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On $\mathcal{S}arw$ continuous and $\mathcal{S}arw$ -Irresolute Maps in Topological Spaces

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Abstract

In this paper, a new class of continuous maps called $\mathcal{S}arw$ -continuous maps in topological spaces are introduced and studied. Also some of their properties have been investigated. We also introduce $\mathcal{S}arw$ -irresolute maps, strongly $\mathcal{S}arw$ -continuous maps, perfectly $\mathcal{S}arw$ -continuous maps and discuss some properties.

Key words: $\mathcal{S}arw$ -closed sets, $\mathcal{S}arw$ -open sets, $\mathcal{S}arw$ -continuous maps, $\mathcal{S}arw$ -irresolute maps, strongly $\mathcal{S}arw$ -continuous maps and perfectly $\mathcal{S}arw$ -continuous maps.

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1. Introduction

The concept of continuous functions plays a very important role in general topology. The regular continuous and completely continuous functions are introduced and studied by Arya S P³. Later, R S walli *et al.*³¹ introduced and investigated α rw-continuous functions in topological space. Recently, Basavaraj M Ittanagi *et al.*⁵ introduced and studied the basic properties of $\mathcal{S}arw$ -closed sets in topological space. The aim of this paper is to introduce $\mathcal{S}arw$ -continuous and $\mathcal{S}arw$ -irresolute maps in topological space. Also, we study some of their basic properties of $\mathcal{S}arw$ -continuous functions.

2. Preliminaries :

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no

separation axioms are assumed unless otherwise mentioned. For a subset A of a space X , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of A and the interior of A respectively. $X-A$ or A^c denotes the complement of A in X .

We recall the following definitions and results.

Definition 2.1: A subset A of a topological space (X, τ) is called,

- 1) Semi-open set¹⁷ if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
- 2) Pre-open set²¹ if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.
- 3) α -open set¹⁴ if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
- 4) Semi-preopen set¹ (β -open) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and a semi-pre closed set ($= \beta$ -closed) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
- 5) Regular open set²⁶ if $A = \text{int}(\text{cl}(A))$ and a regular closed set if $A = \text{cl}(\text{int}(A))$.
- 6) Regular semi open set¹⁰ if there is a regular open set U such that $U \subseteq A \subseteq \text{cl}(U)$.
- 7) Regular α -open set²⁸ (briefly α -open) if there is a regular open set U s.t $U \subseteq A \subseteq \alpha\text{cl}(U)$.

Definition 2.2: A subset A of a topological space (X, τ) is called

- 1) Semi α regular weakly closed (briefly sarw -closed) set⁵ if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is arw -open in X .
- 2) generalized pre regular closed set (briefly gpr -closed)¹² if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- 3) $w\alpha$ -closed set⁷ if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is w -open in X .
- 4) α regular ω -closed (briefly $\text{ar}\omega$ -closed) set³¹ if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is rw -open in X .
- 5) regular generalized α -closed set (briefly, $\text{rg}\alpha$ -closed)²⁸ if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .
- 6) generalized closed set (briefly g -closed)¹³ if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 7) generalized semi-closed set (briefly gs -closed)⁴ if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 8) generalized semi pre regular closed (briefly gspr -closed) set²⁴ if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- 9) strongly generalized closed set²⁴ (briefly g^* -closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- 10) α -generalized closed set (briefly αg -closed)¹⁹ if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 11) w -closed set²⁷ if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- 12) weakly generalized closed set (briefly, wg -closed)²³ if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 13) regular weakly generalized closed set (briefly rwg -closed)²³ if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- 14) semi weakly generalized closed set (briefly swg -closed)²³ if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .
- 15) generalized pre closed (briefly gp -closed) set if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 16) regular w -closed (briefly $\text{r}\omega$ -closed) set⁸ if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .
- 17) g^* -pre closed (briefly g^*p -closed)²⁹ if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- 18) generalized regular closed (briefly gr -closed) set⁹ if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 19) regular generalized weak (briefly rgw -closed) set²² if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X .

