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Unsteady MHD Mixed Convection Flow of Jeffrey Fluid Past a Radiating Inclined Permeable Moving Plate in the Presence of Thermophoresis Heat Generation and Chemical Reaction

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Abstract

In this manuscript a study of the unsteady magneto-hydrodynamic mixed convection Jeffrey fluid flow over an inclined permeable moving plate in presence of thermal radiation, heat generation, thermophoresis effect and homogenous chemical reaction, subjected to variable suction has been undertaken. The governing equations are solved by using a regular perturbation technique. The expressions for the distributions of velocity, temperature and species concentration are obtained. With the aid of these, the expressions for skin friction, Nusselt number and Sherwood number also have been derived. The influences of various physical parameters involved in the problem on the above mentioned quantities are discussed with the help of graphs and tables. From the significant findings, it has been found that the velocity increases with an increase in Soret number in the presence of permeability. It shows reverse effects in the case of heat absorption coefficient, magnetic parameter, radiation parameter and chemical reaction parameter.

Keywords: Non-Newonian fluid, MHD, Radiation, Incompressible flow, Porous media, Convection.

Subject Classification codes: 76A05, 76Wxx, 83C30, 76Dxx, 76Sxx, 76E06.

Nomenclature

A	Amplitude of suction velocity	$t^{\backslash}$	Dimensional time
B_0	Magnetic field strength	t	Dimension less time

C	Concentration	U_0	Scale of free stream velocity
C_p	Specific heat at constant pressure	u', v'	Dimensional velocity components
C_f	Skin friction co-efficient	u, v	Velocity components
D	Mass diffusion co-efficient	V_0	Scale of suction velocity
D_1	Thermal diffusion co-efficient	x', y'	Dimensional distances along and perpendicular to the plate respectively
$e_{b\lambda}$	Plank's function	x, y	Distance along and perpendicular to the plate respectively
F	Radiation parameter		**Greek Symbols**
G	Acceleration due to gravity	χ	Dimension less material parameter
Gr	Thermal Grashof number	α	Inclination parameter
Gm	Solutal Grashof number	ε	Scalar constant
k'	Permeability of the porous medium	Φ	Dimensionless heat absorption coefficient
K_r	Chemical reaction parameter	κ	Thermal conductivity
$K_{\lambda w}$	Absorption co-efficient	σ	Electrical conductivity
M	Magnetic field parameter	ρ	Density of the fluid
N	Dimension less material parameter	μ	Dynamic viscosity
N	Dimension less exponential index	υ	Kinematic viscosity
Nu	Nusselt number	τ	Skin Friction coefficient
Pr	Prandtl number	η	Dimensionless normal distance
Q_0	Heat absorption co-efficient	β_c	Coefficient of volumetric concentration expansion
Re_x	Local Reynolds number	β_T	Coefficient of volumetric thermal expansion
Sc	Schmidt number		**Subscripts and Superscripts**
λ_1	Jeffrey parameter	$'$	Dimensional properties
S_0	Soret Number	P	Plate
β_c	Coefficient of volumetric concentration expansion	W	Wall condition
β_T	Coefficient of volumetric thermal expansion	∞	Free stream condition
S_h	Sherwood number	K_c	Dimensionless Chemical reaction parameter
T	Temperature	u'_p	Moving plate velocity

I) Introduction

The order of differential system in non-Newtonian fluid situation is higher than that of the viscous material. A variety of non-Newtonian fluid models have been proposed in the literature keeping in view of their rheological features. In these fluids, the constitutive relationships between stress and rate of strain are much complicated in comparison to the Navier-Stokes equations. Non-Newtonian fluid model has attracted many researchers. The most common and simplest model of non-Newtonian fluids is Jeffrey fluid. The Jeffrey fluid is a better model for physiological fluids and this fluid model is capable of describing the characteristics of relaxation, because Newtonian fluid model can be deduced from this as a special case by taking $\lambda_1 = 0$. The Jeffrey's fluid model degenerates to a Newtonian fluid at a very high wall shear stress i.e. when the wall stress is much greater than the yield stress. This fluid model also approximates reasonably well the rheological behavior of other liquids including physiological suspensions, foams, geological materials, cosmetics, and syrups. Several researchers have studied Jeffrey fluid flows under different conditions. Interest in the boundary layer flows of non-Newtonian fluids has increased due to its applications in science and engineering including thermal oil recovery, food and slurry transportation, polymer and food processing, etc.

Sandeep and Sulochana[1] proposed a new mathematical model for investigating the momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids over a stretching surface in the presence of transverse magnetic field, non-uniform heat source/sink, thermal radiation and suction effects. The researchers f this paper have found an excellent agreement of the present results by comparing with the published results. Results indicate that the Jeffrey nanofluid has better heat transfer performance when compared to the Maxwell and Oldroyd-B nanofluids. Malik *et al*.[2] presented the Jeffrey fluid flow with a pressure-dependent viscosity. They considered two types of flow problems, *i.e.* Couette and Poiseulle flow. Qasim[3] investigated on heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source. Das *et al*.[4] investigated numerically the influence of melting heat transfer and thermal radiation on MHD stagnation point flow of an electrically conducting non-Newtonian fluid (Jeffrey fluid) over a stretching sheet with partial surface slip. His analysis revealed that the fluid temperature is higher in case of Jeffrey fluid than that of Newtonian fluid. Zeeshan and Majeed[5] investigated the flow and heat transfer of Jeffery fluid past a linearly stretching sheet with the effect of a magnetic dipole. Abdul Gaffar *et al.*[6] examined the nonlinear, steady state boundary layer flow, heat and mass transfer of an incompressible non-Newtonian Jeffrey's fluid past a semi-infinite vertical plate. He stated that this model applications in metallurgical materials processing, chemical engineering flow control, etc.

