)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} f_0 = 0, f_1 = 0, \theta_0' = -1, \theta_1' = 0, C_0' = -1, C_1' = -1, at \eta = 0 \\ f_0 \rightarrow 0, f_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, C_0 \rightarrow 0, C_1 \rightarrow 0, \eta \rightarrow \infty \end{aligned} \right\} \quad (23)$$

The result of the Eqs.(17)-(22) with the relevant boundary condition (23), we get

$$f_0 = K_3 e^{h_2 \eta} + K_1 e^{e_2 \eta} + K_2 e^{d_2 \eta} \quad (24)$$

$$f_1 = K_{18} e^{h_2 \eta} + K_{11} e^{e_2 \eta} + K_{12} e^{2h_2 \eta} + K_{13} e^{2e_2 \eta} + K_{14} e^{2d_2 \eta} + K_{15} e^{(h_2 + e_2) \eta} + K_{16} e^{(d_2 + e_2) \eta} + K_{17} e^{(d_2 + h_2) \eta} \quad (25)$$

$$\theta_0 = -\frac{1}{e_2} e^{e_2 \eta} \quad (26)$$

$$\theta_1 = K_{10} e^{e_2 \eta} + K_4 e^{2h_2 \eta} + K_5 e^{2e_2 \eta} + K_6 e^{2d_2 \eta} + K_7 e^{(h_2 + e_2) \eta} + K_8 e^{(d_2 + e_2) \eta} + K_9 e^{(d_2 + h_2) \eta} \quad (27)$$

$$C_0 = -\frac{1}{d_2} e^{d_2 \eta} \quad (28)$$

$$C_1 = 0 \quad (29)$$

Substituting the solutions of equations (24)-(29) in (16), we obtain

$$f(\eta) = K_3 e^{h_2 \eta} + K_1 e^{e_2 \eta} + K_2 e^{d_2 \eta} + E(K_{18} e^{h_2 \eta} + K_{11} e^{e_2 \eta} + K_{12} e^{2h_2 \eta} + K_{13} e^{2e_2 \eta} + K_{14} e^{2d_2 \eta} + K_{15} e^{(h_2 + e_2) \eta} + K_{16} e^{(d_2 + e_2) \eta} + K_{17} e^{(d_2 + h_2) \eta}) \quad (30)$$

$$\theta(\eta) = -\frac{1}{e_2} e^{e_2 \eta} + E(K_{10} e^{e_2 \eta} + K_4 e^{2h_2 \eta} + K_5 e^{2e_2 \eta} + K_6 e^{2d_2 \eta} + K_7 e^{(h_2 + e_2) \eta} + K_8 e^{(d_2 + e_2) \eta} + K_9 e^{(d_2 + h_2) \eta}) \quad (31)$$

$$C(\eta) = -\frac{1}{d_2} e^{d_2 \eta} \quad (32)$$

The Nusselt number (Nu), which decides the rate of heat transfer as given by

$$N_u = \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = 1 - E[K_{10} e_2 + 2K_4 h_2 + 2K_5 e_2 + 2K_6 d_2 + (e_2 + h_2) K_7 + (d_2 + e_2) K_8 + (d_2 + h_2) K_9] \quad (33)$$

The skin friction (τ), which is defined as :

$$\tau = \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0} = K_3 h_2 + K_1 e_2 + K_2 d_2 + E[K_{18} h_2 + K_{17} e_2 + 2K_{12} h_2 + 2K_{13} e_2 + 2K_{14} d_2 + K_{15} (e_2 + h_2) + K_{16} (d_2 + e_2) + K_{17} (d_2 + h_2)] \quad (34)$$

The Sherwood number (Sh), which decides the rate of mass transfer as given below:

$$S_h = \left(\frac{\partial C}{\partial \eta} \right)_{\eta=0} = -1 + E(-1) = -(1 + E) \quad (35)$$

3. Results and Discussion

The numerical outcomes are presented in flow fields for dissimilar values of diverse physical parameters drawn in figures. The velocity profiles for different of different parameters are depicted in figs. 1-7. Impact of chemical reaction parameter on velocity depicts in fig. 1. It is noticed that velocity decreases with an increase in