- 20) weak generalized regular- α closed (briefly wgra-closed) set¹⁵ if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .
 - 21) regular pre semi-closed (briefly rps-closed) set²⁵ if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is rg- open in X .
 - 22) generalized pre regular weakly closed (briefly gprw-closed) set¹⁶ if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi- open in X .
 - 23) α -generalized regular closed (briefly α gr-closed) set³⁰ if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
 - 24) R^* -closed set¹³ if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi- open in X .
- The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- 1) regular-continuous(r-continuous)³ if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y .
- 2) Completely-continuous³ if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y .
- 3) Strongly-continuous²⁶ if $f^{-1}(V)$ is clopen (both open and closed) in X for every subset V of Y .
- 4) α -continuous¹⁴ if $f^{-1}(V)$ is α -closed in X for every closed subset V of Y .
- 5) strongly α -continuous³² if $f^{-1}(V)$ is α -closed in X for every semi-closed subset V of Y .
- 6) α g-continuous¹⁹ if $f^{-1}(V)$ is α g-closed in X for every closed subset V of Y .
- 7) wg-continuous²³ if $f^{-1}(V)$ is wg-closed in X for every closed subset V of Y .
- 8) rwg-continuous²³ if $f^{-1}(V)$ is rwg-closed in X for every closed subset V of Y .
- 9) gs-continuous⁴ if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y .
- 10) gp-continuous²⁰ if $f^{-1}(V)$ is gp-closed in X for every closed subset V of Y .
- 11) gpr-continuous¹² if $f^{-1}(V)$ is gpr-closed in X for every closed subset V of Y .
- 12) α gr-continuous³⁰ if $f^{-1}(V)$ is α gr-closed in X for every closed subset V of Y .
- 13) $\omega\alpha$ -continuous⁷ if $f^{-1}(V)$ is $\omega\alpha$ -closed in X for every closed subset V of Y .
- 14) gspr-continuous²⁴ if $f^{-1}(V)$ is gspr-closed in X for every closed subset V of Y .
- 15) g-continuous⁷ if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y .
- 16) ω -continuous²⁷ if $f^{-1}(V)$ is ω -closed in X for every closed subset V of Y .
- 17) rga-continuous²⁸ if $f^{-1}(V)$ is rga-closed in X for every closed subset V of Y .
- 18) gr-continuous⁹ if $f^{-1}(V)$ is gr-closed in X for every closed subset V of Y .
- 19) g^*p -continuous²⁹ if $f^{-1}(V)$ is g^*p -closed in X for every closed subset V of Y .
- 20) rps-continuous²⁵ if $f^{-1}(V)$ is rps-closed in X for every closed subset V of Y .
- 21) R^* -continuous¹⁵ if $f^{-1}(V)$ is R^* -closed in X for every closed subset V of Y .
- 22) gprw-continuous¹⁶ if $f^{-1}(V)$ is gprw-closed in X for every closed subset V of Y .
- 23) wgra-continuous¹⁵ if $f^{-1}(V)$ is wgra-closed in X for every closed subset V of Y .
- 24) swg-continuous²³ if $f^{-1}(V)$ is swg-closed in X for every closed subset V of Y .
- 25) $r\omega$ -continuous⁸ if $f^{-1}(V)$ is $r\omega$ -closed in X for every closed subset V of Y .
- 26) rgw-continuous²² if $f^{-1}(V)$ is rgw-closed in X for every closed subset V of Y .

Definition 2.4: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- 1) α -irresolute¹⁴ if $f^{-1}(V)$ is α -closed in X for every α -closed subset V of Y .

- 2) irresolute⁷ if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y .
- 3) contra ω -irresolute²⁷ if $f^{-1}(V)$ is ω -open in X for every ω -closed subset V of Y .
- 4) contra irresolute¹⁴ if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y .
- 5) contra r -irresolute³ if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y
- 6) contra continuous¹¹ if $f^{-1}(V)$ is open in X for every closed subset V of Y .
- 7) $r\omega^*$ -open (resp $r\omega^*$ -closed)⁸ map if $f(U)$ is $r\omega$ -open (resp $r\omega$ -closed) in Y for every $r\omega$ -open (resp $r\omega$ -closed) subset U of X .

*Lemma 2.5:*⁵

- 1) Every closed (resp regular-closed, α -closed) set is $sarw$ -closed set in X .
- 2) Every $sarw$ -closed set is gs -closed set
- 3) Every $sarw$ -closed set is sg -closed (resp gsp -closed, rps -closed, $gspr$ -closed) set in X

*Lemma 2.6:*⁵

If a subset A of a topological space X and

- 1) If A is regular open and $sarw$ -closed then A is α -closed set in X
- 2) If A is open and αg -closed then A is $sarw$ -closed set in X
- 3) If A is open and gp -closed then A is $sarw$ -closed set in X
- 4) If A is regular open and gpr -closed then A is $sarw$ -closed set in X
- 5) If A is open and wg -closed then A is $sarw$ -closed set in X
- 6) If A is regular open and rwg -closed then A is $sarw$ -closed set in X
- 7) If A is regular open and αgr -closed then A is $sarw$ -closed set in X
- 8) If A is ω -open and $\omega\alpha$ -closed then A is $sarw$ -closed set in X

*Lemma 2.7:*⁵

If a subset A of a topological space X , and

- 1) If A is semi-open and sg -closed then it is $sarw$ -closed.
- 2) If A is semi-open and w -closed then it is $sarw$ -closed.
- 3) A is $sarw$ -open iff $U \subseteq aint(A)$, whenever U is $r\omega$ -closed and $U \subseteq A$.

*Definition 2.8:*⁹ A topological space (X, τ) is called α -space if every α -closed subset of X is closed in X .

3. *Sarw continuous Maps in Topological Spaces :*

Definition 3.1: A function $f: X \rightarrow Y$ is called semi α regular weakly continuous ($sarw$ -continuous) if $f^{-1}(V)$ is $sarw$ -closed set in X for every closed set V in Y .

Theorem 3.2: Every continuous function is $sarw$ -continuous but not conversely.

Proof: Let $f: X \rightarrow Y$ be continuous. Let F be any closed set in Y . Then the inverse image $f^{-1}(F)$ is closed set in X . Since every closed set is $sarw$ -closed by Lemma 2.5, $f^{-1}(F)$ is $sarw$ -closed in X . Therefore f is $sarw$ -continuous.