In many industrial applications, external magnetic field is used to control construction of material. Mixed convection flow and heat transfer over a continuously moving surface is applicable to many industrial fields such as hot rolling, paper production, wire drawing, glass fibre production, aerodynamic extrusion of plastic sheets, the boundary-layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Pal and Chatterjee[7] studied various aspects of mixed convection problems such as heat and mass transfer, suction/injection, thermal radiation, MHD flow, porous media, slip flows. Javaherdeh *et al*.[8] explored the natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium. The detailed aspects of this topic have been dealt in the books by Ingham and Pop[9]. Raju *et al*.[10] studied the heat and mass transfer in MHD mixed convection flow on a moving inclined porous plate.

Some kind of chemical reaction is observed if there is a foreign entity in air or water. During a chemical reaction between two species heat is generated. Generally the reaction rate depends on the concentration of the species itself. If the rate of reaction is directly proportional to concentration itself, then it is said to be of first order. Srinivas *et al*.[11] presented an analysis which is of special interest for clinicians of radiation and chemical therapy for cancer and other tumour related diseases.

The influence of heat generation or absorption in moving fluids and its study remains essential while dealing with chemical reactions and separating fluids. Heat generation alters the temperature distribution as a result of the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers, etc. It has been found that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapour deposition (MCVD) process as currently used in the fabrication of optical fiber performs. Thermophoretic deposition of radioactive particles is considered to be one of the important factors causing accidents in nuclear reactors.

Chamkha *et al*.[12] studied magnetic field effects on electrically conducting free convection heat and mass transfer from an inclined plate with heat generation/absorption. Ibrahim *et al.*[13] discussed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. Elbashbeshy *et al.*[14] discussed the impact of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorption. Reddy *et al.*[15] examined effects of variable viscosity and thermal diffusivity on MHD free convection flow along a moving vertical plate embedded in a porous medium with heat generation. Raju *et al.*[16] studied MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule's heating.

Rushi Kumar *et al.*[17] analyzed the double dispersion effects on unsteady, free convective, chemically reacting, MHD viscoelastic fluid (Walters liquid-B model) flow over a vertical cone and a flat plate saturated with non-Darcy porous medium in the presence of Soret and Dufour effects. Rushi Kumar *et al.*[18] studied heat and mass transfer characteristics in the unsteady free convective flow of an incompressible viscoelastic fluid over a moving vertical cone and a flat plate in the presence of magnetic field and higher order chemical reaction. Prakash *et al.*[19] studied the heat and mass transfer characteristics of unsteady mixed convective MHD flow of a heat absorbing fluid in an accelerated vertical wavy plate, subject to varying temperature and mass diffusion, with the influence of buoyancy, thermal radiation and Dufour effect.

Seth *et al.*[20] studied the effects of Hall current and rotation on a hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusion. Seth *et al.*[21] investigated on the unsteady hydromagnetic free convection heat and mass transfer flow with Soret and Hall effects of a viscous, incompressible, electrically conducting and optically thick radiating fluid past an impulsively moving infinite vertical plate with ramped temperature through a uniform porous medium in a rotating system in the presence of first order chemical reaction. Seth *et al.*[22, 23, 24] have worked in the areas, the hydromagnetic natural convection heat and mass transfer flow of a viscous, incompressible, chemical reaction of heat absorbing, ramped temperature through a porous medium. Jasmine Benazir *et al.*[25] study focused on the effects of double dispersion, non-uniform heat source/sink and higher order chemical reaction on unsteady free convective, MHD Casson fluid flow over a vertical cone and flat plate saturated with porous medium. Seth *et al.*[26] investigated on unsteady MHD natural convection flow with heat and mass transfer of an electrically conducting viscous incompressible chemically reactive and optically thin heat radiating fluid through a saturated porous medium past an infinite moving vertical plate with arbitrary ramped temperature. It is found that, for isothermal plate, the fluid temperature and, the rate of heat transfer at isothermal plate approaches steady state when $t > 1.5$. Ramesh Babu *et al.*[27] investigated on unsteady MHD free convective flow of a visco-elastic fluid past an infinite vertical porous moving plate with variable temperature and concentration.

In this paper, the author/researchers have studied, The effects of various characteristics of heat and mass transfer on Jeffrey fluid in the presence of thermal radiation, heat generation, chemical reaction and thermal diffusion past an inclined plate.The objective of this study is to investigate analytically an unsteady MHD mixed convection flow of Jeffrey fluid past a radiating inclined permeable moving plate in the presence of thermophoresis heat absorption and chemical reaction. The uniqueness of this study is the consideration of well known non-Newtonian fluid namely Jeffrey fluid past an inclined plate which is an extension of the work done by Raju *et al.*[20], who studied various flow characteristics of a Newtonian fluid. In the absence of Jeffrey parameter, the results of our study are in good agreement with the results of Raju *et al.*[20].