chemical reaction parameter. From Fig. 2, the effect of permeability parameter (α) on Velocity profiles is described. It is observed that velocity increases with increase in permeability parameter. It is interesting to observe that the fluid velocity increases and reaches its maximum over a very short distance from the plate and then decreases gradually to zero for positive value of η . Velocity profile for different value of Radiation parameter (F) is depicted by figure 3. It is observed that the velocity increases with increase in radiation parameter. Effect of Prandtl Number Pr on the velocity profile is depicted in Figure. 4. It is observed that the velocity increases with increase in Prandtl Number (Pr). Figure. 5 exhibits that an increase in Gm results in decrease in the velocity. From Fig. 6, the effect of Eckert number (E) on the velocity profile is described and found that an increase in E results in decrease in the velocity. The temperature profiles are depicted in figures 8-10. In Figure 8, the influence of Eckert Number (E) on temperature is shown. It seen that the temperature increases with increase in Eckert Number (E). Figure 9 displays the effects of Schmidt Number (Sc) on temperature profile. It is observed that temperature increases with increase in Sc . Effect of Chemical reaction parameter (Kr) on the temperature is depicted in Figure 10. It is observed that the temperature decreases with increase in Kr . Fig. 11 shows the effect of Schmidt number on nusselt number in presence of different type of porous media.

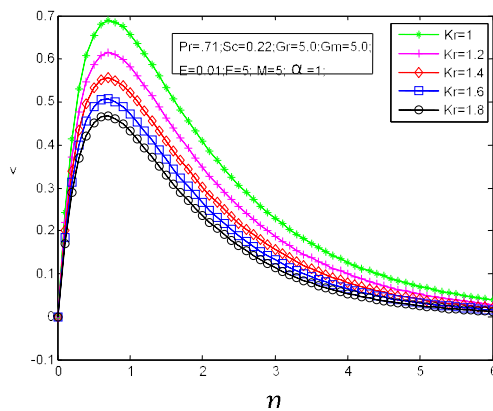


Fig. 1 Velocity profile for different value of chemical reaction parameter (Kr)

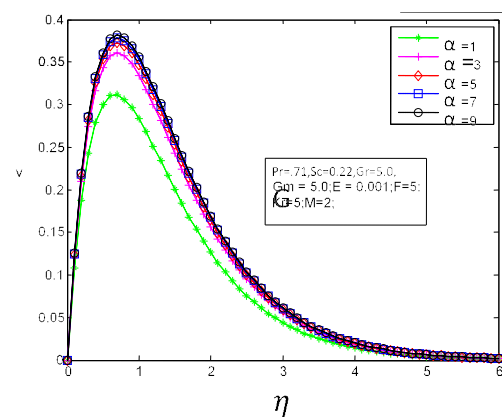


Fig. 2 Velocity profile for different value of permeability parameter (α)

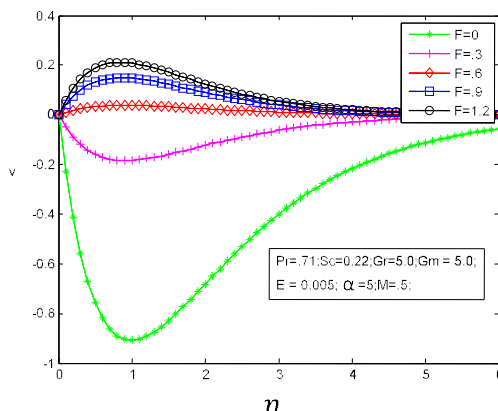


Fig. 3 Velocity profile for different value of Radiation parameter (F)

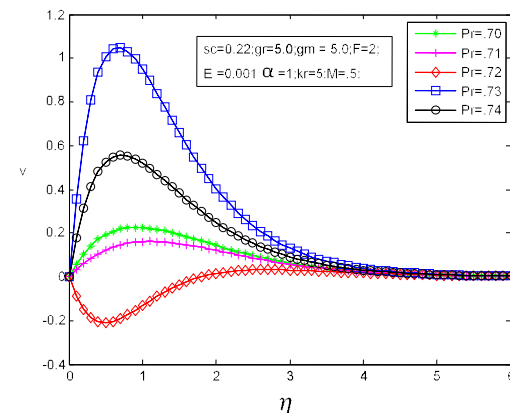


Fig.4 Velocity profile for different value of Prandtl Number (Pr)

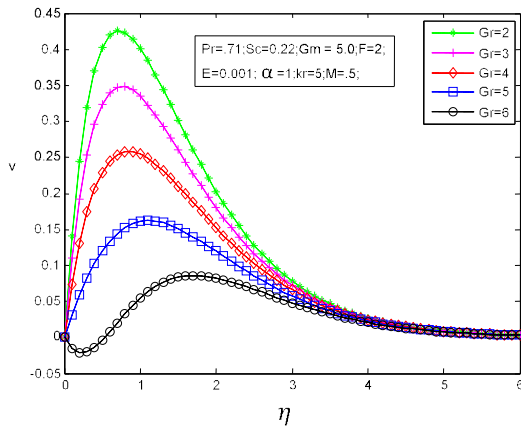


Fig. 5 Velocity profile for different value of Heat Grashof Number (Gr)