Example 3.3: Let $X=Y=\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=c$, then f is $sarw$ -continuous but not continuous, as closed set $F = \{b\}$ in Y , then, $f^{-1}(F) = \{a\}$ in X which is not closed set in X .

Theorem 3.4: Every α -continuous function is $sarw$ -continuous but not conversely.

Proof: Let $f: X \rightarrow Y$ be α -continuous. Let F be any closed set in Y . Then the inverse image $f^{-1}(F)$ is α -closed set in X . Since every α -closed set is sarw -closed by Lemma 2.5, $f^{-1}(F)$ is sarw -closed in X . Therefore f is sarw -continuous.

Example 3.5: Let $X=Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=c$, then f is sarw -continuous but not α -continuous, as closed set $F=\{b\}$ in Y , then, $f^{-1}(F)=\{a\}$ in X which is not α -closed set in X .

Theorem 3.6: Every sarw -continuous function is gs -continuous but not conversely.

Proof: Let f be sarw -continuous. Let F be any closed set in Y . Then the inverse image $f^{-1}(F)$ is sarw -closed set in X . Since every sarw -closed set is gs -closed by Lemma 2.5, $f^{-1}(F)$ is gs -closed in X . Therefore f is gs -continuous.

Example 3.7: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is gs -Continuous. but not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Theorem 3.8: Every sarw -continuous function is gsp -continuous but not conversely.

Proof: The proof follows from the fact that every sarw -closed set is gsp -closed set.

Example 3.9: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is gsp -continuous but not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Theorem 3.10: Every sarw -continuous function is gspr -continuous but not conversely.

Proof: The proof follows from the fact that every sarw -closed set is gspr -closed set.

Example 3.11: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is gspr -continuous but not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Theorem 3.12: Every sarw -continuous function is sg -continuous but not conversely.

Proof: The proof follows from the fact that every sarw -closed set is sg -closed set.

Example 3.13: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is sg -continuous, but not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Theorem 3.14: Every sarw -continuous function is rps -continuous but not conversely.

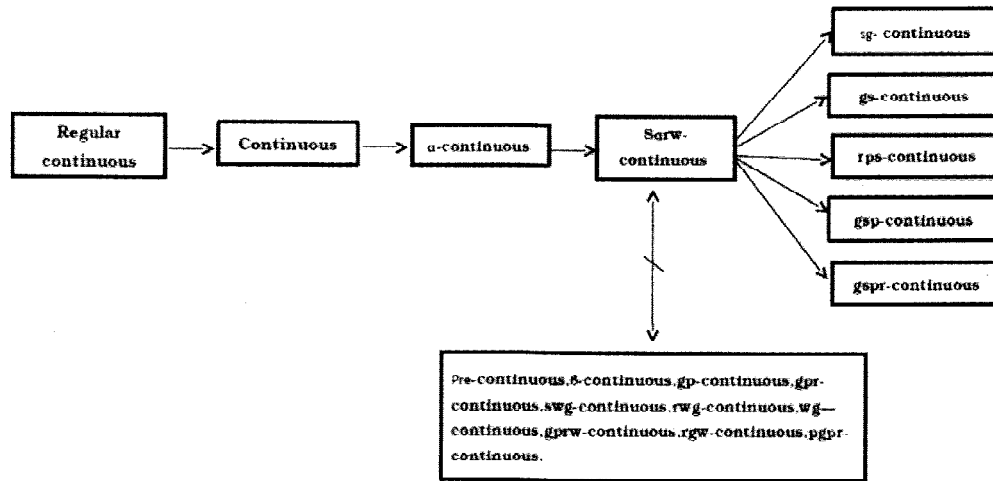
Proof: The proof follows from the fact that every sarw -closed set is rps -closed set.

Example 3.15: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is rps -continuous. but not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Remark 3.16: The following examples shows that sarw -continuous maps are independent of pre -continuous, β -continuous, gp -continuous, gpr -continuous, swg -continuous, rwg -continuous, wg -continuous, gprw -continuous, rgw -continuous, pgpr -continuous.

Example 3.17: Let $X=\{a,b,c,d\}$, $Y=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma=\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$, then f is pre -continuous, β -continuous, gp -continuous, gpr -continuous, swg -continuous, rwg -continuous, wg -continuous, gprw -continuous, rgw -continuous, pgpr -continuous. But not sarw -continuous, as closed set $F=\{c\}$ in Y , then, $f^{-1}(F)=\{b\}$ in X which is not sarw -closed set in X .

Remark 3.18: From the above discussion and know results we have the following implications.



$A \longrightarrow B$ means the continuous function A implies the continuous function B.