II) Formulation of the Problem:

We have considered an unsteady MHD flow of a laminar, Jeffrey, incompressible, electrically conducting, double diffusive and generating fluid past a semi infinite inclined permeable moving plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field B_0 in the presence of thermal radiation and the first order homogeneous chemical reaction. It is assumed that there is no applied voltage which implies the absence of an electrical field. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible (Cramer and Pai[21]). $x'-$ Axis is taken in the upward direction along with the ûow and $y'-$ axis is taken perpendicular to it. At $y'=0$ the plate is initially assumed to be moving with a uniform velocity u'_p in the direction of the ûuid ûow, and the free stream velocity follows the exponentially increasing small perturbation law. Besides that, it is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time. By considering the above assumptions, and the assumptions followed by Raju *et al.*[20], the governing equations are given as follows

$$\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\frac{\partial u'}{\partial t'}+\mathrm{v}'\frac{\partial u'}{\partial y'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\frac{\upsilon}{1+\lambda_1}\frac{\partial^2 u'}{\partial y'^2}+g\beta_T(\mathrm{T}'-\mathrm{T}'_\infty)\cos\alpha+g\beta_C(\mathrm{C}'-C'_\infty)\cos\alpha-\frac{\sigma B_0^{\,2}u'}{\rho}-\frac{\upsilon}{k'}u' \tag{2}$$

$$\frac{\partial T'}{\partial t'}+\mathrm{v}'\frac{\partial T'}{\partial y'}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q_r'}{\partial y'}+\frac{Q'}{\rho C_p}\frac{\partial T'}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+\mathrm{v}'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}+D_1\frac{\partial^2 \mathrm{T}'}{\partial y'^2}-K_c(\mathrm{C}'-C'_\infty) \tag{4}$$

Under the above assumptions, the appropriate boundary conditions for the distributions of velocity, temperature and concentration are given by

$$u'=u'_p,\ T'=T'_w+\varepsilon\,\mathrm{e}^{n't'}(T'_w-T'_\infty),\ \ \mathrm{C}'=\mathrm{C}'_w+\varepsilon\,\mathrm{e}^{n't'}(\mathrm{C}'_w-C'_\infty) \qquad \text{at } y'=0 \tag{5}$$

$$u'\to u'_\infty=U_0(1+\varepsilon\,\mathrm{e}^{n't'}),\quad T'\to T'_\infty,\quad C'\to C'_\infty \qquad \text{as } y'\to\infty \tag{6}$$

it is known from Eq. (1) that the suction velocity at the plate surface is a function of time only and it is assumed in the following form

$$\mathrm{v}'=-V_0(1+\varepsilon A\,\mathrm{e}^{n't'}) \tag{7}$$

Outside the boundary layer Eq. (2) modifies as

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'}=\frac{dU'_\infty}{dt'}+\frac{\upsilon}{k'}U'_\infty+\frac{\sigma}{\rho}B_0^{\,2}U'_\infty \tag{8}$$

we consider a mathematical model, for an optically thin limit gray gas near equilibrium in the form given by.

$$\frac{\partial q_r'}{\partial y'}=4(T'-T'_w)I \tag{9}$$

where $I=\int_0^\infty K_{\lambda w}\left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda$, $K_{\lambda w}$ Is the absorption coefficient at the wall and $e_{b\lambda}$ is the Planck's function.

By introducing the following non-dimensional quantities and parameters

$$u=\frac{u'}{U_0},\upsilon=\frac{\upsilon'}{V_0},\eta=\frac{V_0 y'}{\upsilon},U_\infty=\frac{U'_\infty}{U_0},U_p=\frac{u'_p}{U_0},t=\frac{t'V_0^2}{\upsilon},\theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},$$

$$n=\frac{n'\upsilon}{V_0^2},k=\frac{k'V_0^2}{\upsilon^2},\Pr=\frac{\mu C_p}{K},Sc=\frac{\upsilon}{D},M=\frac{\sigma}{\rho}\frac{B_0^2}{V_0^2},\ S_0=\frac{D_1(T'_w-T'_\infty)}{\upsilon(C'_w-C'_\infty)},K_r=\frac{K'_c\,\upsilon}{V_0^2}, \tag{10}$$

$$F=\frac{4I_1\upsilon}{\rho C_p V_0^2},G_r=\frac{\upsilon\beta_T g(T'_w-T'_\infty)}{U_0V_0^2},\ G_m=\frac{\upsilon\beta_C g(C'_w-C'_\infty)}{U_0V_0^2},\phi=\frac{Q'}{\rho C_p V_0}$$

in view Eqs. (7) to (10), (2) to (4) are reduced to the following non-dimensional form,

$$\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial\eta}=\frac{dU_\infty}{dt}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial\eta^2}+Gr\,\theta\cos\alpha+Gm\,C\cos\alpha+N(U_\infty-u) \tag{11}$$

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\theta}{\partial\eta}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial\eta^2}-\chi\theta \tag{12}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial C}{\partial\eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial\eta^2}-K_rC+S_0\frac{\partial^2\theta}{\partial\eta^2} \tag{13}$$

Where $N=M+\frac{1}{k},\chi=\phi+F$

The dimension less form of the boundary conditions are become

$$u=U_P,\quad \theta=1+\varepsilon Ae^{nt},\quad C=1+\varepsilon Ae^{nt}\qquad \text{at } \eta=0 \tag{14}$$

$$u\to U_\infty=1+\varepsilon Ae^{nt},\ \ \theta\to 0,\quad C\to 0\qquad \text{at } \eta\to\infty \tag{15}$$

III) Solution of the problem:

The set of Eqs. (11) to (13) are partial differential equations which cannot be solved in closed form. However, these can be solved by reducing them into a set of ordinary differential equations using the following perturbation technique. We now represent the velocity, temperature and concentration distributions in terms of harmonic and non-harmonic functions as

$$u=u_0(\eta)+\varepsilon\exp(nt)\ u_1(\eta)+O(\varepsilon^2)+------- \tag{16}$$

$$\theta=\theta_0(\eta)+\varepsilon\exp(nt)\theta_1(\eta)+O(\varepsilon^2)+------- \tag{17}$$

$$C=C_0(\eta)+\varepsilon\exp(nt)C_1(\eta)+O(\varepsilon^2)+------- \tag{18}$$

Substituting Eqs. (16) to (18) into Eqs.(11) to (13) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms of ε, we obtain the following pairs of equations of order zero and order one.