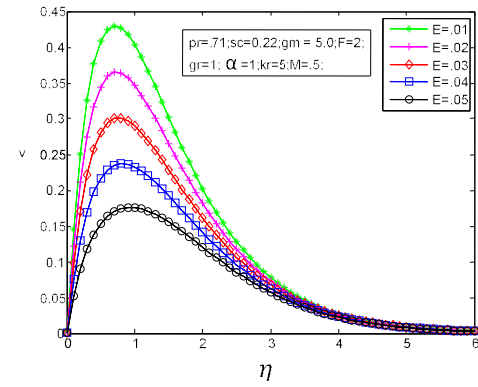


Fig. 6 Velocity profile for different value of Mass Grashof Number (Gm)

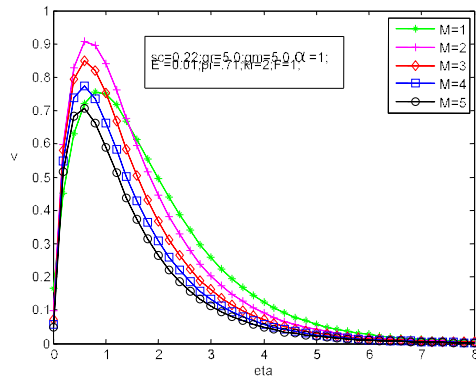


Fig. 7 Velocity profile for different value of Magnetic Parameter (M)

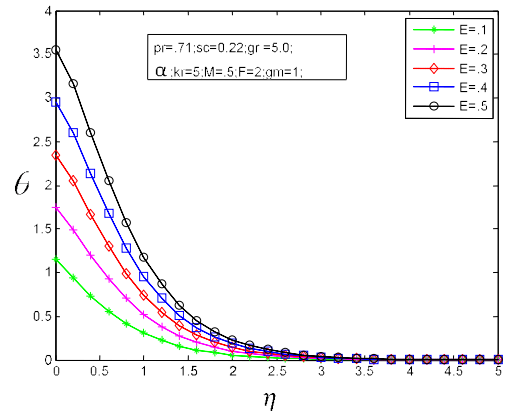


Fig. 8 Temperature profile for different value of Eckert Number (E)

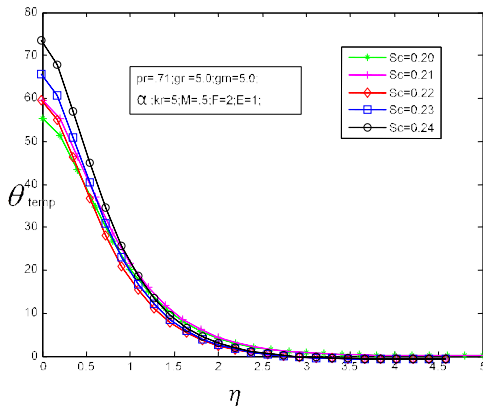


Fig. 9 Temperature profile for different value of Schmidt Number (Sc)

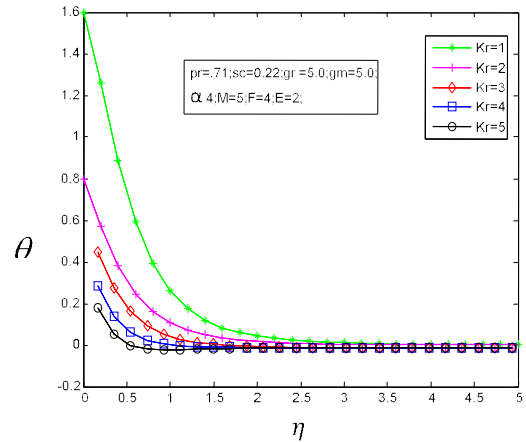


Fig. 10 Temperature profile for different value of chemical reaction parameter (Kr)

4. Conclusion

In this article, we have analyzed the effect of radiation, chemical reaction in presence of heat and mass flux on MHD natural convective flow through a porous medium bounded by a vertical surface. The solution is obtained by perturbation technique. Based on these solutions, numerical computations for various values of the material parameters are carried out.

The following conclusions are drawn from the present study.

1. The velocity of fluid flow decreases with increase in chemical reaction parameter & Eckert number. The velocity profile enhances as the permeability of porous medium and value of radiation parameter and Prandtl number increases and reverse trend has seen in magnetic parameter.
2. The temperature of fluid is increases with increase in Eckert Number (E) and Schmidt Number. The temperature of fluid decreases with increase in chemical reaction parameter (K_r).

Scope of future work :

The present study of a fluid flow over a vertical surface can be used as the basis for many scientific and engineering applications in case of vertical surfaces.

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