$A \longleftrightarrow B$ means the continuous function A and B are independent of each other

Figure 1

Theorem 3.19: Let $f: X \rightarrow Y$ be a map. Then the following statements are equivalent:

- f is sarw-continuous.
- the inverse image of each open set in Y is sarw-open in X

Proof:

- Assume that $f: X \rightarrow Y$ is sarw-continuous. Let U be open in Y . The U^c is closed in Y . Since f is sarw-continuous, $f^{-1}(U^c)$ is sarw-closed in X . But $f^{-1}(U^c) = X - f^{-1}(U)$. Thus $f^{-1}(U)$ is sarw-open in X .
- Assume that the inverse image of each open set in Y is sarw-open in X . Let F be any closed set in Y . By assumption $f^{-1}(F^c)$ is sarw-open in X . But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is sarw-open in X and so $f^{-1}(F)$ is sarw-closed in X . Therefore f is sarw-continuous.

Hence (i) and (ii) are equivalent.

Theorem 3.20: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is map. Then the following holds.

- f is sarw-continuous and contra r -irresolute map then f is α -continuous
- f is ag -continuous and contra continuous map then f is sarw-continuous.
- f is gp -continuous and contra continuous map then f is sarw-continuous
- f is gpr -continuous and contra r -irresolute map then f is sarw-continuous.
- f is wg -continuous and contra continuous map then f is sarw-continuous
- f is rwg -continuous and contra r -irresolute map then f is sarw-continuous
- f is agr -continuous and contra r -irresolute map then f is sarw-continuous
- f is $w\alpha$ -continuous and contra w -irresolute map then f is sarw-continuous

Proof:

- Let V be regular closed set of Y as every regular closed set is closed, V is closed set in Y . Since f is sarw-continuous and contra r -irresolute map, $f^{-1}(V)$ is sarw-closed and regular open in X . Now by

Lemma 2.6, $f^{-1}(V)$ is α -closed in X . Thus f is α -continuous.

- ii) Let V be closed set of Y . Since f is αg -continuous and contra continuous map, $f^{-1}(V)$ is αg -closed and open in X . Now by Lemma 2.6, $f^{-1}(V)$ is sarw -closed in X . Thus f is sarw -continuous. Similarly, we can prove (iii),(iv),(v),(vi),(vii).

Theorem 3.21: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is map. Then the following holds.

- i) f is sg -continuous and contra irresolute map then f is sarw -continuous.
 ii) f is w -continuous and contra irresolute map then f is sarw -continuous

Proof:

- i) Let V be closed set of Y . As every closed set is semi-closed, V is semi-closed set in Y . Since f is sg -continuous and contra irresolute map, $f^{-1}(V)$ is sg -closed and semi-open in X . Now by Lemma 2.7, $f^{-1}(V)$ is sarw -closed in X . Thus f is sarw -continuous.
 ii) The proof is in the similar manner.

Theorem 3.22: Let A be a subset of a topological space X . Then $x \in \text{sarwcl}(A)$ if and only if for any sarw -open set U containing x , $A \cap U \neq \emptyset$.

Proof: Let $x \in \text{sarwcl}(A)$ and suppose that, there is a sarw -open set U in X such that $x \in U$ and $A \cap U = \emptyset$ implies that $A \subseteq U^c$ which is sarw -closed in X implies $\text{sarwcl}(A) \subseteq \text{sarwcl}(U^c) = U^c$. Since $x \in U$ implies that $x \notin U^c$ implies that $x \notin \text{sarwcl}(A)$, this is a contradiction. Conversely,

For any sarw -open set U containing x , $A \cap U \neq \emptyset$. To prove that $x \in \text{sarwcl}(A)$.

Suppose that $x \notin \text{sarwcl}(A)$, then there is sarw -closed set F in X such that $x \notin F$ and $A \subseteq F$. Since $x \notin F$ implies that $x \in F^c$ which is sarw -open in X . Since $A \subseteq F$ implies that $A \cap F^c = \emptyset$, This is a contradiction. Thus $x \in \text{sarwcl}(A)$.

Theorem 3.23: Let $f: X \rightarrow Y$ be a function from a topological space X into a topological space Y . If $f: X \rightarrow Y$ is sarw -continuous, then $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A))$ for every subset A of X .

Proof: Since $f(A) \subseteq \text{cl}(f(A))$ implies that $A \subseteq f^{-1}(\text{cl}(f(A)))$. Since $\text{cl}(f(A))$ is a closed set in Y and f is sarw -continuous, then by definition $f^{-1}(\text{cl}(f(A)))$ is a sarw -closed set in X containing A . Hence $\text{sarwcl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A))$.

Theorem 3.24: Let $f: X \rightarrow Y$ be a function from a topological space X into a topological space Y . Then the following statements are equivalent:

- i) For each point x in X and each open set V in Y with $f(x) \in V$, there is a sarw -open set U in X such that $x \in U$ and $f(U) \subseteq V$.
 ii) For each subset A of X , $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A))$.
 iii) For each subset B of Y , $\text{sarwcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.
 iv) For each subset B of Y , $f^{-1}(\text{int}(B)) \subseteq \text{sarwint}(f^{-1}(B))$.

Proof:

(i) \Rightarrow (ii) Suppose that (i) hold and let $y \in f(\text{sarwcl}(A))$ and let V be any open set of Y .

Since $y \in f(\text{sarwcl}(A))$ implies that there exists $x \in \text{sarwcl}(A)$ such that $f(x) = y$. Since $f(x) \in V$, then by (i) there exists a sarw -open set U in X such that $x \in U$ and $f(U) \subseteq V$.