$$\frac{d^2u_0}{d\eta^2}+(1+\lambda_1)\frac{du_0}{d\eta}-N(1+\lambda_1)u_0=-(1+\lambda_1)N-(1+\lambda_1)Gr\theta_0\cos\alpha-(1+\lambda_1)GmC_0\cos\alpha \tag{19}$$

$$\frac{d^2\theta_0}{d\eta^2}+\Pr\frac{d\theta_0}{d\eta}-\Pr\chi\theta_0=0 \tag{20}$$

$$\frac{d^2C_0}{d\eta^2}+Sc\frac{dC_0}{d\eta}-\mathrm{Sc\ Kc}\ C_0=-Sc\ S_0\ \frac{d^2\theta_0}{d\eta^2} \tag{21}$$

$$\frac{d^2u_1}{d\eta^2}+(1+\lambda_1)\frac{d\,u_1}{d\,\eta}-(N+n)(1+\lambda_1)u_1=(1+\lambda_1)\left[-(N+n)-A\frac{d\,u_0}{d\,\eta}-Gr\theta_1\cos\alpha-GmC_1\cos\alpha\right] \tag{22}$$

$$\frac{d^2\theta_1}{d\eta^2}+\Pr\frac{d\,\theta_1}{d\,\eta}-\Pr(\mathrm{n}-\chi)\theta_1=-A\Pr\frac{d\,\theta_0}{d\,\eta} \tag{23}$$

$$\frac{d^2C_1}{d\eta^2}+\mathrm{S}_c\frac{d\,C_1}{d\,\eta}-(\mathrm{K}_c\,\mathrm{S}_c+\mathrm{S}_c\,n)\mathrm{C}_1=-S_c\,S_0\,\frac{d^2\theta_1}{d\eta^2}-\mathrm{A}\,\mathrm{S}_c\mathrm{C}_0 \tag{24}$$

Where the prime denotes differentiation with respect to η. The corresponding boundary conditions are now given by

$$u_0=U_p,\,u_1=0,\theta_0=1,\theta_1=1,\mathrm{C}_0=1,C_1=1 \qquad \text{at } \eta=0 \tag{25}$$

$$u_0=1,u_1=1,\theta_0\to 0,C_0\to 0,\mathrm{C}_1\to 0 \qquad \text{at } \eta\to\infty \tag{26}$$

Now by using the boundary conditions (25) to (26) and solving the set of Eqs. (19) to (24) we get the following solutions

$$u_0=1+k_5\,\mathrm{e}^{(-m_{10}\eta)}+k_4\,\mathrm{e}^{(-m_2\,\eta)}+k_2\,\mathrm{e}^{(-m_6\,\eta)} \tag{27}$$

$$u_1=\left[1+k_{18}\,\mathrm{e}^{(-m_{12}\,\eta)}+k_8\,\mathrm{e}^{(-m_8\,\eta)}+k_{12}\,\mathrm{e}^{(-m_{10}\,\eta)}+k_{15}\,\mathrm{e}^{(-m_4\,\eta)}+k_{16}\,\mathrm{e}^{(-m_2\,\eta)}+k_{17}\,\mathrm{e}^{(-m_6\,\eta)}\right] \tag{28}$$

$$\theta_0=\mathrm{e}^{(-m_2\,\eta)} \tag{29}$$

$$\theta_1=(1-L_1)\,\mathrm{e}^{(-m_4\,\eta)}+L_1\,\mathrm{e}^{(-m_2\,\eta)} \tag{30}$$

$$\mathrm{C}_0=(1-L_2)\,\mathrm{e}^{(-m_6\,\eta)}+L_2\,\mathrm{e}^{(-m_2\,\eta)} \tag{31}$$

$$\mathrm{C}_1=(1-L_8)\,\mathrm{e}^{(-m_8\,\eta)}+L_7\,\mathrm{e}^{(-m_2\,\eta)}+L_3\,\mathrm{e}^{(-m_4\,\eta)}+L_5\,\mathrm{e}^{(-m_6\,\eta)} \tag{32}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(\eta,t)=1+k_5\,\mathrm{e}^{(-m_{10}\eta)}+k_4\,\mathrm{e}^{(-m_2\eta)}+k_2\,\mathrm{e}^{(-m_6\eta)}+\varepsilon\,\mathrm{e}^{(nt)}\begin{pmatrix}1+k_{18}\,\mathrm{e}^{(-m_{12}\eta)}+k_8\,\mathrm{e}^{(-m_8\eta)}+k_{12}\,\mathrm{e}^{(-m_{10}\eta)}+k_{15}\,\mathrm{e}^{(-m_4\eta)}\\+k_{16}\,\mathrm{e}^{(-m_2\eta)}+k_{17}\,\mathrm{e}^{(-m_6\eta)}\end{pmatrix} \tag{33}$$

$$\theta(\eta,t)=\mathrm{e}^{(-m_2\eta)}+\varepsilon\,\mathrm{e}^{(nt)}\left[(1-L_1)\,\mathrm{e}^{(-m_4\eta)}+L_1e^{(-m_2\eta)}\right] \tag{34}$$

$$\mathrm{C}(\eta,\mathrm{t})=\left[(1-L_2)\,\mathrm{e}^{(-\mathrm{m}_6\,\eta)}+L_2\,\mathrm{e}^{(-\mathrm{m}_2\,\eta)}\right]+\varepsilon\,\mathrm{e}^{(nt)}\left[(1-L_8)\,\mathrm{e}^{(-\mathrm{m}_8\,\eta)}+L_7\,\mathrm{e}^{(-\mathrm{m}_2\,\eta)}+L_3\,\mathrm{e}^{(-\mathrm{m}_4\,\eta)}+L_5\,\mathrm{e}^{(-\mathrm{m}_6\,\eta)}\right] \tag{35}$$

Skin friction:

Very important physical quantity at the boundary is the skin function which is given in the non-dimensional form and derives as

$$\begin{aligned}C_f&=\frac{\tau'_w}{\rho U_0V_0}=\frac{\partial u}{\partial\eta}\Big|_{\eta=0}\\&=(-m_{10}k_5-m_2k_4-m_6k_2)+\varepsilon\,\mathrm{e}^{(nt)}\left[-m_{12}k_{18}-m_8k_8-m_{10}k_{12}-m_4k_{15}-m_2k_{16}-m_6k_{17}\right]\end{aligned} \tag{36}$$

Another physical quantity like rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are also derived and given below respectively

$$N_u = x\frac{\left.\frac{\partial T'}{\partial y'}\right|_{y=0}}{T'_w - T'_\infty}$$

$$N_u \operatorname{Re}_x^{-1} = \left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = -m_2 + \varepsilon\, e^{(nt)}\left[-m_4(1-L_1) - m_2L_1\right] \tag{37}$$

$$S_h = x\frac{\left.\frac{\partial C'}{\partial y'}\right|_{y=0}}{C'_w - C'_\infty}$$

$$S_h \operatorname{Re}_x^{-1} = \left.\frac{\partial C}{\partial\eta}\right|_{\eta=0} = \left[-m_6(1-L_2) - m_2L_2\right] + \varepsilon\, e^{(nt)}\left[-m_8(1-L_7) - m_2L_8 - m_4L_3 - m_6L_5\right] \tag{38}$$

IV) Results and Discussion

In this paper, the thermophoresis effect on unsteady magneto-hydrodynamic mixed convection Jeffrey fluid flow over an inclined permeable moving plate in presence of thermal radiation, heat generation and homogenous chemical reaction, subjected to variable suction are discussed and presented in detail through Figs. 1-19. The governing equations are solved by analytical method. The velocity, temperature and concentration distributions, Nusselt number, Sherwood number and skin friction are analyzed. The effects of Jeffrey parameter (λ_1), Angle of inclination (α), Prandtl number (Pr), solutial Grashof number (Gm) and Heat generation coefficient(ϕ) on the flow has been discussed are shown in Figse. (1) - (5). The effects Heat generation coefficient (ϕ), Prandtl number (Pr) and on temperature are shown in Figs. (6)-(7). The effects of Schmidt number (Sc), Sorret number (So) and Chemical reaction (Kr) on concentration are shown in Figs. (8)-(10). Also the variations in skin friction, rate of heat transfer and Sherwood number for various parameters are shown in Figs. (11) - (19).

Validity of our results :

For the validity our results, we have compared our results with the existing results of Raju *et al*.[25] in the absence of inclined angel and Jeffrey fluid on skin friction in table 1. From this table it is found that our results are in good agreement with the existing results Raju *et al*.[20].

Figs.1-5, represent the variations in velocity distribution for diffrent values of Jeffrey parameter λ_1, inclined angle α , Prandtl number Pr , modified grash of number Gm and heat absorption coefficient ϕ. From Fig.1, it is seen that velocity increases with the increasing values of λ_1. Fig.2 shows that the fluid velocity increases with a decrease of inclined angle α , it is fact that the inclination decreases automatically fluid velocity increases

Table 1. Comparison of skin friction values with Raju *et al.*[25] and present study when $\lambda_1 = 0$ and $\alpha = 0$.

Study of Raju *et al.*[20], $\lambda_1 = 0$, $\alpha = 0$		Present study $\lambda_1 = 0$, $\alpha = 0$	
M	$\lvert\tau\rvert$	M	$\lvert\tau\rvert$
0	22.3714	0	22.3914
0.1	17.9509	0.1	17.8909
0.2	14.9067	0.2	14.9167
0.3	12.6155	0.3	12.6195
0.4	10.7685	0.4	10.7885
0.5	9.1913	0.5	9.1900
0.6	7.7736	0.6	7.7636
0.7	6.4374	0.7	6.3974
0.8	5.1189	0.8	5.1089
0.9	3.7583	0.9	3.7823
1.0	2.2895	1.0	2.2995

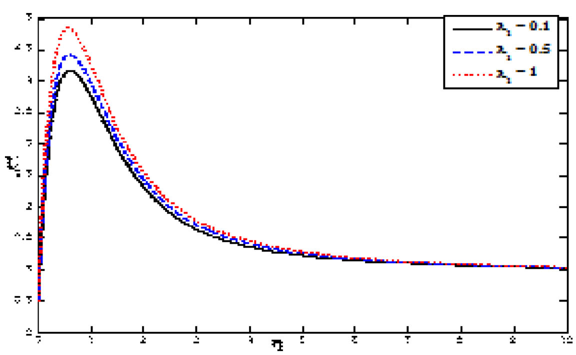
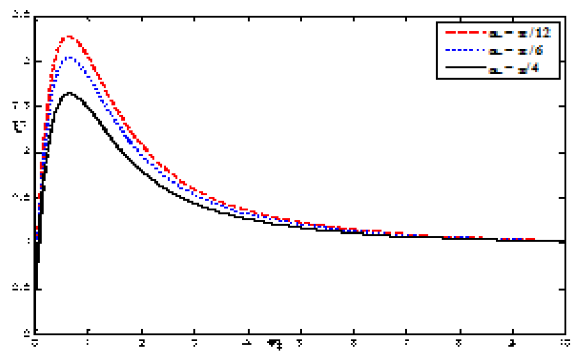


Figure 1: The velocity profiles for different values of Jeffrey parameter (λ_1).