Since $x \in \text{sarwcl}(A)$, then by theorem 3.22 $U \cap A \neq \emptyset$. $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$, then $V \cap f(A) \neq \emptyset$. Therefore we have $y = f(x) \in \text{cl}(f(A))$. Hence $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A))$.

(ii) \Rightarrow (i) Let if (ii) holds and let $x \in X$ and V be any open set in Y containing $f(x)$. Let $A = f^{-1}(V^c)$ this implies that $x \notin A$. Since $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A)) \subseteq V^c$ this implies that

$\text{sarwcl}(A) \subseteq f^{-1}(V^c) = A$. Since $x \notin A$ implies that $x \notin \text{sarwcl}(A)$ and by theorem 3.22 there exists a sarw -open

set U containing x such that $U \cap A = \emptyset$, then $U \subseteq A^c$ and hence $f(U) \subseteq f(A^c) \subseteq V$.

(ii) \Rightarrow (iii) Suppose that (ii) holds and Let B be any subset of Y . Replacing A by $f^{-1}(B)$ we get from (ii) $f(\text{sarwcl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\text{sarwcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iii) \Rightarrow (ii) Suppose that (iii) holds, let $B = f(A)$ where A is a subset of X .

Then we get from (iii), $\text{sarwcl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{cl}(f(A)))$ implies $\text{sarwcl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$.

Therefore $f(\text{sarwcl}(A)) \subseteq \text{cl}(f(A))$.

(iii) \Rightarrow (iv) Suppose that (iii) holds. Let $B \subseteq Y$, then $Y - B \subseteq Y$. By (iii) $\text{sarwcl}(f^{-1}(Y - B)) \subseteq f^{-1}(\text{cl}(Y - B))$ this implies $X - \text{sarwint}(f^{-1}(B)) \subseteq X - f^{-1}(\text{int}(B))$. Therefore $f^{-1}(\text{int}(B)) \subseteq \text{sarwint}(f^{-1}(B))$.

(iv) \Rightarrow (iii) Suppose that (iv) holds Let $B \subseteq Y$, then $Y - B \subseteq Y$.

By (iv), $f^{-1}(\text{int}(Y - B)) \subseteq \text{sarwint}(f^{-1}(Y - B))$ this implies that $X - f^{-1}(\text{cl}(B)) \subseteq X - \text{sarwcl}(f^{-1}(B))$.

Therefore $\text{sarwcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

Definition 3.25: Let (X, τ) be topological space and $\tau_{\text{sarw}} = \{V \subseteq X : \text{sarw-cl}(V^c) = V^c\}$, τ_{sarw} is topology on X .

Definition 3.26:

1) A space (X, τ) is called Tsarw-space if every sarw-closed is closed.

2) A space (X, τ) is called sarwTsc - space if every sarw-closed set is semi-closed set.

Remark 3.27: The Composition of two sarw-continuous maps need not be sarw-continuous map

Example 3.28: Let $X=Y=Z = \{a, b, c\}$. Let $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ be a topology on X , $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ be a topology on Y and $\eta = \{Z, \emptyset, \{a\}, \{b, c\}\}$ be a topology on Z . Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be the identity map. Both f and g are sarw-continuous but their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not a sarw-continuous map as the closed set $F = \{a\}$ in (Z, η) , but $(g \circ f)^{-1}(F) = \{a\}$ is not sarw-closed set in X .

Theorem 3.29: Let $f: X \rightarrow Y$ is sarw-continuous function and $g: Y \rightarrow Z$ is continuous function then $g \circ f: X \rightarrow Z$ is sarw-continuous.

Proof: Let g be continuous function and V be any open set in Z then $g^{-1}(V)$ is open in Y . Since f is sarw-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sarw-open in X . Hence $g \circ f$ is sarw-continuous.

Theorem 3.30: Let $f: X \rightarrow Y$ is sarw-continuous function and $g: Y \rightarrow Z$ is sarw-continuous function and Y is Tsarw-space, then $g \circ f: X \rightarrow Z$ is sarw-continuous.

Proof: Let g be sarw-continuous function and V is any open set in Z then $g^{-1}(V)$ is sarw-open in Y and Y is Tsarw-space, then $g^{-1}(V)$ is open in Y . Since f is sarw-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is sarw-open in X . Hence $g \circ f$ is sarw-continuous.

Theorem 3.31: If a map $f: X \rightarrow Y$ is completely-continuous, then it is sarw-continuous.

Proof: Suppose that a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is completely-continuous. Let F closed set in Y . Then $f^{-1}(F)$ is regular closed in X and hence $f^{-1}(F)$ is sarw-closed in X . Thus f is sarw-continuous.

Definition 3.32: A function f from a topological space X into a topological space Y is called perfectly semi α regular weakly continuous (briefly perfectly sarw-Continuous) if $f^{-1}(V)$ is clopen (closed and open) set in X for every sarw-open set V in Y .

Theorem 3.33: If a map $f: X \rightarrow Y$ is continuous, then the following holds.

- i) If f is perfectly sarw-continuous, then f is sarw-continuous.
- ii) If f is perfectly sarw-continuous, then f is gs-continuous.
- iii) If f is perfectly sarw-continuous, then f is gsp-continuous (resp sg-continuous, gspr-continuous, rps-continuous).