Figur.2: The velocity profiles for different values of inclined angle α .

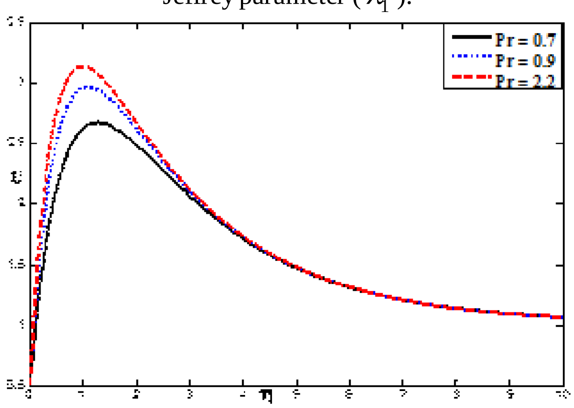


Figure3: The velocity profiles for different values of prandtal number Pr .

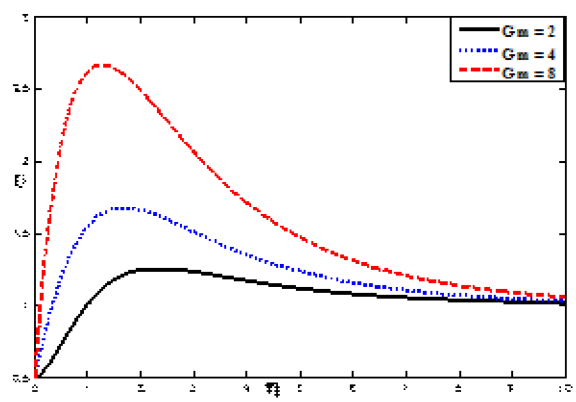


Figure 4: The velocity profiles for different values of modified grashof number Gm .

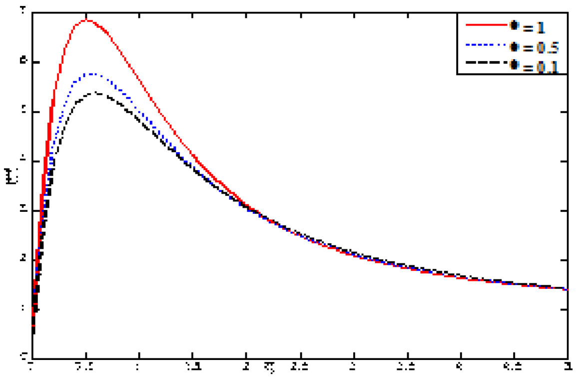


Figure 5 :The velocity profiles for different values of Heat absorption coefficient ϕ

Fig.3, indicates that a rise in Pr substantially increases the velocity. A similar effect is noticed from Fig. 4, in the presence of modified Grashof number, which also increases fluid velocity. Fig. 5 illustrates the influence of heat absorption coefficient ϕ on the velocity. From this figure, it is observed that the velocity increases with decreases of ϕ. It is also observe that the velocity reaches finaly around unity, when $\eta \to \infty$.

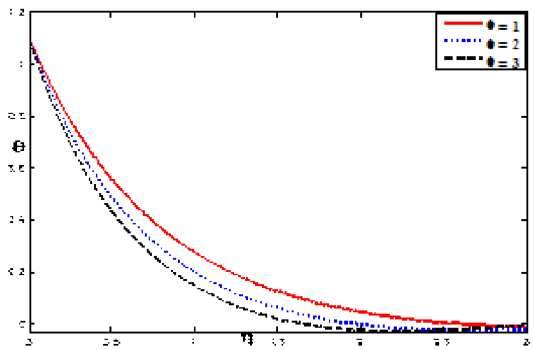


Figure 6 : The temperature profile for different values of Heat absorption coefficient ϕ

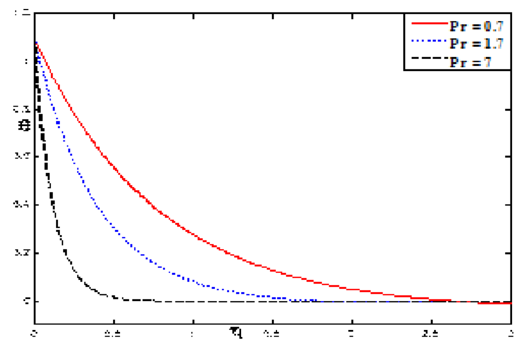


Figure 7 : The temperature profile for different values of Prandtl number Pr

Fig. 6, shows that The temperature profile for different values of Heat absorption coefficient ϕ, here we observe that the temperature decreases with an increase of ϕ. Fig. 7, indicates that a rise in Pr substantially reduces the temperature in the viscous fluid. It can be found from Fig. 7 that the solutal boundary layer thickness of the fluid enhances with the increase of Pr.

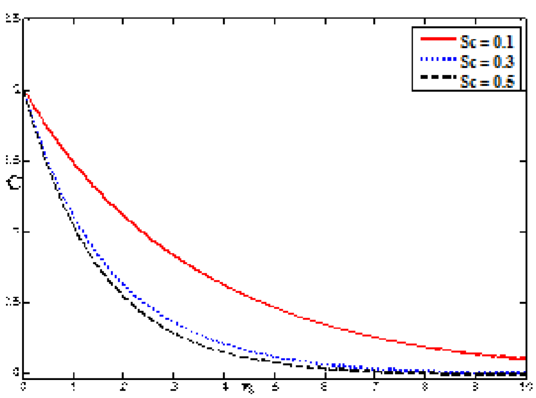


Figure 8 :The concentration profile for different values of Schmidt number *Sc*

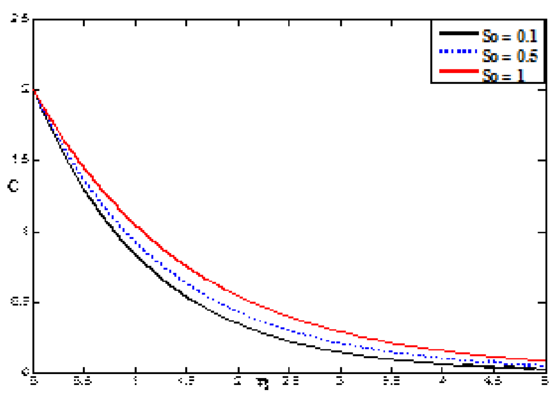


Figure 9 : The concentration profile for different values of Soret number *So*

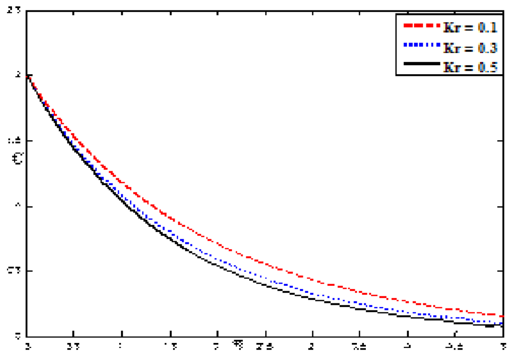


Figure 10 :The concentration profile for different values of chemical reation parameter K_r

Influence of Schmidt number on concentration is shown in Fig. 8, from this figure it is noticed that concentration decreases with an increase in Schmidt number. Because, Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes.

Fig. 9, shows that the Concentration profile for different values of Soret number So, here we observe that the Concentration increases with an increase of So. decreases as chemical reaction Kr increases. From Fig. 10, we observe that the Concentration profile for different values of chemical reation parameter K_r, here we observe that the Concentration decreases with an increas of K_r.

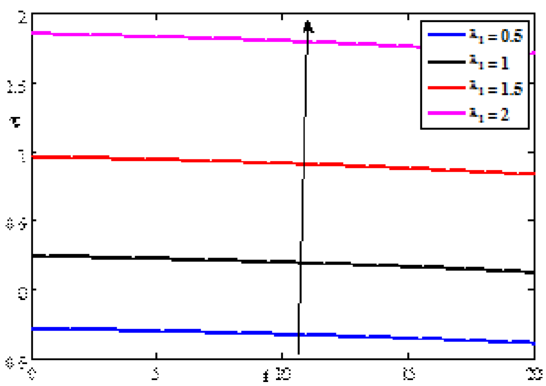


Figure 11: The skin friction profile for different values of Jeffrey parameter λ_1

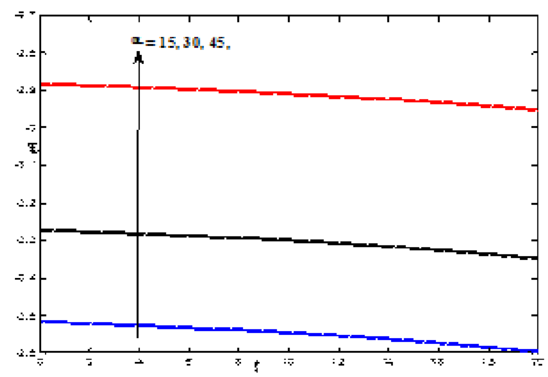


Figure 12: The skin friction profile for different values of inclined angle α.

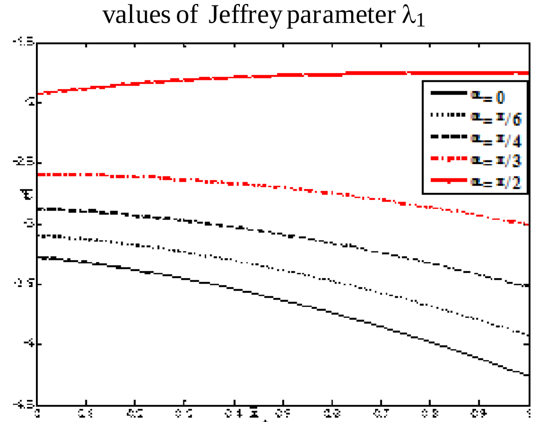


Figure 13 : The skin friction profile for different values of inclined angle α.

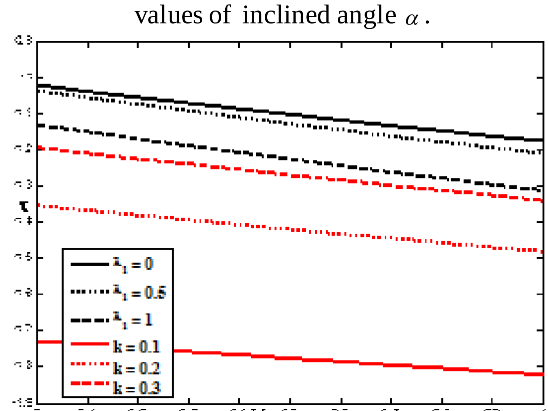


Figure 14 : The skin friction profile for different values of Jeffrey parameter λ_1 and permeability parameter k

Fig. 11. illustrates the influence of Jeffrey parameter λ_1 on the Skin friction various as time. From this figure it is observed that 'τ' increases with an increase in λ_1. The influences of the inclination angle α on the Skin friction τ various as time t are shown in Fig. 12. From this figure it is seen that 'τ' increases with an increase in α. Figure 13 represents the effects of inclination angle α on Skin friction τ with various as Jeffrey parameter. From this figure it is observed that 'τ' increases with an increase in α with various as Jeffrey parameter from 0 to 1. The influences of the Jeffrey parameter λ_1 and permeability parameter k with various as magnetohydrodynamics M on the Skin friction τ, are shown in Fig. 14. From this figure it is seen that 'τ' increases with an increase in k, but the opposite behavior appeared for Jeffrey parameter λ_1, 'τ' increases with an decrease in λ_1.