Proof:

- i) Let F be open set in Y , as every open is sarw -open in Y since f is perfectly sarw -continuous then $f^{-1}(F)$ is both closed and open in X , as every open is sarw -open, $f^{-1}(F)$ is sarw -open in X . Hence f is sarw -continuous.
- ii) Let F be open set in Y , as every open is sarw -open in Y , since f is perfectly sarw -continuous, then $f^{-1}(F)$ is both closed and open in X , as every open is sarw -open that implies is gs -open, then $f^{-1}(F)$ is gs -open in X . Hence f is gs -continuous.

Similarly we can prove (iii).

Definition 3.34: A function f from a topological space X into a topological space Y is called semi α regular weakly*- continuous (briefly sarw^* -continuous) if $f^{-1}(V)$ is sarw -closed set in X for every semi-closed set V in Y .

Theorem 3.35: If A map $f: (X, \tau) \rightarrow (Y, \sigma)$ be function,

- i) f is sarw -irresolute then it is sarw^* -continuous.
- ii) f is sarw^* -continuous then it is sarw -continuous.

Proof:

- i) Let $f: X \rightarrow Y$ be sarw -irresolute. Let F be any semi-closed set in Y . Then F is sarw -closed in Y . Since f is sarw -irresolute, the inverse image $f^{-1}(F)$ is sarw -closed set in X . Therefore f is sarw^* -continuous.
- ii) Let $f: X \rightarrow Y$ be sarw^* -continuous. Let F be any closed set in Y . Then F is semi-closed in Y . Since f is sarw^* -continuous. The inverse image $f^{-1}(F)$ is sarw -closed set in X . Therefore f is sarw -continuous.

Theorem 3.36: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following statements are equivalent:

- i) f is strongly sarw -continuous.
- ii) f is perfectly sarw -continuous.

Proof:

(i) \Rightarrow (ii) Let U be any sarw -open set in (Y, σ) . By hypothesis $f^{-1}(U)$ is open in (X, τ) . Since (X, τ) is a discrete space, $f^{-1}(U)$ is also closed in (X, τ) . $f^{-1}(U)$ is both open and closed in (X, τ) . Hence f is perfectly sarw -continuous.

(ii) \Rightarrow (i) Let U be any sarw -open set in (Y, σ) . Then $f^{-1}(U)$ is both open and closed in (X, τ) . Hence f is strongly sarw -continuous.

4. Sarw -Irresolute and Strongly Sarw -Continuous Maps in Topological Spaces :

Definition 4.1: A function f from a topological space X into a topological space Y is called semi α regular weakly irresolute (sarw -irresolute) map if $f^{-1}(V)$ is sarw -closed set in X for every sarw -closed set V in Y .

Definition 4.2: A function f from a topological space X into a topological space Y is called strongly semi α regular weakly continuous (strongly sarw -continuous) map if $f^{-1}(V)$ is closed set in X for every sarw -closed set V in Y .

Theorem 4.3: If A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is sarw -irresolute, then it is sarw -continuous but not conversely.

Proof: Let $f: X \rightarrow Y$ be sarw -irresolute. Let F be any closed set in Y . Then F is sarw -closed in Y . Since f is sarw -irresolute, the inverse image $f^{-1}(F)$ is sarw -closed set in X . Therefore f is sarw -continuous. The converse of the above theorem need not be true as seen from the following example.

Example 4.4: Let $X=Y=\{a, b, c\}$. Let $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ be a topology on X , $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}\}$ be a topology on Y . Then the identity map $f: (X, \tau) \rightarrow (Y, \sigma)$ is sarw -continuous but not sarw -irresolute, as the inverse image of sarw -closed set $\{a, b\}$ in Y is $\{a, b\}$ which is not sarw -closed set in X .

Theorem 4.5: If A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is sarw -irresolute, if and only if the inverse image $f^{-1}(V)$ is sarw -

open set in X for every sarw -open set V in Y .

Proof: Assume that $f: X \rightarrow Y$ is sarw -irresolute. Let G be sarw -open in Y . The G^c is sarw -closed in Y . Since f is sarw -irresolute, $f^{-1}(G^c)$ is sarw -closed in X . But $f^{-1}(G^c) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is sarw -open in X . Conversely, Assume that the inverse image of each open set in Y is sarw -open in X . Let F be any sarw -closed set in Y . By assumption $f^{-1}(F^c)$ is sarw -open in X . But $f^{-1}(F^c) = X - f^{-1}(F)$.

Thus $X - f^{-1}(F)$ is sarw -open in X and so $f^{-1}(F)$ is sarw -closed in X . Therefore f is sarw -irresolute.

Theorem 4.6: If A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is sarw -irresolute, then for every subset A of X , $f(\text{sarwcl}(A)) \subseteq \text{acl}(f(A))$.