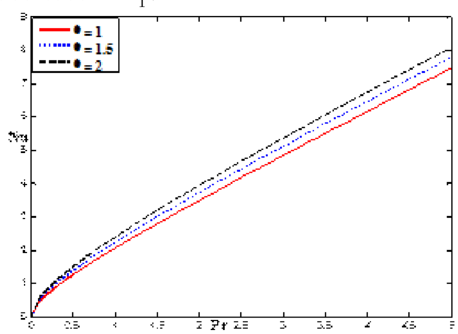


Figure 15 : The Nusselt number profile for different values of heat absorption coefficient ϕ

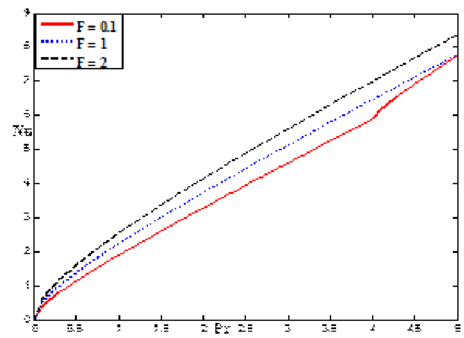


Figure 16 : The Nusselt number profile for different values of Radiation parameter F

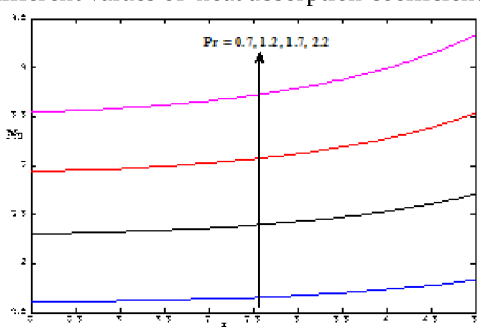


Figure 17 : The Nusselt number profile for different values of prandtl number Pr

Nusselt number Nu is presented in Figs.15 and 16 against Prandtl number Pr for different values of heat absorption coefficient ϕ and radiation parameter F. From these figures, it is found that the Nusselt number increases with an increase in heat absorption coefficient ϕ and radiation parameter F. Nusselt number Nu is presented in Figs.17 against time t for different values of Prandtl number Pr. From this figure, it is observed that the Nusselt number increases with an increase in Prandtl number Pr.

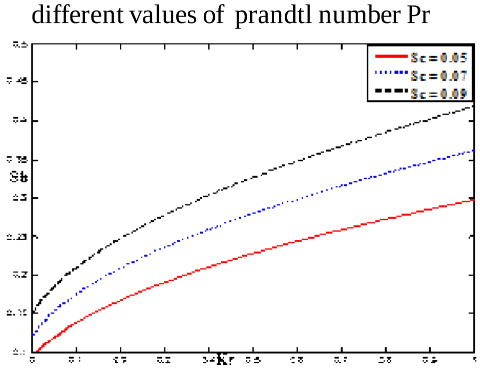


Figure 18 : The sherwood number profile for different values of Schmidt number Sc.

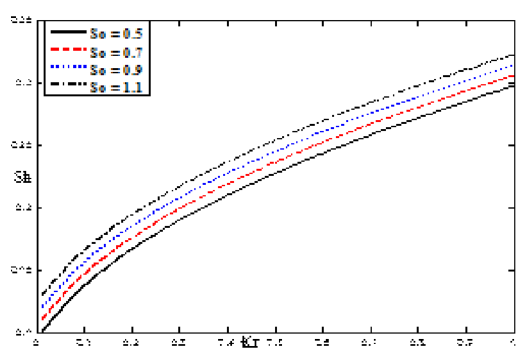


Figure 19: The sherwood number profile for different values of Soret number So.

From figs. 18 and 19, we studied about Sherwood number against Chemical reaction parameter Kr for various values of Schmidt number Sc and Soret number So . From these figures, it is observed that Share wood number increases with an increase in Schmidt numberand Soret number So . The presence of chemical reaction enhances the rate of mass transfer.

V) Conclusions

Unsteady MHD mixed convection flow of Jeffrey fluid past a radiating inclined permeable moving plate in the presence of thermophoresis heat absorption and chemical reaction. The following is the summery of conclusions.

a. Velocity distribution is observed to increase with an increase in magnetic field parameter, inclined angle, Grashoff number and modified Grashoff number, where as it shows reverse effect in the case of Jeffrey fluid, permeability of the porous medium, Prandtl number and heat generation coefficient.

b. Temperature distribution increases with an increase in radiation parameter, Prandtl number and dimension less heat absorption coefficient.

c. Concentration distribution decreases as Schmidt number, rate of chemical and Soret number increase.

Scope of future work:

Now we present some informal observation problems pertaining to possible extension of this paper to different problems. We briefly outline some interesting problems which can be taken up in the future.

1. Non-Newtonian fluid flow over a cone with gyrotactic microorganisms and flux conditions.
2. Heat and mass transfer investigations of Magnetohydrodymaics(MHD) over a Casson fluid over a rotating cone in a rotating frame with heat flux.
3. Free convection of MHD non-Newtonian over a rotating plate with partial slip, gyrotactic microorganisms and flux conditions.

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