Proof: If $A \subseteq X$ then consider $\text{acl}(f(A))$ which is sarw -closed in Y . since f is sarw -irresolute, $f^{-1}(\text{acl}(f(A)))$ is sarw -closed in X . Furthermore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{acl}(f(A)))$. Therefore by sarw -closure, $\text{sarwcl}(A) \subseteq f^{-1}(\text{acl}(f(A)))$, consequently, $f(\text{sarwcl}(A)) \subseteq f(f^{-1}(\text{acl}(f(A)))) \subseteq \text{acl}(f(A))$.

Theorem 4.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

- i) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is sarw -continuous if g is r -continuous and f is sarw -irresolute.
- ii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is sarw -irresolute if g is sarw -irresolute and f is sarw -irresolute.
- iii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is sarw -continuous if g is sarw -continuous and f is sarw -irresolute.

Proof:

- i) Let U be open set in (Z, η) . Since g is r -continuous, $g^{-1}(U)$ is r -open set in (Y, σ) . Since every r -open is sarw -open then $g^{-1}(U)$ is sarw -open in Y , since f is sarw -irresolute $f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) . Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) and hence $g \circ f$ is sarw -continuous.
- ii) Let U be a sarw -open set in (Z, η) . Since g is sarw -irresolute, $g^{-1}(U)$ is sarw -open set in (Y, σ) . Since f is sarw -irresolute, $f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) . Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) and hence $g \circ f$ is sarw -irresolute.
- iii) Let U be open set in (Z, η) . Since g is continuous, $g^{-1}(U)$ is open set in (Y, σ) . As every open set is sarw -open, $g^{-1}(U)$ is sarw -open set in (Y, σ) . Since f is sarw -irresolute $f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) . Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) and hence $g \circ f$ is sarw -continuous.

Theorem 4.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous then it is continuous, but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous. Let F be closed set in Y . As every closed is sarw -closed, F is sarw -closed in Y . since f is strongly sarw -continuous then $f^{-1}(F)$ is closed set in X . Therefore f is continuous.

Example 4.9: Let $X=Y = \{a, b, c\}$. Let $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ be a topology on X , $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$ be a topology on Y . Then the map $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b$, $f(b)=a$, $f(c)=c$, is continuous but not strongly sarw -continuous, as the inverse image of sarw -closed set $\{b\}$ in Y is $\{a\}$ which is not closed set in X .

Theorem 4.10: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous then it is strongly α -continuous but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous. Let F be α -closed set in Y . As every α -closed is sarw -closed, F is sarw -closed in Y , since f is strongly sarw -continuous. Then $f^{-1}(F)$ is closed set in X . Therefore f is strongly α -continuous.

Example 4.11: Let $X=Y = \{a, b, c\}$. Let $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ be a topology on X , $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$ be a topology on Y . Then the map $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b$, $f(b)=a$, $f(c)=c$, is strongly α -continuous but not strongly sarw -continuous, as the inverse image of sarw -closed set $\{b\}$ in Y is $\{a\}$ which is not α -closed set in X .

Theorem 4.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous if and only if $f^{-1}(G)$ is open set in X for every sarw -open set G in Y .

Proof: Assume that $f: X \rightarrow Y$ is strongly sarw -continuous. Let G be sarw -open in Y . The G^c is sarw -closed in Y . Since f is strongly sarw -continuous, $f^{-1}(G^c)$ is closed in X .

But $f^{-1}(G^c) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is open in X . Conversely, Assume that the inverse image of each open set in Y is sarw -open in X . Let F be any sarw -closed set in Y . By assumption F^c is sarw -open in Y . But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X . Therefore f is strongly sarw -continuous.

Theorem 4.13: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly continuous then it is strongly sarw -continuous

Proof: Assume that $f: X \rightarrow Y$ is strongly continuous. Let G be sarw -open in Y and also it is any subset of Y since f is strongly continuous, $f^{-1}(G)$ is open (and also closed) in X . $f^{-1}(G)$ is open in X Therefore f is strongly sarw -continuous.

Theorem 4.14: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous then it is sarw -continuous.

Proof: Let G be open in Y , every open is sarw -open, and G is sarw -open in Y , since f is strongly sarw -continuous, $f^{-1}(G)$ is open in X . and therefore $f^{-1}(G)$ is sarw -open in X . Hence f is sarw -continuous.

Example 4.15: Let $X=Y = \{a, b, c\}$. Let $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ be a topology on X , $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$ be a topology on Y . Then the map $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b, f(b)=a, f(c)=c$, is sarw -continuous but not strongly sarw -continuous, as the inverse image of sarw -closed set $\{b\}$ in Y is $\{a\}$ which is not closed set in X .

Theorem 4.16: In discrete space, a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly sarw -continuous then it is strongly continuous.

Proof: F any subset of Y , in discrete space, Every subset F in Y is both open and closed, then subset F is both sarw -open or sarw -closed, i) let F is sarw -closed in Y , since f is strongly sarw -continuous, then $f^{-1}(F)$ is closed in X . ii) let F is sarw -open in Y , since f is strongly sarw -continuous, then $f^{-1}(F)$ is open in X . Therefore $f^{-1}(F)$ is closed and open in X . Hence f is strongly continuous.

Theorem 4.17: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

- i) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is strongly sarw -continuous if g is strongly sarw -continuous and f is strongly sarw -continuous.
- ii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is strongly sarw -continuous if g is strongly sarw -continuous and f is continuous.
- iii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is sarw -irresolute if g is strongly sarw -continuous and f is sarw -continuous.
- iv) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is continuous if g is sarw -continuous and f is strongly sarw -continuous

Proof:

- i) Let U be a sarw -open set in (Z, η) . Since g is strongly sarw -continuous, $g^{-1}(U)$ is open set in (Y, σ) . As every open set is sarw -open, $g^{-1}(U)$ is sarw -open set in (Y, σ) . Since f is strongly sarw -continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) .

Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence $g \circ f$ is strongly sarw -continuous.

- ii) Let U be a sarw -open set in (Z, η) . Since g is strongly sarw -continuous, $g^{-1}(U)$ is open set in (Y, σ) . Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) .

Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence $g \circ f$ is strongly sarw -continuous.

- iii) Let U be a sarw -open set in (Z, η) . Since g is strongly sarw -continuous, $g^{-1}(U)$ is open set in (Y, σ) . Since f is sarw -continuous $f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) .

Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an sarw -open set in (X, τ) and hence $g \circ f$ is sarw -irresolute

- iv) Let U be open set in (Z, η) . Since g is sarw -continuous, $g^{-1}(U)$ is sarw -open set in (Y, σ) . Since f is strongly sarw -continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) .

Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence $g \circ f$ is continuous.

Theorem 4.18: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

- i) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is strongly sarw -continuous if g is perfectly sarw -continuous and f is continuous.
- ii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is perfectly sarw -continuous if g is strongly sarw -continuous and f is perfectly sarw -continuous.

Proof:

- i) Let U be a sarw -open set in (Z, η) . Since g is perfectly sarw -continuous, $g^{-1}(U)$ is clopen set in (Y, σ) . $g^{-1}(U)$ is open set in (Y, σ) . Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) . Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence $g \circ f$ is strongly sarw -continuous.
- ii) Let U be a sarw -open set in (Z, η) . Since g is strongly sarw -continuous, $g^{-1}(U)$ is open set in (Y, σ) . $g^{-1}(U)$ is open set in (Y, σ) . Since f is perfectly sarw -continuous, $f^{-1}(g^{-1}(U))$ is a clopen set in (X, τ) . Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a clopen set in (X, τ) and hence $g \circ f$ is perfectly sarw -continuous.

Theorem: 4.19: Let (X, τ) be any topological space and (Y, σ) be a Tsarw -space and

$f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following are equivalent:

- i) f is strongly sarw -continuous.
- ii) f is continuous.

Proof: (i) \Rightarrow (ii) Let U be any open set in (Y, σ) . Since every open set is sarw -open, U is sarw -open in (Y, σ) . Then $f^{-1}(U)$ is open in (X, τ) . Hence f is continuous.

(ii) \Rightarrow (i) Let U be any sarw -open set in (Y, σ) . Since (Y, σ) is a Tsarw -space, U is open in (Y, σ) . Since f is continuous. Then $f^{-1}(U)$ is open in (X, τ) . Hence f is strongly sarw -continuous.

Theorem 4.20: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following statements are equivalent:

- i) f is strongly sarw -continuous.
- ii) f is perfectly sarw -continuous.

Proof: (i) \Rightarrow (ii) Let U be any sarw -open set in (Y, σ) . By hypothesis $f^{-1}(U)$ is open in (X, τ) . Since (X, τ) is a discrete space, $f^{-1}(U)$ is also closed in (X, τ) . $f^{-1}(U)$ is both open and closed in (X, τ) . Hence f is perfectly sarw -continuous.

(ii) \Rightarrow (i) Let U be any sarw -open set in (Y, σ) . Then $f^{-1}(U)$ is both open and closed in (X, τ) . Hence f is strongly sarw -continuous.

Theorem 4.21: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map. Both (X, τ) and (Y, σ) are Tsarw -space. Then the following are equivalent:

- i) f is sarw -irresolute.
- ii) f is strongly sarw -continuous
- iii) f is continuous.
- iv) f is sarw -continuous.

Proof: Straight forward.

Theorem 4.22: Let X and Y be sarwTsc -spaces, then for a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

- i) f is sc -irresolute.
- ii) f is sarw -irresolute.

Proof:

(i) \Rightarrow (ii): Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a sc -irresolute. Let V be a sarw -closed set in Y . As Y sarwTsc -space, then V is a semi-closed set in Y . Since f is sc -irresolute, $f^{-1}(V)$ is semi-closed in X . But every semi-closed set is sarw -closed in X and hence $f^{-1}(V)$ is a sarw -closed in X . Therefore, f is sarw -irresolute.

(ii) \Rightarrow (i): Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a sarw -irresolute. Let V be a semi-closed set in Y . But every semi-closed set is sarw -closed set and hence V is sarw -closed set in Y and f is sarw -irresolute

implies $f^{-1}(V)$ is $sarw$ -closed in X . But X is $sarwTsc$ -space and hence $f^{-1}(V)$ is semi-closed set in X . Thus, f is sc -irresolute.

Future work:

The extension of the paper will be carried as the future work with $sarw$ -closed and open maps, homeomorphism in topological spaces.

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